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Introduction

Rare and uncommon risks and the financial meltdown

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2008–2009 will be remembered as an era where extreme events have come on their own, ex-ante ignored, but factual and painful ex-post. Ex ante we have a tendency to ignore rare events, seeking comfort in numbers we can point out to and ignoring the others (Paul Samuelson). For example, insurance firms concentrate on aggregate risks, avoiding the existence of outliers and non-quantifiable rare events. Yet, risks – true risks, are outliers! Jean Pierre Landau (Vice Governor of the Bank of France), points out that modern finance is based in practice if not in theory on an "implied" ignorance of extreme risks (and implied complexity, my addition) of financial markets. There is thus an overall weakness when confronted with the theoretical implications of rare and extreme events. History and cemeteries are filled with their consequences, however, that have for many reasons and conveniently been forgotten (both due to our inability to confront these events and our own "finiteness" – presuming that it will not happen to us!). Extreme risks are increasingly common and recurring, however. On the one hand climatic changes, population and concentrations growths are causing previously "unthinkable" disasters to be recurrent. Extreme weather is now a TV show while Terror is ever present and everywhere. Extreme and Rare risks, are now at the center stage of our working probability distributions. Yet, they are not always appreciated at their just importance.

Financial mathematics for example, presumes both the "predictability" of future prices, interest rates as well as other and related time series emphasizing that "financial uncertainty" is a Martingale, fair and expectedly constant. Such processes have been presumed by Bachelier already in 1900 and underlie the Random Walk Hypothesis (and the Brownian motion) in finance (Cootner, 1964) and in physics (Einstein, 1906). These processes have special characteristics and consist of independent increments, independently and identically distributed Gaussian (thin tail) random variables, contributing to our continued fixation on linearly growing variance (as a measure of uncertainty) over time. This facet of "the growth of uncertainty" has been severely criticized and numerous statistical tests have been based on it to demonstrate that the underlying process need not be Brownian motion.

Empirical evidence has shown that financial series are not "well behaved" and cannot be always predicted. They may exhibit unpredictable and "chaotic behavior" which underscores "nonlinear science" approaches to finance. Rather, in many cases, it is observed that data can behave "unpredictably" at time and at others, it may exhibit regular variations. "Bursts" of activity, "feedback volatility" and broadly varying behaviors by stock market agents, "memory" (both long and short, exhibiting persistent behaviors) etc. are characteristics that contribute to the "nonlinearity of uncertainty growth" and thereby to challenging fundamental finance. Further, even aggregation of time series that are mildly auto-regressive can turn out to have long run memory and thereby to serious contentions regarding the assumptions of fundamental finance. By the same token, Vallois and myself [42,43] have indicated that persistence in pure random walks has a short memory and lead to a nonlinear evolution of the process volatility.

The study of real time series have motivated a number of approaches falling under a number of themes spanning: fat tails, Leptokurtic distributions, Pareto–Levy stable distributions, long run memory – fractional Brownian models, dependence, persistent processes, Chaotic analysis, Lyapunov stability analysis; Complexity analysis; R/S analysis etc. (see [29] and [26], for example).

Both theoretical and practical reasons underlie the importance of research dealing with departure of the "random walk hypothesis" or "linear stochastic modeling" and in developing the required mathematical and statistical apparatus needed to detect when and extreme events arise. While models are always a simplified of some part of a presumed or desired reality, our greatest mistakes will be to believe in closed-ended models – economic or not. While models are extremely useful, and needed, models will always be also a source of grief – realities are far more complex and always surprising. The challenges for risk analysis and financial and economic assessments are thus always challenged and challenging.

The purpose of this special issue is to focus some attention to alternative modeling and the mathematical treatment of "uncommon risks" and address in part some of the issues to focus their attention. There are many approaches to dealing with these problems however. For example, there are many forms and approaches to defining what is "Chaos". For some, chaos expresses a "stochastic like behavior" of deterministic systems. Typically, a dynamic system may have a number of "types of attractors". These may include equilibrium points, periodic orbits or cycles, quasi-periodic orbits as well as chaotic or "strange attractors". In a financial framework, a chaotic process is a process which is not predictable. In other words, even though a process may be well defined in the sense that its equations are well specified, all variables are endogeneous, its' actual and long run behaviors are hardly predictable. Such processes, even when they are well specified, provide always "new" information that redefine the steady state of such processes. Chaos is often used to characterize a-periodic and non-explosive dynamical systems that are completely (deterministic and) determined but unpredictable! In the statistical study of time series, chaos is also used to express a number of properties summarized by nonlinearity and unpredictable volatility; long term or strong dependence; "fat tails" distribution (or Pareto–Levy stable distributions with an exponent other than 2) and complexity [5,13,34].

If a process is dependent on the initial condition, and the state of equilibrium determined in terms of the initial condition, then perturbation in this state will perturb the "equilibrium" state which renders such a process unpredictable. In this sense, even though a process may be deterministic, it can exhibit a "stochastic like behavior" (since a change in a process state will alter its initial condition and therefore, can lead in some cases, to multiple long term equilibria).

The presence of long run memory or persistence has important implications for many of the paradigms used in modern risk assessment, economics and finance. For example, optimal consumption-savings and portfolio decisions may become extremely sensitive to the investment horizon if stock returns were long term dependent. Problems also arise in the pricing of options and futures since the class of models used are incompatible with long term memory. Traditional tests of the capital asset pricing model and the Arbitrage Pricing Theory are no longer valid since the usual forms of statistical inference do not apply to time series exhibiting such persistence. Conclusions of more recent tests of "efficient" markets hypotheses or stock market rationality also hang precariously on the presence or absence of long term memory [25]. Further, if speculative prices exhibit dependency then the existence of such dependency would be inconsistent with rational expectations and would thus make a strong case for technical forecasting on stock prices. In other words, the Martingale approach to finance will be of little use. Empirically, it is well documented that interest rates and future prices can have long term memory [5,11,12,14, 17–20,25,36].

The commonality of "uncommon risks" calls for a concerted effort to increase our awareness that such risks exist and motivate the attention they deserve. This special issue is one step in this direction.

This special issue is focused on a number of papers, each approaching another aspect of extreme and rare risk mathematics.

Tyrone Duncan, provides a tractable introduction to fractal Brownian mathematics providing both a clear understanding of what they are and how they may be applied. Fractional Brownian motion denotes a family of Gaussian processes whose applicability has been demonstrated empirically for a wide variety of physical and financial phenomena. For more than five decades, these processes have described risky outcomes or physical uncertainties and in particular for more than four decades these processes have been used to model fluctuations in economic data. These processes have a self-similarity or fractal property in probability law. They can provide a model for long range dependence, rare events and "bursting" behavior. Mandelbrot motivated by his study of fractals studied fractional Brownian motions to model economic data as well as turbulence and coined the name fractional Brownian motion and attraction attention to Hurst's index. The empirical evidence for models with a fractional Brownian motion has continued in hydrology, finance and turbulence. More recently the empirical evidence for the use of fractional Brownian motions (FBMs) as random models broadened to internet, traffic medicine and cognition (see Duncan's paper). The particular contribution of this paper is in its introduction of the essential mathematical properties and in particular the definition of a stochastic calculus with stochastic differential equations with fractal Brownian.

The second paper is devoted to "Power-Law Distributions: Beyond Paretian Fractality" by Iddo Eliazar and Joseph Klafter. In their paper, the notion of fractality, in the context of positive-valued probability distributions, is conventionally associated with the class of Paretian probability laws. In their research, they show that the Paretian class is merely one out of six classes of probability laws, all equally entitled to be ordained fractal, all possessing a characteristic power-law structure, and all being the unique fixed points of renormalizations acting on the space of positive-valued probability distributions. These six fractal classes are further shown to be one-dimensional functional projections of underlying fractal Poisson processes governed by: (i) a common elemental power-law structure; and, (ii) an intrinsic scale which can be either linear, harmonic, log-linear, or log-harmonic. This research provides a panoramic and comprehensive view of fractal distributions, backed by a unified theory of their underlying Poissonian fractals.

The third paper is by Yaniv Dover, Sonia Moulet, Sorin Solomon and Gur Yaari, members of a joint Israeli and Italian research team, "Do all economies grow equally fast?" uses a generalized Lotka–Volterra approach to the study of interactions between economic sectors, countries and blocks. The theory they used predicts robustly in a very wide range of conditions systematic regularities in the growth rates evolution of various subsystems. They describe the 'Growth Alignment Effect' (GAE), it's theoretical basis and demonstrate it empirically for numerous cases in the international and intra-national economies. The GAE is the concept that in steady state the growth rates of the GDP per capita of the various system components align. The particular importance of their research is in the intensity of their use of a theoretical framework imbedded in large amounts of data and the manner they are able to overcome the complexity of economic interdependent systems. Such complexity, studied from a microstructural (atomic) viewpoint is revealing and provides a useful realization that there are after all "aggregate patterns" that arise from basic and simple "atomic"-interactions.

Finally, the paper of Philip Maymin provides a different and psychological approach to the "creation" of "uncommon risks" based on Prospect Theory. Explicitly, Maymin considers a behavioral representative investor who evaluates a single risky asset and shows that such an investor will often induce high kurtosis, negative skewness, and persistent autocorrelation into the distribution of market returns. This would be the case even if the asset payoffs are merely a sequence of independent coin tosses and the investor is simply loss averse.

This special issue does not cover all the issues and alternative modeling approaches to "uncommon risks" but seeks simply to emphasize that a in a world as global as it is today, as dangerous as it has become, risk and decision analysis are confronted with unusual challenges that require unusual approaches and solutions.

References

[1] R. Baillie and T. Bollerslev, Prediction in dynamic models with time dependent conditional variances, *Journal of Business Economics and Statistics* **5** (1992), 91–113.

[2] J. Beran, *Statistics for Long-Memory Processes*, Chapman and Hall, London, 1994.

[3] T. Bollerslev, Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics* **31** (1986), 307–327.

[4] G. Booth, F. Kaen and P. Koveos, R/S analysis of foreign exchange rates under two international monetary regimes, *Journal of Monetary Economics* **10** (1982), 407–415.

[5] W.A. Brock, D.A. Hsieh and B. LeBaron, *Nonlinear Dynamics, Chaos and Instability*, MIT Press, Cambridge, MA, 1992.

[6] F. Diebold and G. Rudebusch, Long memory and persistence in aggregate output, *Journal of Monetary Economics* **24**(2) (1989), 189–209.

[7] C. Donati-Martin and M. Yor, Fubini's theorem for double Wiener integrals and the variance of the Brownian path, *Annales Institut H. Poincaré* **27** (1991), 181–200.

[8] C. Donati-Martin and M. Yor, On some examples of quadratic functionals of Brownian motion, *Advances in Applied Probability* **25** (1993), 570–584.

[9] R. Engle, Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation, *Econometrica* **50** (1982), 987–1008.

[10] W. Feller, The asymptotic distribution of the range of sums of independent random variables, *Annals of Math. Stat.* **22** (1951), 427–432.

[11] H.-G. Fung and W.-C. Lo, Memory in interest rate futures, *The Journal of Futures Markets* **13** (1993), 865–873.

[12] H.-G. Fung, W.-C. Lo and J.E. Peterson, Examining the dependency in intra-day stock index futures, *The Journal Futures Markets* **14** (1994), 405–419.

[13] J. Gleick, *Chaos: Making a New Science*, Viking Press, New York, NY, 1987.

[14] J. Grandmont and P. Malgrange, Nonlinear economic dynamics: Introduction, *Journal of Economic Theory* **40** (1986), 3–12.

[15] C. Granger, Long memory relationships and the aggregation of dynamic models, *Journal of Econometrics* **14** (1980), 227–238.

[16] C.W. Granger and T. Trasvirta, *Modelling Nonlinear Economic Relationships*, Oxford University Press, Oxford, 1993.

[17] M.T. Green and B. Fielitz, Long term dependence in common stock returns, *J. Fin. Econ.* **4** (1977), 339–349.

[18] M.T. Green and B. Fielitz, Long term dependence and least squares regression in investment analysis, *Management Science* **26**(10) (1980), 1031–1038.

[19] B. Helms, F. Kaen and R. Rosenman, Memory in commodity futures contracts, *Journal of Futures Markets* **4** (1984), 559–567.

[20] D.A. Hsieh, Chaos and nonlinear dynamics application to financial markets, *Journal of Finance* **46** (1991), 1839–1877.

[21] H.E. Hurst, Long terms storage of reservoirs, *Transaction of the American Society of Civil Engineers* **116** (1951), 770–808.

[22] J.P. Imhoff, On the range of Brownian motion and its inverse process, *Ann. Prob.* **13**(3) (1985), 1011–1017.

[23] P. Levy, *Théorie de l'addition des variables aléatoires*, Gauthier-Villars, Paris, 1937.

[24] P. Levy, Wiener random functions and other Laplacian random functions, in: *Proc. 6th Berkeley Symp. Prob. Theor. Math. Stat.*, Berkeley, CA, 1951, pp. 171–186.

[25] A.W. Lo, Long term memory in stock market prices, *Econometrica* **59**(5) (1991), 1279–1313.

[26] A.W. Lo, Fat tails, long memory and the stock market since 1960's, *Economic Notes* **26** (1997), 213–245.

[27] B. Mandelbrot, When can price be arbitraged efficiently? A limit to the validity of the random walk and Martingale models, *Review of Economics and Statistics* **53** (1971), 225–236.

[28] B. Mandelbrot, Statistical methodology for non-periodic cycles: from the covariance to R/S analysis, *Annals of Economic and Social Measurement* **1** (1972), 259–290.

[29] B. Mandelbrot, Three fractal models in finance: Discontinuity, concentration, risk, *Economic Notes* **26** (1997), 171–212.

[30] B. Mandelbrot and M. Taqqu, Robust R/S analysis of long run serial correlation, *Bulletin of the International Statistical Institute* **48** (1979), 59–104 (Book 2).

[31] B. Mandelbrot and J. Van Ness, Fractional Brownian motion, fractioanl noises and applications, *SIAM Review* **10** (1968), 422–437.

[32] B. Mandelbrot and J. Wallis, Noah, Joseph and operational hydrology, *Water Resources Research* **4** (1968), 909–918.

[33] B. Mandelbrot and J.R. Wallis, Computer experiments with fractional noises, *Water Resources Res.* **5** (1969), 228–267.

[34] A. Medio, *Chaotic Dynamics*, Cambridge University Press, Cambridge, 1992.

[35] T.H. Otway, Records of the Florentine proveditori degli cambiatori: An example of an antipersistent time series in economics, *Chaos, Solitons and Fractals* **5** (1995), 103–107.

[36] E.E. Peter, *Chaos and Order in Capital Markets*, Wiley, New York, NY, 1995.

[37] Z. Shi and M. Yor, On an identity in law for the variance of the Brownian bridge, Prepublication no. 283 of the Laboratoire des Probabilités de l'Université de Paris VI, May, 1995.

[38] C.S. Tapiero, *Applied Stochastic Models and Control for Finance and Insurance*, Kluwer Academic, Boston, MA, April, 1998.

[39] C.S. Tapiero and P. Vallois, Run length statistics and the Hurst exponent in random and birth–death random walks, *Chaos, Solitons and Fractals* **7**(9) (1996), 1333–1341.

[40] P. Vallois, Diffusion arrêtée au premier instant où le processus de l'amplitude atteint un niveau donné, *Stochastics and Stochastic Reports* **43** (1993), 93–115.

[41] P. Vallois and C.S. Tapiero, The range process in random walks: Theoretical results and applications, in: *Advances in Computational Economics*, H. Ammans, B. Rustem and A. Whinston, eds, Kluwer Publications, Boston, MA, 1996.

[42] P. Vallois and C.S. Tapiero, Range reliability in random walks, *Mathematical Methods of Operational Research* **45** (1997), 325–345,

[43] P. Vallois and C.S. Tapiero, Memory-Based persistence in a counting random walk process, *Physica A* **386** (2007), 303–317.

[44] P. Vallois and C.S. Tapiero, A claims persistence process and insurance, *Insurance Economics and Mathematics* (2009), forthcoming.