Work xx (20xx) x–xx
DOI:10.3233/WOR-210452
IOS Press

RETRACTED: Providing a system of continuous recovery during athletes' rehabilitation

Zhao Hua Tang[a], Tang Xun Yang[b,*] and Valentin Oleshko[b]
[a]*School of Sports, University of Electronic Science and Technology of China, Chengdu, Sichuan, People's Republic of China*
[b]*Department of Combat Sports and Power Sports, National University of Physical Education and Sport of Ukraine, Kyiv, Ukraine*

Received 16 March 2021
Accepted 28 May 2021

1. Introduction

As noted by Llorens et al., the early studies conducted in the field of system identification related to time series were based on the scientific papers of K.F. Gauss "Theoria motus corporum coelestium in sectionibus conicis solem ambientium" (1809) and G.E. Fisher's "On an Absolute Criterion for Fitting Frequency Curves" (1912), and were called statistical estimations [1]. Most of the identification procedures were based on observing the reactions of controlled objects with the presence of some control actions and, depending on what type of information about the object was used, identification methods were divided into frequency and time [2]. They represent a description of a controlled system in the form of a state space, which made it possible to work with multidimensional systems [3].

System identification methods for control problems have been developed and described as a subspace method based on the use of projections in Euclidean space, as well as a prediction error method based on minimising a criterion that depends on model parameters [4]. Many papers are devoted

*Address for correspondence: Tang Xun Yang, Department of Combat Sports and Power Sports, National University of Physical Education and Sport of Ukraine, 1 Fizkultury Str., Kyiv 03150, Ukraine. E-mail: xun_yang5769@unesp.co.uk.

to the search for a model of the object under study in the state space, which has the smallest order of the state vector, based on information about the impulse transient response [5]. This problem, in the presence of random process realisation, where the Markov model is formed, became the basis of the subspace method [6]. Various works present a maximum likelihood method developed by time series specialists to estimate model parameters in the form of difference equations [7]. These models, known as autoregressive moving average and autoregressive moving average with input, formed the basis for the prediction error method [8]. The use of identification methods was described from the moment the collection of information about an object began to the receipt and verification of the model [9].

When solving the identification problem, the structure of the system and the class of models to which it belongs are considered known [10]. A priori information about the system is quite wide [11]. A wider class of different identification methods was considered and material on the sensitivity of system characteristics to identification errors was presented [12]. All papers considered are related to three well-known problems of signal theory [13]: identification problem, when, based on known signals at the input and output of the system, a conclusion is made about the composition of the system and its characteristics; a control task when the characteristics of the system and the input signal are known and the law of change of the signal at the output of the system or such an input signal that at the output brings the system to a given state is determined; measurement task when the original signal and system characteristics are known and the characteristics of the input signal are determined. The methods are applicable to sequential parameter identification [14].

The relationship between the forms of representation of random processes in state variables and using the covariance function is often recorded [15]. Also, the foundation for the study of dynamical systems has been laid [16–18]. The "input-state-output" approach differs from the conventional "input-output" structure in that definitions are made without distinguishing a priori between causes (inputs) and effects (results) [19]. The "input-state-output" structure acts as a special type of system representation [20]. It has been argued that this framework is much better suited to provide a satisfactory conceptual framework for modelling physical systems as the language of mathematical systems theory [21]. From this perspective, various qualitative systemic properties are introduced (such as linearity, time invariance, and the like) [22]. The results of the spatial representation of these systems are presented. These ideas became the basis for the development of information technology for use in continuous production.

2. Materials and methods

Management is understood as an organisational activity that carries out management functions for any work aimed at achieving certain goals. The management process consists of analysing the decision-making process on the most appropriate actions in appropriate situations. The person in charge makes decision by assessing the environment with the help of information received from his sensory organs, measuring instruments, other persons and technical devices. In many cases, this information is insufficient for an unambiguous assessment of the situation. Then experience, knowledge, memory, intuition are used. A remarkable property of a person is the ability to make decisions in conditions of significant uncertainty about the environment with obtaining appropriate information. To carry out the management functions, mathematical methods have been developed that allow analysing existing types of information, filtering out unnecessary information and highlighting its most significant part, we should use the necessary information to assess the situation and develop recommendations that ensure the most effective implementation of control objectives. The choice of a model when solving control problems and studying continuous processes is due to both the conditions of implementation and requirements of adequacy. In conditions of uncertainty, algorithms and methods of information analysis of data from various structures come to the fore, and information analysis is an integral part of the identification system.

To solve the problem of control in conditions of uncertainty, the methods of the adaptive systems theory are used, which allow: to ensure high accuracy of control with a significant change in the dynamic properties of the object; optimise the operating modes of the object in conditions of changes in its characteristics; improve the reliability of the system, unify individual control subsystems and their blocks; reduce the development and debugging time of the system. Adaptive methods are used to solve problems in which there is no information about the nature and conditions of the object operation, as well as in the case of impossibility or insufficiently complete

formalisation of a priori data. In general, the process of synthesising an adaptive system can be divided into several stages. First, the goal of management is formed and requirements are set for the structure of the mathematical model of the object. For this, a priori and/or experimental information is used. In a more general case, the synthesis of a system is associated with solving the problem of structural identification. At the second stage, the structure of the control device is determined. Next, the algorithm for adapting the controller parameters is selected. At the final stage, the adopted algorithm is justified. Object identification is reduced to determining the structure and parameters of the model based on the observed data (input and output of the object) and the available a priori information. All existing approaches to identification are divided into two groups – statistical and multiple-functional (deterministic). The indicated classes differ by taking account of the nature of disturbances (noises) acting on the system and by the estimates obtained. Despite the wide variety of algorithms and identification methods, there are no procedures for regular synthesis of the model structure, which is explained by the complexity and variety of control objects, poor knowledge of the processes occurring in the object.

3. Results and discussion

The set of dynamic processes in control objects can be described using differential Equation (1) with one input u and output y:

$$a_0 y^{(m)} + a_1 y^{(m-1)} + \ldots + a_m y = b_0 u^{(k)} + b_1 u^{(k-1)} + \ldots + b_{ku} + \xi \quad (1)$$

where ξ – random disturbance; a, b – weight coefficients of the differential equation.

From equation (1), we can go to the finite-difference representation. Assuming $t = n\Delta t$, where $n = 0, 1, ..., \Delta t$ – data collection interval, and introducing backward shift operator $z - zy(n) = y(n-1)$, obtain (2):

$$D_y(z)\, y(n) = D_u(z)\, u(n) + \xi(n) \quad (2)$$

where $D_y(z) = a_0 z^m + a_1 z^{m-1} + \ldots + a_m$, $D_u(z) = b_0 z^k + b_1 z^{k-1} + \ldots + b_k$.

If $\xi(t)$ is a random sequence, then (2) is an autoregression equation – moving average, and at $D_u(z) = 1$ – moving average model. In general, the autoregression equation is a moving average with dynamic specification for $\xi(t)$ in space $\{U, Y\}$ has the form (3):

$$Y(t) = F(A, Y(\tau_1), U(\tau_2), \xi(\tau_3)), \quad \tau_i \in \left[t_{\tau_i}, t\right], i = \overline{1,3} \quad (3)$$

where $t_{\tau_i} \geq t_0$.

From (3) it can be seen that the dynamic properties of an object can be determined both by its internal structure and by the dynamic properties of the input $U(t)$ and noise $\xi(t)$. Equations (1, 2) can be written in matrix form (in the form of state space). For linear stationary plant, the equation in the state space has the form (4):

$$\begin{aligned} \dot{X} &= AX + BU + \xi \\ Y &= CX + DU + \zeta \end{aligned} \quad (4)$$

where $X \in R^m$ – state vector; $A \in R^{m \times m}$ – state matrix; $U \in R^{n \times k}$ – input vector; $Y \in R^n$ – output vector; $B \in R^{m \times k}$, $C \in R^{n \times m}$, $D \in R^{n \times k}$; $\zeta \in R^n$ – non-retentive vector of measurement errors; $\xi \in R^m$ – noise vector.

The first Equation in (4) is called the equation of state, and the second is called the measurement (observation) equation. In identification problems, matrix D is usually equal to zero. Any object can be characterised by a set of variables $\vec{u}(t) \in U$, arriving at its input, and a set of variables $\vec{x}(t) \in X$, that are the object's response to the impact and reflect its state. The object is immersed in an external environment, the influence of which manifests itself in the form of controlled $\zeta(t) \in \Xi$ and uncontrolled $\xi(t) \in \Sigma$ disturbances and also affects its state. In identification problems, disturbances that are controlled are included in the input vector $\vec{u}(t)$. Uncontrolled disturbances $\xi(t)$ are manifested through the object output $\vec{y}(t) \in Y$, that belongs to X and is only part of the state vector, that is $Y \subseteq X$. The information space of the object is represented in the form (5):

$$I = U \times X \times \Psi \times J \times S \quad (5)$$

where $\Psi = \Sigma \times \Xi$ – perturbation space; J – object observation interval; S – space of system parameters (6):

$$S \subseteq R^m \times R^n \quad (6)$$

$K \subseteq R^n$ – space of structural features of the system "object + environment", $K \subseteq S$. The informative space I in real conditions is incompletely observable, therefore, it is covered by some information set

and containing the sets U and X available for measurement. In this case, the set I is a set of vectors $\vec{u}(t) \in U \subset U, \vec{y}(t) \in Y \subseteq X$, observed on J (7):

$$I_e = \left\{U \in R^m, Y \in R^n | \vec{u}(t), \vec{y}(t), t \in J\right\} \quad (7)$$

Since the elements of I are obtained in the process of measuring the observed state variables, the set I contains only numerous images of the elements of the spaces U and X. For a complete characteristic of the object in the set I, a priori information is included, therefore I is represented in the form (8):

$$I = \{I_a, I_e\} \quad (8)$$

where I_a, I_e – a priori and experimental information, respectively.

Information set I (8) is used to identify control objects. The structure and properties of the set I_e greatly influence the choice of the method for synthesising the mathematical model and the type of the parameter estimation algorithm used. The information I_e has the form (7) and includes the results of measurements of input and observed (initial) state variables, as well as indirect parameters depending on the output variables. Depending on the identification problem being solved, the set I_e can have a different structure. If the problem of one-time identification of the control object is solved, then the measurement process has a fixed end t_e, moreover $t_e > t_0$, where t_0 – the start time of observation. In this case, the information I_e has the form (7) and is represented in the form (9):

$$I_e(J) = \{H_U(J), H_Y(J)\} \quad (9)$$

where $H_U(J) = [u(t_0), u(t_1), ..., u(t_e)]$, $H_Y(J) = \left[y(t_0), y(t_1), ..., y(t_e)\right]$ – matrices of the corresponding dimensions.

If the current identification is performed, then the information I_e has the form (10):

$$I_e = \{\vec{u}(t), \vec{y}(t)\} \quad (10)$$

where $t = t_o + (i-1)\tau, i = 1, 2, ..., t$ – current moment in time; τ – data collection interval.

With the current identification, i grows indefinitely with the growth of the object's operation time Thus, the set I_e in specific applications has a different form. The information I_e contains data on the parameters and characteristics of the object and the limits of their measurement. I_e implicitly displays the constraints on the parametric space of processes occurring in the object. Therefore, the set I_e, having the form of a digital array, can have a rather complex structure. The element $w(t) \in I_e$ can be represented as (11):

$$w(t) = w_c(t) + w_g(t) + w_x(t) + \xi(t) \quad (11)$$

where $w_c(t)$ – a constant or a function that changes arbitrarily; $w_g(t)$ – harmonic function with a fixed period; $w_x(t)$ – some process; $\xi(t)$ – measurement interference.

The component $w_c(t)$ reflects the main mode of operation of the object, $w_g(t)$ – any periodic phenomena and processes also associated with the main process. Functions $w_x(t)$ and $\xi(t)$ reflect the influence of various perturbations: $w_x(t)$ characterise internal, and $\xi(t)$ external influences. The set I_e will not always have such a structure. Depending on the properties of the object, some of the components of w may be absent or have a more specific form, that is, the function $w(t)$ can be written in the form (12):

$$w(t) = w_s(t) + \xi(t) \quad (12)$$

where $w_s(t)$ – function that displays the structural features of the object; $\xi(t)$ – environment.

Processes $w_s(t)$ and $\xi(t)$ can have both stochastic and regular structures. In the case of regular structure, various descriptions are used that allow an algorithmic representation. In the case of stochastic nature of the processes $\xi(t)$ various static and probabilistic characteristics are used for presentation. A random process is a change in a random variable over time. Random processes include most of the processes occurring in bioelectrical systems, like organism, as well as interference that accompanies the transmission of signals through neural communication networks. Random processes can be continuous or discrete, depending on which random variable is continuous or discrete in time. There are also several methods for describing signals: signal values in the time domain; signal values in the frequency domain; representation of signals and interferences by orthogonal series.

Each time an experiment is carried out, the result is a function defined over a time interval. It is necessary to characterise the entire random signal (process). There is one property that a task method must involve. If we consider a number of points in time $t_1, t_2, \ldots, t_n$ on the interval of assigning processes, then they correspond to n random variables of $x_{t1}, x_{t2}, \ldots, x_{tn}$. Any complete task should be able to determine the overall probability density $p_{xt1,xt2,...,xtn}(X_1, X_2, ..., X_n)$. In addition, it should allow to determine this density for any set of points in time on a given interval (for any finite n). Let us consider operations on a random

process that can be studied without actually completely specifying the process. For such operations, only a partial solution to the problem of the process being analysed is required. A large number of partial tasks are possible. The two most common ones are: representing a process value at only one point in time; presentation of process values by second points. When applied at one point in time, only the first-order probability density $p_{xi}(X)$ at a point in time t is determined. It is a function of time. When defining a process by its second moments, only the first and second moments of the process are set. The process mean value function is determined by the formula (13):

$$m_x(t) = E(x_i) = \int_{-\infty}^{\infty} X_i p_{xi}(X_i)\, dX_i \tag{13}$$

It is also a function of time. The correlation function is determined by the expression (14):

$$R_x(t,u) = E(x_i x_u) = E(x_i x_u) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X_i X_u p_{x_i x_u}(X_i, X_u)\, dX_i dX_u \tag{14}$$

The covariance function is defined as follows (15):

$$K_x(t,u) = E\{[x_t - m_x(t)][x_u - m_x(u)]\} = R_x(t,u) - m_x(t)\, m_x(u) \tag{15}$$

The partial process task is well suited to linear operations on random processes. The covariance function has several useful properties (16). One of the properties is the symmetry property, which follows from the definition (15):

$$K_x(t,u) = K_x(u,t) \tag{16}$$

If we multiply the sample function $x(t)$ by some deterministic function $f(t)$, integrable in a square, and integrate on the interval $[0, T]$, then we get a random variable (17):

$$x_j = \int_0^T x(t)\, f(t)\, dt \tag{17}$$

The average of this random variable is (18), and the variance, after the introduction of expectation under the integral sign, has the form (19):

$$E\left(x_j\right) = \overline{x_j} = E\int_0^T x(t)\, f(t)\, dt = \int_0^T m_x(t)\, f(t)\, dt \tag{18}$$

$$\sigma^2\left(x_j\right) = \int_0^T \int_0^T f(t)\, K_x(t,u)\, f(u)\, dt\, du \tag{19}$$

The variance must be greater than or equal to zero. Thus (20) true for $f(t)$ with a finite energy value. This property is called inherent certainty. If inequality (20) holds strictly for $f(t)$ with nonzero finite energy, then $K_x(t,u)$ is positive definite.

$$\int_0^T \int_0^T f(t)\, K_x(t,u)\, f(u)\, dtdu \geq 0 \tag{20}$$

If the process is defined on an infinite interval and its covariance function depends only on $|t-u|$, and not on t or u separately, then the process is covariance-stationary and can be written in the form (21):

$$K_x(t,u) = K_x(t-u) = K_x(\tau) \tag{21}$$

Similarly, if the correlation function depends only on $|t-u|$, and the process is correlation-stationary, and it can be written in the form (22):

$$R_x(t,u) = R_x(t-u) = R_x(\tau) \tag{22}$$

Consider Gaussian stochastic processes. Random variables $x_1, x_2, \ldots, x_N$ are jointly Gaussian if (23) is a Gaussian random variable for any set g_i:

$$y = \sum_{i=0}^{N} g_i x_i \tag{23}$$

If N – countable infinite, then it is necessary that g_i would be such that $E\left[y^2\right] < \infty$. In a random process, it is necessary to obtain a linear functional of the random function. To do this, assume that $x(t)$ is a random process defined on a certain interval $\left[T_\alpha, T_\beta\right]$ with a mean value of $m_x(t)$ and a covariance function $K_x(t,u)$. If each linear functional of $x(t)$ – is a Gaussian random variable, then $x(t)$ it is a Gaussian random process. That is, if (24) and $g(u)$ – any function meeting the condition $E\left[y^2\right] < \infty$, then for $x(u)$) to be a Gaussian random process, in must be a Gaussian random variable for each $g(u)$ in the above class.

$$y = \int_{T_\alpha}^{T_\beta} g(u)\, x(u)\, du \tag{24}$$

The output quantity of a linear system is a given linear functional. Let us denote the impulse response – the output value at the moment of time t, caused by the action on the input of a single impulse at the moment of time u through $h(t,u)$. If the input quantity is equal to $x(t)$ and is a sample function of a Gaussian

random process, then the output quantity $y(t)$ is also a sample function of a Gaussian random process. Thus (25):

$$y(t) = \int_{T_\alpha}^{T_\beta} h(t,u)\, x(u)\, du,\ T_\gamma \le t \le T_\Delta \qquad (25)$$

where $h(t,u)$ – impulse transient response with time-varying parameters.

An interval $\left[T_\gamma, T_\Delta\right]$ is the area where the function $y(t)$ is defined. Linear systems are characterised by an impulse response $h(t,u)$ Linear systems are characterised by an impulse response $h(\tau)$ in the case of time-constant parameters. A distinctive feature of this description is that the input signal is considered known over the interval $-\infty < t < \infty$. The impulse response $h(t,u)$ is simply a solution to a differential equation where the input signal is a pulse at the point in time u.

There are three solutions for describing systems using differential equations. The first solution is related to initial conditions and state variables when considering dynamic systems. The state of the system is defined as the minimum amount of information regarding the influences of previous signals at the input of the system, necessary to fully describe the output signal at $t \ge 0$. Variables containing this information are state variables. If the state of the system at the moment of time t_0 and the input signal in the interval from t_0 to t_1, are given, then both the initial signal and the state of the system at the moment of time t_1 can be found. The second solution is reduced to the implementation (or modelling) of the differential equation using an analogue calculator. It can be thought of as a system consisting of integrators, circuits with time-varying transmission coefficients, adders and nonlinear inertialess transmissions of signals and organ states, combined in such a way as to reproduce the required ratio between input and output signals. The initial condition $y(t_0)$ acts here as a bias at the output of the integrator. The offset output voltage of the integrator is a system state variable. The third solution relates to the issue of generating a random process. If $u(t)$ is a random process or $y(t_0)$ is a random variable (or both of them are random), then $y(t)$ – is also a random process. Let us consider a system described by a differential equation of the form (26):

$$y^{(n)}(t) + p_{n-1}(t)\, y^{(n-1)}(t) + \ldots + p_0(t)\, y(t) = b_0 u(t) \qquad (26)$$

where $y^{(n)}(t)$ – n-th derivative of $y(t)$; $p_i(t)$ – operator of differentiation; $u(t)$ – signal at the system input; b_0 – weight coefficient.

To determine the solution of the n-th order equation, it is necessary to know the value $y(t), \ldots y^{(n-1)}(t)$ at the moment in time t_0. The first step in finding an analogue computer form implementation is to model the terms on the left side of this equation. The next step is to combine these quantities in a way that the specified equation is satisfied. The differential equation determines the input voltage to the integrator. We introduce the initial conditions by setting certain offsets at the outputs of the integrator. State variables are biased voltages at the output of the integrator. It is easier to work with a first order vector differential equation than with a n-th order scalar differential equation. Let there be given (27, 28):

$$\begin{aligned} x_1(t) &= y(t) \\ x_2(t) &= \dot{y}(t) = \dot{x}_1(t) \\ &\vdots \\ x_n(t) &= y^{(n-1)}(t) = \dot{x}_{n-1}(t) \end{aligned} \qquad (27)$$

$$\dot{x}_n(t) = y^n(t) = -\sum_{k=1}^{n} p_{k-1} y^{(k-1)}(t) + b_0$$

$$= -\sum_{k=1}^{n} p_{k-1} x_k(t) + b_0 u(t) \qquad (28)$$

Denoting the system $x_i(t)$ by the column matrix, we note that the scalar equation of the n-th order is equivalent to the n-dimensional vector equation of the first order (29):

$$\frac{dx(t)}{dt} = \dot{x}(t) = Ax(t) + Bu(t) \qquad (29)$$

where A – system state matrix; B – matrix of control (input).

The vector $x(t)$ is called the state vector for a given linear system (29) – the state equation of the system. Any non-singular linear transformation of the vector $x(t)$ gives another state vector. The output voltage $y(t)$ is related to the state vector by equation (30):

$$y(t) = Cx(t) \qquad (30)$$

where C – measurement matrix.

Equation (30) is the initial equation of the system. Equations (29) and (30) completely define the system. For systems with time-varying parameters, as the main representation, we consider vector Equations

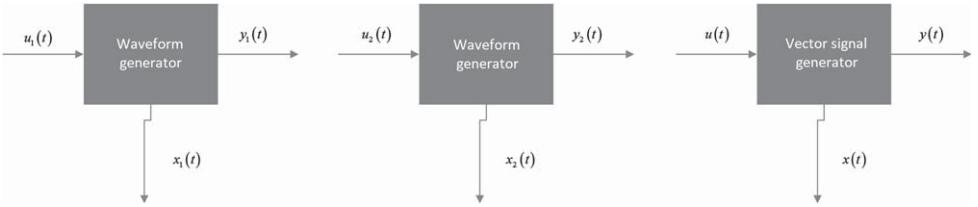

Fig. 1. Generation of two messages.

(31, 32):

$$\frac{dx(t)}{dt} = A(t)x(t) + B(t)u(t) \tag{31}$$

$$y(t) = C(t)x(t) \tag{32}$$

where $x(t)$ – state vector; $A(t)$, $B(t)$ – variable matrices of the differential equation; $u(t)$ – system input signal, white noise-type; $C(t)$ – measurement matrix.

Equation (31) is the state equation of the system, and (32) is the original equation of the system. Using white noise as input (33) some non-stationary stochastic processes can be simulated.

$$E[u(t)u(\tau)] = q\delta(t-\tau) \tag{33}$$

A non-stationary process can appear even when the matrices A and B are constants, and $x_0(t)$ – deterministic value. Consider a system that generates two initial messages $y_1(t)$ and $y_2(t)$ (Fig. 1).

The state of the first system is described by Equations (34, 35):

$$\dot{x}_1(t) = A_1(t)x_1(t) + B_1(t)u_1(t) \tag{34}$$

$$y_1(t) = C_1(t)x_1(t) \tag{35}$$

where $x_1(t)$ – n-dimensional state vector.

The representation of the second system is similar to the first and has the form (36, 37):

$$\dot{x}_2(t) = A_2(t)x_2(t) + B_2(t)u_2(t) \tag{36}$$

$$y_2(t) = C_2(t)x_2(t) \tag{37}$$

where $x_2(t)$ – m-dimensional state vector.

The only vector system of equations with a two-dimensional state vector is a more convenient way to describe these two systems (38, 43):

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \tag{38}$$

$$A(t) = \begin{bmatrix} A_1(t) & 0 \\ 0 & A_2(t) \end{bmatrix}, \tag{39}$$

$$B(t) = \begin{bmatrix} B_1(t) & 0 \\ 0 & B_2(t) \end{bmatrix}, \tag{40}$$

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \tag{41}$$

$$C(t) = \begin{bmatrix} C_1(t) & 0 \\ 0 & C_2(t) \end{bmatrix}, \tag{42}$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \tag{43}$$

The resulting differential equations have the form (44, 45):

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{44}$$

$$y(t) = C(t)x(t) \tag{45}$$

The exciting function is a vector. To simulate the process, we assume that the exciting function is white noise with a matrix covariance function (46):

$$E[u(t)u(\tau)] = Q\delta(t-\tau) \tag{46}$$

where Q – inherently definite matrix.

For random initial conditions, it is necessary to specify the covariance function and the mean value $E[x(t_0)]$ at the initial time moment t_0 (47):

$$K_x(t_0, t_0) = E\left[x(t_0)x^T(t_0)\right] \tag{47}$$

It is possible to model related processes by replacing the diagonal matrices in (44, 45), and (47) with matrices of a general form. If Equation (44) is a homogeneous equation with constant coefficients, then (48) with the initial condition $x(t_0)$.

$$x(t) = Ax(t) \tag{48}$$

If $x(t)$ and A are scalars, then the solution has the form (49):

$$x(t) = e^{A(t-t_0)}x(t_0) \tag{49}$$

For the vector case, it can be shown that (50), where e^{At} is determined by the infinite series (51), where I – identity matrix.

$$x(t) = e^{A(t-t_0)}x(t_0) \tag{50}$$

$$e^{At} = I + At + \frac{A^2t^2}{2!} + ... \tag{51}$$

We denote the function $e^{A(t-t_0)}$ by (52):

$$\Phi(t-t_0) = \Phi(\tau) \tag{52}$$

The function $\Phi(t-t_0)$ is the transition matrix of the state of the system, which is defined as a function of two variables $\Phi(t, t_0)$, that satisfy the differential equation (53) with an initial condition $\Phi(t_0, t_0) = I$.

$$\dot{\Phi}(t, t_0) = A(t)\Phi(t, t_0) \tag{53}$$

The solution at any moment of time has the form (54):

$$x(t, t_0) = \Phi(t, t_0)x(t_0) \tag{54}$$

For the inhomogeneous case, the general solution contains a homogeneous and particular solution of the form (55):

$$x(t) = \Phi(t, t_0)x(t_0) + \int_{t_0}^{t} \Phi(t, \tau)B(\tau)u(\tau)d\tau \tag{55}$$

Linear systems with time-varying parameters are characterised by an impulse response $h(t, \tau)$ provided that the input quantity is known over the interval from $-\infty$ to t. Thus (56):

$$y(t) = \int_{-\infty}^{t} h(t, \tau)u(\tau)d\tau \tag{56}$$

In most cases, the influence of the initial condition $x(-\infty)$ does not appear in (55), which means that it can be taken equal to zero. Then we obtain (57):

$$y(t) = C(t)\int_{-\infty}^{t} \Phi(t, \tau)B(\tau)u(\tau)d\tau \tag{57}$$

Comparing (56) and (57), we obtain (58):

$$h(t, \tau) = \begin{cases} C(t)\Phi(t, \tau)B(\tau) \\ 0npu\ \partial pysuxt \end{cases} \tag{58}$$

Matrices $C(t)$, $\Phi(t, \tau)$, $B(\tau)$ depend on the representation of the system, but the matrix impulse response is unique. Let us establish some statistical properties of vector processes $x(t)$ and $y(t)$, provided that $u(t)$ is a sample function of a vector random process of white noise-type (59):

$$E\left[u(t)u^T(\tau)\right] = Q\delta(t-\tau) \tag{59}$$

The cross-correlation between the state vector $x(t)$ of the system, excited by white noise $u(t)$ with zero mean, and the input quantity $u(\tau)$ equal to (60):

$$K_{xu}(t, \tau) = E\left[x(t)u^T(\tau)\right] \tag{60}$$

This discontinuous function has the form (61):

$$K_{xu}(t, \tau) = \begin{cases} 0 & \tau > t \\ \frac{1}{2}B(t)Q & \tau = t \\ \Phi(t, \tau)B(\tau)Q & t_0 < \tau < t \end{cases} \tag{61}$$

Substituting (54) into definition (60), we obtain (62):

$$K_{xu}(t, x) = E\left\{\left[\Phi(t, t_0)x(t_0) + \int_{t_0}^{t} \Phi(t, \alpha)B(\alpha)u(\alpha)d\alpha\right]u^T(\tau)\right\} \tag{62}$$

where α – delay time.

Let us introduce the mathematical expectation under the integral sign and assume that the initial state $x(t_0)$ does not depend on $u(\tau)$ at $\tau > t_0$. Then (63):

$$K_{xu}(t, \tau) = \int_{t_0}^{t} \Phi(t, \alpha)B(\alpha)E\left[u(\alpha)u^T(\tau)\right]d\alpha = \int_{t_0}^{t} \Phi(t, \alpha)B(\alpha)Q\delta(\alpha-\tau)d\alpha \tag{63}$$

At $\tau > t$ the expression is equal to zero. If $\tau = t$, and the delta function is symmetric, since it is the limit of the covariance function, then it is necessary to take only half of the area near the right boundary point of the interval. Thus (64):

$$K_{xu}(t, t) = \frac{1}{2}\Phi(t, t)B(t)Q \tag{64}$$

Using the result following from (46), we get the expression located in the second line (54). If $\tau < t$, we obtain (65), which corresponds to the third line (62).

$$K_{xu}(t, \tau) = \frac{1}{2}\Phi(t, \tau)B(\tau)Q, \tau < t \tag{65}$$

A special case (65) is obtained by putting $\tau \to t$

$$\lim_{\tau \to t} K_{xu}(t, \tau) = B(\tau) Q \tag{66}$$

Hence the cross-correlation function of the output vector $y(t)$ and $u(t)$ (67):

$$K_{yu}(t, \tau) = C(t) K_{xu}(t, \tau) \tag{67}$$

Denote (68):

$$\Lambda_x(t) = K_x(t, t) \tag{68}$$

As a result (69):

$$\Lambda_x(t) = E\left[x(t) x^T(t)\right] \tag{69}$$

Differentiating (69), we obtain (70):

$$\frac{d\Lambda_x(t)}{dt} = E\left[\frac{dx(t)}{dt} x^T(t)\right] + E\left[x(t) \frac{dx^T(t)}{dt}\right] \tag{70}$$

Substituting (44) into the first term (69), we obtain (71):

$$E\left[\frac{dx(t)}{dt} x^T(t)\right] = E\left\{\left[A(t) x(t) + B(t) u(t)\right] x^T(t)\right\} \tag{71}$$

Using property (65) to the second term (71), we obtain (72):

$$E\left[\frac{dx(t)}{dt} x^T(t)\right] = A(t) \Lambda_x(t) + \frac{1}{2} B(t) Q B^T(t) \tag{72}$$

Then the dispersion matrix of the state vector $x(t)$ of system (44) satisfies the differential Equation (73) with the initial condition (74):

$$\dot{\Lambda}_x(t) = A(t) \Lambda_x(t) + \Lambda_x(t) A^T(t) + B(t) Q B^T(t) \tag{73}$$

$$\Lambda_x(t_0) = E\left[x(t_0) x^T(t_0)\right] \tag{74}$$

The dispersion equation does not contain the received signal; therefore, it can be solved to receive any information and used to solve the transmission coefficients. The dispersion equation is the Riccati matrix equation.

4. Conclusions

The recovery of athletes refers to the system of structural support for the entire rehabilitation period. Every athlete inevitably rolls back after injury, since the parameters of activity are always reduced and can be restored only under the condition of high-quality medical care, monitoring of athletes in the course of rehabilitation and determination of threshold indicators that can be designated as achieved for the recovery process.

The main goal of the study was to solve the problem of identifying continuous rehabilitation processes to determine the structure of a dynamic object by the initial signal, the structure of its operator based on the structural properties of linear operators, and ordering the set of output signals of the continuous rehabilitation process. The solution of problems of identification and prediction of the course of continuous rehabilitation was considered for a typical sports injury of athletes – stretching and fracture of the knee joint, namely the final stage of its rehabilitation – extension. One of the main rehabilitation parameters of knee joint and fracture is the conditions of temperature.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] Llorens R, Noé E, Alcañiz M, Deutsch JE. Time since injury limits but does not prevent improvement and maintenance of gains in balance in chronic stroke. Brain Injury. 2018;32(3):303-9. https://doi.org/10.1080/02699052.2017.1418905

[2] Ahmadabadi F, Avandi SM, Aminian-Far A. Correlation between sway parameters of center of pressure in static and dynamic balance indices with eyes closed after four weeks static warm up in skilled gymnast athletes. Koomesh. 2017;19(1):102-12.

[3] Zhavoronkova LA, Maksakova OA, Shevtsova TP, Moraresku SI, Kuptsova SV, Kushnir EM, Iksanova EM. Dual tasks as an indicator of the characteristics of cognitive deficit in patients after craniocerebral trauma. Neuroscience and Behavioral Physiology. 2020;50(5):546-51. https://doi.org/10.1007/s11055-020-00935-w

[4] Noah S, Gibson-Horn C, Vincenzo JL. Four months of wearing a balance orthotic improves measures of balance and mobility among a cohort of community-living older adults. Journal of Geriatric Physical Therapy. 2019;42(4):216-23. https://doi.org/10.1519/JPT.0000000000000174

[5] Hadi M, Sadrodin SS, Amirhossein B, Malihe H, Shahram A. The effect of balance training on postural sensory organization against external disturbances in athletes with chronic ankle instability. Annals of Tropical Medicine and Public Health. 2019;20:2028-38.

[6] Toots A, Wiklund R, Littbrand H, Nordin E, Nordström P, Lundin-Olsson L, Gustafson Y, Rosendahl E. The effects of exercise on falls in older people with dementia living in nursing homes: a randomized controlled trial. Journal of the American Medical Directors Association. 2019;20(7):835-42. https://doi.org/10.1016/j.jamda.2018.10.009

[7] Armstrong R, Brogden CM, Milner D, Norris D, Greig M. Effect of fatigue on functional movement screening performance in dancers. Medical Problems of Performing Artists. 2018;33(3):213-9. https://doi.org/10.21091/mppa.2018.3032

[8] Szmeja Z, Rostek J. Late follow up examination of the state of hearing and of the organ of equilibrium after injury of the brain stem (Polish). Patologia Polska. 1974;25(3):475-9.

[9] Xiao K, Zhou JH, Gao XF, Zhao JJ, Fang ZH, Yang L. Effects of ligustrazine on free radicals and enzyme metabolism after static postural load in rat models. Chinese Journal of Tissue Engineering Research. 2019;23(35):5600-4. https://doi.org/10.3969/j.issn.2095-4344.1979

[10] Langeard A, Pothier K, Chastan N, Marcelli C, Chavoix C, Bessot N. Reduced gait and postural stability under challenging conditions in fallers with upper limb fracture. Aging Clinical and Experimental Research. 2019;31(4):483-9. https://doi.org/10.1007/s40520-018-0992-z

[11] Arora N, Graham RB, Grenier SG. Effect of whole body vibration on the postural control of the spine in sitting. Human Movement Science. 2015;40:77-88. https://doi.org/10.1016/j.humov.2014.11.014

[12] Wise AK, Hromatka KA, Miller KR. Energy expenditure and protein requirements following burn injury. Nutrition in Clinical Practice. 2019;34(5):673-80. https://doi.org/10.1002/ncp.10390

[13] Nagai T, Lovalekar M, Wohleber MF, Perlsweig KA, Wirt MD, Beals K. Poor anaerobic power/capability and static balance predicted prospective musculoskeletal injuries among Soldiers of the 101st Airborne (Air Assault) Division. Journal of Science and Medicine in Sport. 2017;20:11-6. https://doi.org/10.1016/j.jsams.2017.10.023

[14] Meier TB, Giraldo-Chica M, España LY, Mayer AR, Harezlak J, Nencka AS, Wang Y, Koch KM, Wu Yu-C, Saykin AJ, Giza CC, Goldman J, DiFiori JP, Guskiewicz KM, Mihalik JP, Brooks A, Broglio SP, McAllister T, McCrea MA. Resting-state fMRI metrics in acute sport-related concussion and their association with clinical recovery: A study from the NCAA-DOD CARE consortium. Journal of Neurotrauma. 2020;37(1):152-62. https://doi.org/10.1089/neu.2019.6471

[15] Hadamus A, Białoszewski D. Objective assessment of knee proprioception and sensorimotor function in patients with primary gonarthrosis before and after knee replacement. Ortopedia Traumatologia Rehabilitacja. 2017;19(5):403-14. https://doi.org/10.5604/01.3001.0010.5820

[16] Trubina IE, Novoderzhkina IS, Zarzhetsky YuV, Volkov AV. The influence of hemodilution by rheopolyglycin on the state of haemodynamics and oxygen utilisation in the early post-resuscitation period. Anesthesiology and Reanimatology. 1980;1:61-6.

[17] Yu X. Fatigue and recovery of wushu athletes based on fatigue damage model. IOP Conference Series: Materials Science and Engineering. 2020; 914(1):012016. https://doi.org/10.1088/1757-899X/914/1/012016.

[18] Domingues MDF, Rosa V, Rosa V, Nepomuceno AC, Nepomuceno AC, Tavares C, Alberto N, Andre P, Radwan A, Da Costa Antunes PF, Da Costa Antunes PF. Wearable devices for remote physical rehabilitation using a Fabry-Perot optical fiber sensor: Ankle joint kinematic. IEEE Access. 2020;8:109866-75.

[19] Nikityuk IE, Kononova EL, Nikitin MS, Afonichev KA. Abnormal hypersynchronization of body balance control system in children with post-burn foot deformity. Pediatric Traumatology, Orthopaedics and Reconstructive Surgery. 2019;7(2):61-8. https://doi.org/10.17816/PTORS7261-68

[20] Karch SJ, Lawson BD, Milam LS. Defining normal balance for army aviators. Military Medicine. 2019;184(7/8):e296-e300. https://doi.org/10.1093/milmed/usz064

[21] Szabó G. Relationship between tissue fluid and lymph in pathological conditions. Clinical Hemorheology. 1982;2(5/6):671-82.

[22] Prusch JS, Kleiner AFR, Salazar AP, Pinto C, Marchese RR, Galli M, Pagnussat AS. Automated mechanical peripheral stimulation and postural control in subjects with Parkinson's disease and freezing of gait: A randomized controlled trial. Functional Neurology. 2018;33(4):206-12. https://doi.org/10.11138/FNeur/2018.33.4.206