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Solving patient referral problems by using bat algorithm

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Abstract.
BACKGROUND: A two-hospital patient referral problem intends to calculate an optimal value of referral patients between two hospitals and to evaluate whether or not the current number of referral patients is too low.
OBJECTIVE: The goal of this study is to develop a simulation-based optimization algorithm to find the optimal referral between two hospitals with the unfixed daily patient referral policy.
METHODS: This study applied system simulation and a bat algorithm (BA) to build a simulation model in accordance with the status of the two hospitals case and to calculate an optimal value of daily referral patients.
RESULTS: Based on the 20 test instances, we verified the stability of this algorithm. The results show that the average magnetic resonance imaging (MRI) patient wait time reduced from 16 days to eight days. The hospital should increase the average total monthly MRI referral patients to 370 under the limitation of the daily referral patients to 25.
CONCLUSIONS: This research investigated the two-hospital patient referral problems. We conducted and analyzed a simulation model and improved the case hospital's conditions, enhancing the quality of its medical care. The findings of this study can extend to other departments or hospitals.

Keywords: Patient referral problem, simulation optimization, bat algorithm

1. Introduction

Information was collected on the total expenses of National Health Insurance since 1995 and the total expense was shown to increase each year. Because the public's need for medical resources continues to increase, so does the cost of medical resources. It is therefore essential to know whether or not medical resources are adequately allocated and used.

In Taiwan, people tend to believe that large hospitals offer more comprehensive medical services and more advanced medical techniques than small hospitals. Consequently, people prefer to visit large hospitals [1,2]. Because small hospitals have fewer patients, their medical utilization rates and incomes are lower than those of large hospitals. Consequently, there is an increasing difference between the medical utilization rates of large and small hospitals. To reduce this difference and to shorten patients'

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wait times, hospitals can form cooperative relationships and refer waiting patients to other cooperating hospitals, thus making more efficient use of medical resources. This enables patients in large hospitals to receive treatment faster and increases the medical utilization rates of small hospitals, enhancing the quality of medical care [3–5].

We examined the case hospital and built a simulation model. Each day, a hospital must determine how many patients should be referred to other hospitals. However, determining this number is difficult because if a hospital manager refers a large amount of patients to another hospital, patients of the recipient hospital will have to wait longer. This study therefore developed a bat algorithm (BA) to solve the number of referral patients. We conducted and analyzed a simulation model and improved the case hospital's conditions, thus enhancing the quality of its medical care.

2. Literature review

Patient referral problems have practical and academic importance. Researchers have studied patient referral problems based on solution methodology, including constructing simulation models and developing simulation optimization methods. Chen and Juan [2] applied simulation optimization to solve a two-hospital computed tomography (CT) patient referral problem by using Arena OptQuest software to calculate the optimal referral patients under the fixed-daily-referral policy. Pendharkar et al. [6] studied the problem of referring patients with sleep disorders and used Arena simulation software to compare ways of shortening their wait time. Based on the extension of reference [2], Chen et al. [3] solved the two-hospital referral patient problems under the fixed- and unfixed-daily-referral policies based on two objective functions: the first one is to minimize patients' average wait time and the second one is to maximize hospitals revenues. Chen et al. [3] applied a simulation optimization method (e.g., Arena OptQuest software) to solve the proposed problems.

Chen and Lin [4] extended the two-hospital to multi-hospital patient referral problems and developed three simulation optimization methods, each of which integrated a heuristic algorithm with the particle swarm optimization (PSO), to solve the magnetic resonance imaging (MRI) patient referral problems among three hospitals.

Caimo et al. [7] used a rigorous Bayesian computation method to derive the value of key parameters for exponential random graph models, leading to reduce considerable effort in the construction of a simulation model. Finally, Li et al. [5] conducted a patient referral problem between two hospitals. They focused on designing a threshold control policy to decide whether or not a patient should be referred from one hospital to another. They then developed a heuristic algorithm and applied simulation to verify their results.

Based on the literature review, most researchers considered patient referral problems between two hospitals in order to calculate an optimal value of referred patients under the fixed- or unfixed-daily-referral policies by using simulation optimization software (e.g., Arena OptQuest) or metaheuristic algorithms (e.g., PSO). Chen et al. [3] were the only ones to focus on exploring multi-hospital patient referral problems. The main purpose of their research was to determine to which recipient hospital a patient should be referred when more than one recipient hospital existed. Chen et al. [3] developed three heuristic algorithms, compared their results, and recommended the best one. However, they assumed that the number of the daily referral patients ranged between 0 and 5, which is a small-scale daily patient referral cooperation. Hence, the current study aims to develop a simulation-based optimization algorithm to find the optimal referring patients between two hospitals, which includes outpatients with multiple MRI scans, as well as explore much larger cooperation with the unfixed daily patient referral policy.

To this end, the study uses a case study, which will enable us to validate the feasibility of the proposed methodology and verify its robustness by using different test instances.

3. Methodology

3.1. Research problem and procedures

Researchers have studied two-hospital patient referral problems, which are defined that one hospital with more patients requires to refer a certain number of patients to another with less patients. Managers of the referring hospital must determine how many patients per day should be referred to the recipient hospital. If the daily referral patients are constant for the entire planning period, the referral policy is called the fixed referral policy. Otherwise, the referral policy is called the unfixed referral policy. Furthermore, the results of reference [3] show that based on the Arena simulation results, the unfixed referral policy outperforms the fixed referral policy on the average patient wait time in the two hospitals. Therefore, this study aims to search for the better unfixed referral policy in the two-hospital environment by using an objective function of minimizing the average patient wait time.

The research procedures consist of seven steps. Step 1 is to define a research problem and research scope of the two-hospital patient referral problem. Step 2 is to construct a simulation model, which requires to collect the related data, such as patient arrival time, patient treatment time, the number of physicians and equipment, the number of daily referral patients of the two hospitals, and traveling time between two hospitals. Step 3 is to verify the logic of the constructed simulation model. If the logic is correct, the simulation model passes the verification. The procedure will go to Step 4. Otherwise, the procedure will go back to Step 2 and modify the simulation model. Step 4 is to run the simulation model. Step 5 is to validate the simulation results. If the simulation results are close to those collected by the case two-hospital referral system, the simulation model passes the validation. The procedure will go to Step 6. Otherwise, the procedure will go back to Step 2 and modify the simulation model. Step 6 is to integrate the BA into the validated simulation model and use the BA to calculate the daily patients transferred from the referring hospital to the recipient one. Finally, Step 7 is to obtain a better unfixed referral policy in the two hospitals based on the BA with the simulation model, which is called a simulation optimization method. The details of the BA with the simulation model are presented in Section 3.2.

3.2. Bat algorithm (BA) with the simulation model

The BA is a metaheuristic algorithm based on the simulation of the bats' echolocation mechanism for locating and hunting prey [8]. Yang [8] verified the BA by adapting it to solve multiple testing functions, including multi-peak fitting, the Michalewicz function, and the Rosenbrock function. Yang [8] and Yang and He [9] confirmed that this algorithm can solve optimization problems. The proposed method for integrating the BA with referring patient simulation is presented next.

Step 1. Generate the initial number of referral patients: The BA is used to generate an initial solution. The position, velocity, and frequency of each bat are then randomly configured. Each bat denotes a feasible solution, indicating the daily referral patients for the entire planning periods (e.g., a month).

Step 2. Update bat velocity, frequency, and position: Eqs (1) to (3) are used to update each bat's frequency, velocity, and position. In Eq. (1), f_i represents the frequency of the i^{th} bat, and f_{max} and f_{min} pertain to the maximum and minimum frequencies, respectively. β ranges from 0 and 1 randomly. Equation (2) represents that at iteration t, v_i^t is the i^{th} bat's velocity, and X_i^t is the i^{th} bat's position; v_i^{t-1}

is the i^{th} bat's velocity at iteration $t-1$; and X^* is the optimal position. Equation (3) presents that X_i^{t-1} is the i^{th} bat's position at iteration $t-1$.

$$f_i = (f_{\max} - f_{\min}) \times \beta. \tag{1}$$

$$v_i^t = v_i^{t-1} + (X_i^t - X^*) \times f_i. \tag{2}$$

$$X_i^t = X_i^{t-1} + v_i^t. \tag{3}$$

Step 3. Calculate fitness value (average patient wait time): The simulation model was used to generate patients requiring MRI for each hospital. For example, Hospital A determines whether the current number of referral patients for Hospital B has already reached the upper limit. If it has not been reached, patients are transferred to Hospital B to receive MRI scans; otherwise, the patients remain and are examined in Hospital A. After the patients referred to Hospital B complete their MRI scans, they return to Hospital A. The following patient data are recorded: arrival time at Hospital A, starting and completion times of MRI scans, and patient wait time. The BA then calculates the average patient wait time.

Step 4. Update the current optimal solution: The average patient wait time corresponding to a feasible solution is compared with the average patient wait time corresponding to the current optimal solution. If the updated solution is preferable to the current optimal solution, the updated position replaces the current optimal position.

Step 5. Conduct local and global searches: The BA adopts the pulse rate of each bat as the criterion to determine a search strategy, generating a value between 0 and 1 randomly. If the generated value is larger than the pulse rate of a bat, then a local search is conducted using Eq. (4); otherwise, random search is employed by randomly generating a new feasible solution. Here, X_{new} is the new solution acquired through a local search. X_{old} is the current position of the bat. ε is randomly generated between -1 and 1, and A^t denotes the average bat's loudness.

$$X_{new} = X_{old} + \varepsilon \times A^t. \tag{4}$$

Step 6. Update the current optimal solution: The average loudness is the criterion for determining whether to replace the solutions with that obtained from local and global searches. Generating a random value falls between 0 and 1. If the generated value is smaller than the average loudness, the updated solution is regarded as the current optimal solution, and replaces the previous optimal solution.

Step 7. Update loudness and pulse rate: If the optimal solution is updated, using Eqs (5) and (6) then updated the loudness and pulse rate, respectively. Equation (5) represents that at iteration $t+1$, A_i^{t+1} is the i^{th} bat's loudness; A_i^t is the i^{th} bat's loudness at iteration t; and α is a constant between 0 and 1. Equation (6) represents that r_i^{t+1} is the i^{th} bat's pulse rate at iteration $t+1$; r_i^0 is an initial i^{th} bat's pulse rate; and γ is a positive constant.

$$A_i^{t+1} = \alpha \times A_i^t. \tag{5}$$

$$r_i^{t+1} = r_i^0 \times [1 - \exp^{(-\gamma t)}]. \tag{6}$$

Step 8. Termination criterion: Completing Steps 1 to 7 increases the iteration number by 1. If the iteration number reaches the upper limit, the proposed method is terminated, and the current optimal solution and average patient wait time are listed. If the iteration number has not reached the upper limit, Steps 2 to 7 are repeated.

Table 1
Paired t-tests on bat numbers

Paired t-test	95% C.I.	t-value	p-value
H_0: μ_{10}–$\mu_{20} = 0$	[13.0, 98.9]	2.61	0.012
H_0: μ_{10}–$\mu_{30} = 0$	[114.8, 215.8]	6.56	0.000
H_0: μ_{20}–$\mu_{30} = 0$	[59.3, 159.4]	4.39	0.000

3.3. Variable and parameter setting with the proposed method

This study aims to optimize the average patient wait time of the two hospitals. In this study, the decision variable denotes the number of the daily MRI patients referred from Hospital A to Hospital B, yielding a total of 30 variables. Because the case hospital did not formulate an upper limit for the number of patients referred to Hospital B, this study used the number of patients entering the system each day.

The essential parameters of the BA were loudness and pulse rate, both of which affect the search behavior and efficiency. As suggested in reference [8], the initial loudness, A_i^0, was set between 1 and 2, the initial pulse rate, r_i^0, was set as 1, and the pulse rate was updated by using Eq. (6).

Two parameters, the iteration number and the number of bats, affected solution quality. The larger the parameters, the higher the solution quality. However, the larger of the two parameters would increase the solution time of the BA. Hence, there was a trade-off between solution quality and solving time. This research uses the BA to search for a good solution within a reasonable time. After performing pilot runs, solution quality of the BA converged at a certain value when the iteration number was between 150 and 180. Thus, the iteration number was set as 200.

Furthermore, based on the literature review [10–12], between 10 and 30 bats were used in the medical application papers. To determine an appropriate number of bats, this research set the number of bats as 10, 20, and 30 to conduct 30 separate search processes under identical patient data conditions. As all datasets passed the normality test, the three paired t-tests could be conducted. The paired t-test by Minitab, a statistics package used to perform statistical analyses [13], was used to determine whether the numbers of bats resulted in significantly different target values (H_0: μ_{10}–$\mu_{20} = 0$; H_1: μ_{10}–$\mu_{20} \neq 0$). Here, μ_{10} indicated the average patient wait time in the two hospitals obtained by using the BA with 10 bats. α was the significance level to reject the null hypothesis (H_0: μ_{10}–$\mu_{20} = 0$) and was set to 0.05 in this study; hence, if the p value was less than α (i.e., 0.05), then H_0 was rejected. If the value of the 95% confidence interval (C.I.) was positive, it meant that the average patient wait time in the two hospitals obtained by using the BA with 10 bats was larger than that obtained by using the BA with 20 bats, indicating the BA with 20 bats outperformed the BA with 10 bats (see Table 1).

In Table 1, the result shows that, for all three hypotheses, the p value was less than 0.05 and the value of the 95% C.I. was positive, indicating that the BA with 20 bats outperformed the BA with 10 bats, the BA with 30 bats outperformed the BA with 10 bats, and the BA with 30 bats outperformed the BA with 20 bats. Therefore, the BA with 30 bats showed the best performance on the average patient wait time in the two hospitals. The number of bats used in the BA of this study was set at 30.

4. Results and discussion

This research collected three months of data from two hospitals, such as Hospital A and Hospital B. Before building a simulation model, we performed the data-fitting method. The results showed that he p-value of the Chi-square and Kolmogorov-Smirnov (K-S) tests was greater than 0.05, indicating an appropriate probabilistic distribution (see Table 2). Therefore, this research used the patient arrival time

Table 2
Patient arrival data from the two hospitals

Data	Probabilistic distribution	Chi-square test	p-value	K-S test	p-value
Patient arrival time in Hospital A	Exponential (11)	3.89	0.159	0.0742	> 0.05
Patient arrival time in Hospital B	Exponential (26.7)	0.103	0.0535	0.103	> 0.05

Table 3
MRI scan data from the two hospitals

MRI scans	Probabilistic distribution	Chi-square test	p-value	K-S test	p-value
Spine MRI scan in Hospital A	20 + Exponential (11)	3.44	0.196	0.0562	> 0.05
Neck MRI scan in Hospital A	20 + Exponential (22)	2.69	0.265	0.0758	> 0.05
Abdomen MRI scan in Hospital A	Triangular (20, 21.4, 80)	2.87	0.242	0.115	> 0.05
Spine MRI scan in Hospital B	15 + Exponential (6.26)	0.103	0.0535	0.103	> 0.05
Neck MRI scan in Hospital B	15 + Weibull (6.83, 0.93)	1.28	0.262	0.0638	> 0.05

Table 4
Simulation and the case hospitals' results of the average patient wait time

	Average patient wait time in the current case hospitals (day)	Average patient wait time by the simulation model (day)	SD of the patient wait time by the simulation model (day)	95% C.I. of the average patient wait time by the simulation model (day)
Hospital A	25.17	24.96	0.39	[24.81, 25.11]
Hospital B	5.10	5.48	0.34	[4.88, 5.14]
Average	16.41	16.43	0.24	[16.34, 16.52]

in Hospital A as the exponential distribution with the mean equaling 11 minutes; it used the patient arrival time in Hospital B as the exponential distribution with the mean equaling 26.7 minutes.

For Hospital A, there were three major areas targeted by the MRI scan patients: spine (52%), neck (42%), and abdomen (6%). For Hospital B, there were two major areas: spine (70%) and neck (30%). Based on the data-fitting method, Table 3 shows that the probabilistic distribution of each MRI scan of the two hospitals passed the Chi-square and K-S tests. As there was no abdomen MRI scan data in Hospital B, this research assumed that referral patients (from Hospitals A to B) with abdomen MRI scan would follow the probabilistic distribution as Triangular (20, 21.4, 80) (minutes) in Hospital A. The remaining probabilistic distributions of the two hospitals are shown in Table 3.

Based on the results shown in Tables 2 and 3, this research used the patient arrival time and MRI scan time of the two hospitals to construct a simulation model in order to explore a two-hospital patient referral problem. After examining the logic of the built simulation model, this research ensured that the logic was correct, leading to the verification of the built simulation model. The next procedure was to validate the simulation results. Table 4 shows that the average and standard deviation (SD) of the patient wait time by the simulation model were 24.96 and 0.39 for Hospital A and 5.48 and 0.34 for Hospital B. The longer average patient wait time, which was more than three weeks, was the reason for referring MRI scan patients from the referring hospital (Hospital A) to the recipient hospital (Hospital B).

The corresponding 95% C.I. of the patient wait time according to the simulation model was then calculated, as shown in Table 4. Comparing the results of the current case hospitals and simulation model, Table 4 shows that the average patient wait time in the current case hospitals falls into the range of the 95% C.I. of the average patient wait time by the simulation model, which means that the simulation model passes validation. Therefore, this research could use the validated simulation model to integrate with the BA to calculate the daily referral patients.

Table 5
Test instance results

Test instance	Average patient wait time (day)	SD of patient wait time	Worse-case solution (day)	Best solution (day)	CV
1	8.09	0.16	8.48	7.77	0.02
2	8.37	0.18	8.67	7.99	0.02
3	8.39	0.17	8.77	7.98	0.02
4	7.86	0.18	8.18	7.51	0.02
5	8.10	0.18	8.41	7.63	0.02
6	8.34	0.21	8.68	7.89	0.03
7	8.08	0.18	8.49	7.56	0.02
8	8.55	0.20	8.83	8.06	0.02
9	8.65	0.23	9.04	7.96	0.03
10	8.14	0.19	8.41	7.46	0.02
11	8.07	0.18	8.43	7.80	0.02
12	8.45	0.21	8.82	8.07	0.02
13	8.34	0.18	8.77	8.02	0.02
14	8.47	0.15	8.76	8.19	0.02
15	8.37	0.22	8.79	7.90	0.03
16	8.65	0.24	9.08	7.97	0.03
17	8.45	0.16	8.71	8.13	0.02
18	8.46	0.20	8.89	7.99	0.02
19	8.14	0.19	8.56	7.69	0.02
20	8.37	0.20	8.75	7.76	0.02

Note: A simulated day is set to 480 minutes.

Table 6
The daily referral patients of the best solution of test instance 4

Day	1	2	3	4	5	6	7	8	9	10
No.	14	25	0	4	17	23	4	23	8	14
Day	11	12	13	14	15	16	17	18	19	20
No.	5	1	10	2	3	19	4	20	21	8
Day	21	22	23	24	25	26	27	28	29	30
No.	19	7	8	8	8	22	12	1	15	6

To verify the robustness of the proposed methodology, this research generated 20 test instances according to the case image center's data. Then, the BA was used to determine the patient wait times of the 20 test instances. Because the average patient wait time of each test instance differed, the coefficient of variation (CV) method was used to investigate the results. Here, the CV value was calculated by using the SD of the patient wait time divided by the average patient wait time. When the CV method is used to compare two or more sets of data, the SD and mean of each data set are adopted to determine the dispersion of the data set. A large CV value indicates a high level of data dispersion. The results of this study (Table 5) showed that the CV values of the 20 test instances are all within 0.03, and the differences between the CV values are small, indicating that the calculated patient wait times exhibit a small dispersion level. This verified the robustness of the BA used in this study.

Test instance 4 is used as an example. Table 6 shows that the daily referral patients of the best solution of test instance 4 is 14 on Day 1, 25 on Day 2, 0 on Day 3, . . ., and 6 on Day 30. The range of the daily referral patients is from 0 to 25. The number of the 30-day referral patients is 331. Based on Table 5, the average patient wait time in test instance 4 is 7.86 days.

The simulation optimization results of this study were compared with the actual average patient wait time and total referral patients in the case hospital. The 20 test instances were solved, and the solution sets represented the simulated daily numbers of referral patients in a month; a total of 20 simulation

Table 7
The results between the case hospital and the simulated model

	Average patient wait time	Average total monthly referral patients
Current case hospital condition	About 16 days	219 patients
Simulation-based optimization model	About 8 days	372 patients
Improvement	50%	–

optimization solution sets was therefore obtained. The daily numbers of referral patients in each solution set were added to calculate the total monthly referral patients. In Table 7, the average total monthly referral patients for the 20 test instances is 372, and average wait time is 3,992.59 minutes. Assuming that the MRI machines run 480 minutes per day, the average wait time is approximately eight days.

The number of referral patients and average patient wait time obtained through simulation optimization were compared with those of the case hospital data. In the case hospital, the average patient wait time was 16 days, and total monthly referral patients were 219 people, whereas those obtained through simulation optimization were eight days and 372 people, respectively. Therefore, the 20 test instances' results of simulation optimization indicate that the current total monthly referral patients in the case hospital is too low. If the case hospital increases the total monthly referral patients to 372 and ensures that the daily referral patients do not exceed 24, the average wait time can be reduced to eight days, resulting in a 50% improvement.

The case hospital does not clearly define the number of referral patients, and patient referral decisions are based on patients' wishes. Because a ride on a hospital shuttle between the two hospitals takes about 40 minutes, the case hospital noted that patients might refuse referral because of travel considerations or a preference for Hospital A. However, this adds to patients' wait time and affects the medical quality of the hospital. More importantly, patients in urgent need of MRIs cannot immediately have them because too many other patients are waiting. Therefore, it is recommended that the case hospital gradually increase MRI patients transferred from Hospital A to Hospital B to approximately 370. This can shorten wait times and improve the quality of medical care.

According to the MRI patient data of the case hospital, we built a patient referral model to solve for a favorable number of referral patients. The following two research implications are formulated.

Before system simulation is conducted, data on the actual conditions of the research subject must be collected to ensure accurate research outcomes derived from the subsequent analyses and decisions.

This study adopted simulation optimization, enabling the case hospital to understand the patient waiting list. The results of this study indicate that the patient wait time of Hospital A can still be improved.

By comparing the results of this research to existing literature on patient referral problems discussed in Section 2, this study demonstrated the feasibility of applying a metaheuristic algorithm, the bat algorithm, to solve the patient referral problem in the two-hospital environment instead of directly applying the simulation optimization software [2,3]. With the exception of the PSO method in reference [4], this research has showed the possibility of applying different metaheuristic algorithms to solve the patient referral problem.

For the unfixed-daily-referral policy studies, this research extended the small-sized application of reference [4] to the medium-size application. Thus, researchers can extend a medium-sized application to a large-sized application in future studies.

Furthermore, the results of the unfixed-daily-referral policy in this study can be viewed as a threshold control policy for referring hospitals, similar to the results of reference [5]. Hospital managers can use the proposed methodology in this study to make daily patient referral decisions in order to improve the quality of medical care. Based on these discussions, this research has filled the research gap relating to the two-hospital patient referral problem, thereby accomplishing the goal of this study.

5. Conclusions and future research

The primary contribution of this study involves applying system simulation coordinated with the BA to formulate a simulation optimization method. By collecting data and performing the data-fitting method, the appropriate simulation parameters could be obtained. Verifying and validating the built simulation model ensured that the built model could be used to replace the actual two-hospital patient referral system for exploring different policies. By integrating the BA with the simulation model, it was possible to solve for a favorable number of referral patients for the two hospitals, cutting the average patient wait time in half: from 16 days to eight days. Because the case hospital, Hospital A, does not define a clear number of referral patients, the results of this study can serve as a reference for the hospital to formulate a standard for determining the number of referral patients. The hospital is suggested to increase the average total monthly referral patients to 370 under the limitation of the daily referral patients to 25.

This research investigated the MRI referral patients of the case hospital. Applying the proposed research method to other departments or hospitals would require analyzing the corresponding patient data (e.g., patient arrival intervals and examination durations). Moreover, limitations to the number of referral patients should be addressed according to the patient referral mechanism of the target institution.

Three suggestions are proposed for subsequent studies:

(1) Accounting for various types of referral mechanisms
This study investigated the referral problem between two hospitals of the case hospital. If subsequent studies target multiple hospitals [4], the effects of other referral mechanisms on patient wait time can be investigated.

(2) Applications to patient referral
This study did not examine revenue concerns between the cooperating hospitals [3]. If subsequent studies extend the research to the ways in which hospital revenue affects the number of referral patients, associated factors (e.g., patient wait time, medical costs, and hospital revenue surplus) can be explored.

(3) Integration of other metaheuristic algorithms
This study used the BA. Arora and Singh [14] asserted that the BA, cuckoo search method, and firefly algorithm involve favorable integrated search strategies. Therefore, subsequent studies can consider using the cuckoo search method and firefly algorithm to solve research problems.

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Conflict of interest

None to report.

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