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Application of neutrosophic optimal network using operations

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Abstract. Neutrosophic graphs deals with more complex, uncertain problems in real-life applications which provides more flexibility and compatibility than Intuitionistic fuzzy graphs. The aim of this paper is to enrich the efficiency of the network in accordance with productivity and quality. Here we develop two Neutrosophic graphs into a fully connected Neutrosophic network using the product of graphs. Such a type of network is formed from individuals with unique aspects in every field of work among them. This study proposes extending the other graph products and forming a single valued Neutrosophic graph to find the efficient productivity in the flow of information on a single source network of a single valued Neutrosophic network. An Optimal algorithm is proposed and illustrated with an application.

Keywords: Neutrosophic graph, graph operation, domination number, optimal network, score function

1. Introduction

Graph Theory, a convenient mathematical tool has a broad spectrum of uses in various fields of Science and Technology. The graph is usually a graphical representation of practical, real-world problems. A graph is a collection of sets ($\mathbb{V}$, $\mathbb{E}$) where $\mathbb{V}$ is a non-empty set of vertices connected by $\mathbb{E}$, whose constituents are edges or links. Representing a problem as a graph provides a significant perspective and clarifies the situation. A network is typically a graph model with a set of nodes connected by edges or links. The network gives us a flexible framework for identifying and observing complex systems. The study of complex networks is a crucial concept that comprises several disciplines. Complex systems network theory provides techniques for analyzing systems of interaction structure, represented as networks [10]. These networks are generally defined by simple graphs that consist of vertex representing the objects under exploration, that are linked together by edges if there exists a relationship between them. Lofti Zadeh [19] introduced a novel concept of fuzzy set theory in 1965 to model real-world problems that are efficient, which are generally uncertain. Fuzzy is an upper version of the crisp set with varying membership value grades between [0, 1]. The membership value is a specific

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single value that lies between zero and one. To deal with uncertainties of complex problems, with single membership grade is important. Atanassov [5] extended the fuzzy set to an Intuitionistic fuzzy set by including a non-membership value. Smarandache [3] has presented the idea of a Neutrosophic set to capture problems that are uncertain, imprecise, and vague. A Neutrosophic set is an extension of the crisp set, fuzzy set, and Intuitionistic fuzzy set which has three different types of membership values such as truth, indeterminacy, and falsity values that are not dependent on each other and lies between [0, 1].

There exists many different of information in real-world problems that can be modeled using several types of graphs such as fuzzy graphs, Intuitionistic fuzzy graphs and Neutrosophic graphs [13, 18]. Shannon and Atanassov introduced the concept of Intuitionistic fuzzy graphs [14]. Parvathi et al. [9] proposed some operations between two Intuitionistic fuzzy graphs. Rashmanlou et al. [13] proposed graph operations such as Direct product, semi-strong product, strong product, and Lexicographic on Intuitionistic fuzzy graphs. Mahapatra et al. [15] introduced the fuzzy fractional chromatic number for calculating lexicographic product on two fuzzy graphs, also investigated m-polar fuzzy graphs and their applications [16, 17]. Neutrosophic graphs are used to model real-world problems which consist of in-consistent information. Many Scientists such as Broumi et al. [7], Yang et al. [1] and Akram [3, 4] have researched under a Neutrosophic environment. Single-valued Neutrosophic sets introduced by Haibin Wang are a subclass of Neutrosophic sets that are independent of membership values ranging from [0, 1]. Related work in the extension of the single-valued Neutrosophic set is found in [1, 6].

The main motivation of this research work is to find the most efficient optical network using different operations on single-valued Neutrosophic Graphs-(SVNG) such as Lexicographic, Symmetric difference, Residue product, and Max product based on the domination parameter presented. Further extended our study on its applications and finding the effective minimal spanning tree. In section 2 the motivation and research background is listed with preliminaries for the study. In section 3 we define the different types of operations such as Lexicographic, Symmetric difference, Residue product, Max product and examine the efficiency of the network using the score function. In section 4 the optimal network of symmetric difference is identified and its applications are given for better sales training technology.

2. Preliminaries

Definition 2.1. [2] Let $\mathbb{X}$ be a universe of discourse. A single-valued Neutrosophic set N is defined on $\mathbb{X}$ is given by

$$N = \{(x, T_N(x), I_N(x), F_N(x) : x \in \mathbb{X}\}$$

where

$$T_N(x) : \mathbb{X} \to [0, 1]$$
$$I_N(x) : \mathbb{X} \to [0, 1] \quad \text{and}$$
$$F_N(x) : \mathbb{X} \to [0, 1]$$

are called the degree of truth membership value, degree of indeterminancy value and degree of falsity membership value of x on N, respectively satisfying the condition

$$0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3, \quad \forall\, x \in \mathbb{X}.$$

Definition 2.2. [2] Let $N_G = (\mathbb{V}_G, \mathbb{E}_G)$ be a graph, where $\mathbb{V}_G$ be the set of vertices and $\mathbb{E}_G$ be the set of edges. Then the single-valued Neutrosophic graph of N_G is denoted by $\mathbb{N}_G^{'} = (\mathbb{V}_G, \sigma, \mu)$ where $\sigma = (T_\sigma, I_\sigma, F_\sigma)$ is a single-valued Neutrosophic set on $\mathbb{V}_G$ and $\mu = (T_\mu, I_\mu, F_\mu)$ is a single-valued Neutrosophic symmetric relation on $\mathbb{E}_G \subseteq V_G \times V_G$ is defined as follows:

i) $T_\mu(x, y) \leq T_\sigma(x) \wedge T_\sigma(y), \quad \forall (x, y) \in V_G \times V_G$.
ii) $I_\mu(x, y) \leq I_\sigma(x) \wedge I_\sigma(y), \quad \forall\ (x, y) \in V_G \times V_G$.
iii) $F_\mu(x, y) \geq F_\sigma(x) \vee F_\sigma(y), \quad \forall\ (x, y) \in V_G \times V_G$.

Definition 2.3. [8] The SVNG $\mathbb{N}'_G$ is called strong single-valued Neutrosophic graph if $\forall\, (x, y) \in \mathbb{E}_G$,

$$T_{\mu_{\mathbb{N}'_G}}(x, y) = T_{\sigma_{\mathbb{N}'_G}}(x) \wedge T_{\sigma_{\mathbb{N}'_G}}(y),$$
$$I_{\mu_{\mathbb{N}'_G}}(x, y) = I_{\sigma_{\mathbb{N}'_G}}(x) \wedge I_{\sigma_{\mathbb{N}'_G}}(y)$$
$$F_{\mu_{\mathbb{N}'_G}}(x, y) = F_{\sigma_{\mathbb{N}'_G}}(x) \vee F_{\sigma_{\mathbb{N}'_G}}(y)$$

Definition 2.4. [8] The SVNG $\mathbb{N}'_G$ is said to be complete if $\forall\, x, y \in \mathbb{V}$,

$$T_{\mu_{\mathbb{N}'_G}}(x, y) = T_{\sigma_{\mathbb{N}'_G}}(x) \wedge T_{\sigma_{\mathbb{N}'_G}}(y)$$
$$I_{\mu_{\mathbb{N}'_G}}(x, y) = I_{\sigma_{\mathbb{N}'_G}}(x) \wedge I_{\sigma_{\mathbb{N}'_G}}(y)$$
$$F_{\mu_{\mathbb{N}'_G}}(x, y) = F_{\sigma_{\mathbb{N}'_G}}(x) \vee F_{\sigma_{\mathbb{N}'_G}}(y).$$

Definition 2.5. [8] Let u_d be a vertex in a $\mathbb{N}_G$, $G_{\mathbb{N}_G} = (\mathbb{A}_N, \mathbb{B}_N)$. The degree of a vertex u_d is defined as the sum of the weight of the strong arcs incident at u_d and is denoted by $deg(u_d)$. The neighborhood of u_d is denoted by $\mathbb{N}_G = \{v_d \in \mathbb{A}_N/(u_d, v_d)\}$ is a strong arc.
The minimum degree of $G_{\mathbb{N}_G}$ is
$\delta(G_{\mathbb{N}_G}) = \min\{d_{G_{\mathbb{N}_G}}(u_d)/u_d \in \mathbb{A}_N\}$.
The maximum degree of $G_{\mathbb{N}_G}$ is
$\Delta(G_{\mathbb{N}_G}) = \max\{d_{G_{\mathbb{N}_G}}(u_d)/u_d \in \mathbb{A}_N\}$.

Definition 2.6. [12] The cardinality of a vertex $v_i \in \mathbb{V}$ in a SVNG, $\mathbb{N}_G = (\mathbb{A}, \mathbb{B})$ is defined by

$$|\mathbb{V}_i| = T_{\mathbb{A}}(v_i) + I_{\mathbb{A}}(v_i) + F_{\mathbb{A}}(v_i).$$

The cardinality of an edge $v_i v_j \in \mathbb{E}$ in a SVNG, $\mathbb{N}_G = (\mathbb{A}, \mathbb{B})$ is defined by

$$|v_i v_j| = T_{\mathbb{B}}(v_i v_j) + I_{\mathbb{B}}(v_i v_j) + F_{\mathbb{B}}(v_i v_j).$$

Definition 2.7. [2] Let $\mathbb{C}P_1 = (\mathbb{A}_{\mathbb{C}P_1}, \mathbb{B}_{\mathbb{C}P_1})$ and $\mathbb{C}P_2 = (\mathbb{A}_{\mathbb{C}P_2}, \mathbb{B}_{\mathbb{C}P_2})$ be the two SVNG of $\mathbb{G}_1 = (\mathbb{V}_1, \mathbb{E}_1)$ and $\mathbb{G}_2 = (\mathbb{V}_2, \mathbb{E}_2)$ respectively.

The cartesian product $\mathbb{C}P_1 \times \mathbb{C}P_2$ is defined such that

i)

$$T_{\mathbb{A}}(x_1, x_2) = \min\left(T_{\mathbb{A}_{\mathbb{C}P_1}}(x_1), T_{\mathbb{A}_{\mathbb{C}P_2}}(x_2)\right);$$

$$I_{\mathbb{A}}(x_1, x_2) = \min\left(I_{\mathbb{A}_{\mathbb{C}P_1}}(x_1), I_{\mathbb{A}_{\mathbb{C}P_2}}(x_2)\right);$$

$$F_{\mathbb{A}}(x_1, x_2) = \max\left(F_{\mathbb{A}_{\mathbb{C}P_1}}(x_1), F_{\mathbb{A}_{\mathbb{C}P_2}}(x_2)\right),$$

$$\forall\,(x_1, x_2) \in \mathbb{V}_1 \times \mathbb{V}_2.$$

ii)

$$T_{\mathbb{B}}\left((x, x_2)(x, y_2)\right) = \min\left(T_{\mathbb{A}_{\mathbb{C}P_1}(x)}, T_{\mathbb{B}_{\mathbb{C}P_2(x)}}\right);$$

$$I_{\mathbb{B}}\left((x, x_2)(x, y_2)\right) = \min\left(I_{\mathbb{A}_{\mathbb{C}P_1}(x)}, I_{\mathbb{B}_{\mathbb{C}P_2}(x_2, y_2)}\right);$$

$$F_{\mathbb{B}}\left((x, x_2)(x, y_2)\right) = \max\left(F_{\mathbb{A}_{\mathbb{C}P_1}(x)}, F_{\mathbb{B}_{\mathbb{C}P_2}(x_2 y_2)}\right),$$

$$\forall\, x \in \mathbb{V}_1,\ x_2 y_2 \in \mathbb{E}_2.$$

iii)

$$T_{\mathbb{B}}\left((x_1, z)(y_1, z)\right) = \min\left(T_{\mathbb{B}_{\mathbb{C}P_1}(x_1 y_1)}, T_{\mathbb{A}_{\mathbb{C}P_2}(z)}\right)$$

$$I_{\mathbb{B}}\left((x_1, z)(y_1, z)\right) = \min\left(I_{\mathbb{B}_{\mathbb{C}P_1}(x_1, y_1)}, I_{\mathbb{A}_{\mathbb{C}P_2}(z)}\right)$$

$$F_{\mathbb{B}}\left((x_1, z)(y_1, z)\right) = \max\left(F_{\mathbb{B}_{\mathbb{C}P_1}(x_1 y_1)}, F_{\mathbb{A}_{\mathbb{C}P_2}(z)}\right),$$

$$\forall\, z \in \mathbb{V}_2 \text{ and } x_1 y_1 \in \mathbb{E}_1.$$

The cartesian product $\mathbb{G}_1 \times \mathbb{G}_2$ of two graphs $\mathbb{G}_1$ and $\mathbb{G}_2$ is denoted by $\mathbb{V}(\mathbb{G}_1) \times \mathbb{V}(\mathbb{G}_2)$ such that two vertices $(\mathbb{V}_1, \mathbb{V}_2)$ and $(\mathbb{V}_1', \mathbb{V}_2')$ are adjacent in $\mathbb{G}_1 \times \mathbb{G}_2$ iff

1) $\mathbb{V}_1 = \mathbb{V}_1'$ and $\mathbb{V}_2$ is adjacent to $\mathbb{V}_2'$ in $\mathbb{G}_2$.
2) $\mathbb{V}_2 = \mathbb{V}_2'$ and $\mathbb{V}_1$ is adjacent to $\mathbb{V}_1'$ in $\mathbb{G}_1$.

Definition 2.8. [2] The Lexicographic product $LP_1 \cdot LP_2$ of two graphs $LP_1 = (M_1, N_1)$ and $LP_2 = (M_2, N_2)$ is such that

i) The vertex set of $LP_1 \cdot LP_2$ is the cartesian product $V(LP_1) \times V(LP_2)$.
ii) Any two vertices (m_1, n_1) and (m_2, n_2) are adjacent in $LP_1 \cdot LP_2$ iff either m_1 is adjacent to m_2 in LP_1 or $m_1 = m_2$ and n_1 is adjacent to n_1 in LP_2

Definition 2.9. [8] The residue product of $RP_1 \cdot RP_2$ two graphs RP_1 and RP_2 is defined as
$\sigma_{RP_1 \cdot RP_2}(u_1, v_1) = \sigma_{RP_1}(u_1) \vee \sigma_{RP_2}(v_1)$
and $\mu_{RP_1 \cdot RP_2}((u_1, v_1)(u_2, v_2)) = \mu_{RP_1}(u_1 u_2)$,
$\forall\,(u_1, v_1) \in \mathbb{V}$ and $(u_1, v_1)(u_2, v_2) \in \mathbb{E}$.
If $u_1 u_2 \in \mathbb{E}_1 and v_1 \neq v_2$ then,
$\mu_{RP_1 \cdot RP_2}((u_1, v_1)(u_2, v_2)) \leq \sigma_{RP1(u_1, v_1)} \wedge \sigma_{RP2(u_2, v_2)}$.

Definition 2.10. [2] Let $SD_1 = (\sigma_1, \mu_1)$ and $SD_2 = (\sigma_2, \mu_2)$ be two SVNGs of the graphs $G_{SD_1} = (V_1, E_1)$ and $G_{SD_2} = (V_2, E_2)$ respectively. Then the symmetric difference of SD_1 and SD_2 is defined as

$$SD_1 \oplus SD_2$$

$$= (\sigma_1 \oplus \sigma_2, \mu_1 \oplus \mu_2) \text{ is defined as follows}$$

i) $\forall\,(x, y) \in V_1 \times V_2$.

$$T_{\sigma_{SD_1}} \oplus T_{\sigma_{SD_2}}(x, y) = T_{\sigma_{SD_1}}(x) \wedge T_{\sigma_{SD_2}}(y);$$

$$I_{\sigma_{SD_1}} \oplus I_{\sigma_{SD_2}}(x, y) = I_{\sigma_{SD_1}}(x) \wedge I_{\sigma_{SD_2}}(y) \quad \text{and}$$

$$F_{\sigma_{SD_1}} \oplus F_{\sigma_{SD_2}}(x, y) = T_{\sigma_{SD_1}}(x) \vee F_{\sigma_{SD_2}}(y);$$

ii) $\forall\, x \in V_1$ and $(y, z) \in E_2$,

$$(T_{\mu_{SD_1}} \oplus T_{\mu_{SD_2}})((x, y), (x, z))$$

$$= T_{\sigma_{SD_1}}(x) \wedge T_{\mu_{SD_2}}(y, z);$$

$$(I_{\mu_{SD_1}} \oplus I_{\mu_{SD_2}})((x, y), (x, z))$$

$= I_{\sigma_{SD_1}}(x) \wedge I_{\mu_{SD_2}}(y, z);$

$(F_{\mu_{SD_1}} \oplus F_{\mu_{SD_2}})((x, y), (x, z))$

$= F_{\sigma_{SD_1}}(x) \vee F_{\mu_{SD_2}}(y, z);$

iii) $\forall\, x \in V_2$ and $(y, z) \in E_1$,

$(T_{\mu_{SD_1}} \oplus T_{\mu_{SD_2}})((y, x), (z, x))$

$= T_{\mu_{SD_1}}(y, z) \wedge T_{\sigma_{SD_2}}(x);$

$(I_{\mu_{SD_1}} \oplus I_{\mu_{SD_2}})((y, x), (z, x))$

$= I_{\mu_{SD_1}}(y, z) \wedge I_{\sigma_{SD_2}}(x);$

$(F_{\mu_{SD_1}} \oplus F_{\mu_{SD_2}})((y, x), (z, x))$

$= F_{\mu_{SD_1}}(y, z) \vee F_{\sigma_{SD_2}}(x);$

iv) $\forall\, (x, y) \notin E_1$ and $(z, w) \in E_2$,

$(T_{\mu_{SD_1}} \oplus T_{\mu_{SD_2}})((x, z), (y, w))$

$= T_{\sigma_{SD_1}}(x) \wedge T_{\sigma_{SD_1}}(y) \wedge T_{\mu_{SD_2}}(z, w);$

$(I_{\mu_{SD_1}} \oplus I_{\mu_{SD_2}})((x, z), (y, w))$

$= I_{\sigma_{SD_1}}(x) \wedge I_{\sigma_{SD_1}}(y) \wedge I_{\mu_{SD_2}}(z, w);$

$(F_{\mu_{SD_1}} \oplus F_{\mu_{SD_2}})((x, z), (y, w))$

$= F_{\sigma_{SD_1}}(x) \wedge F_{\sigma_{SD_1}}(y) \wedge F_{\mu_{SD_2}}(z, w);$

v) $\forall\, (x, y) \in E_1$ and $(z, w) \notin E_2$,

$(T_{\mu_{SD_1}} \oplus T_{\mu_{SD_2}})((x, z), (y, w))$

$= T_{\mu_{SD_1}}(x, y) \wedge T_{\sigma_{SD_2}}(z) \wedge T_{\sigma_{SD_2}}(w);$

$(I_{\mu_{SD_1}} \oplus I_{\mu_{SD_2}})((x, z), (y, w))$

$= I_{\mu_{SD_1}}(x, y) \wedge I_{\sigma_{SD_2}}(z) \wedge I_{\sigma_{SD_2}}(w);$

$(F_{\mu_{SD_1}} \oplus F_{\mu_{SD_2}})((x, z), (y, w))$

$= F_{\mu_{SD_1}}(x, y) \vee F_{\sigma_{SD_2}}(z) \vee F_{\sigma_{SD_2}}(w).$

Definition 2.11. [8] Let
$MP_1 = ((\sigma_{mp_1}, \sigma_{mp_2}, (\mu_{mp_1}, \mu_{mp_2})),$
$MP_2 = ((\sigma'_{mp_1}, \sigma'_{mp_2}), (\mu'_{mp_1}, \mu'_{mp_2}))$ be two Intuitionistic fuzzy graph.
The Max product of two Intuitionistic fuzzy graph MP_1, MP_2 and is denoted by
$MP_1 * MP_2\ (V''_1 \times_M V''_2, E''_1 \times_M E''_2)$
where $E''_1 \times_M E''_2 = \{(p''_1, q''_1)((p''_2, q''_2)/\ \ p''_1 = p''_2;$ $q''_1 q''_2 \in E''_2$ or $q''_1 = q''_2;\ p''_1 p''_2 \in E''_1\}$
$\sigma^{MP_1 * MP_2}_{mp_1}(p''_1, q''_1) = \sigma_{mp_1}(p''_1) \vee \sigma'_{mp_1}(q''_1)$ for all $(p_1, q_1) \in (V''_1 \times_M V''_2)$
$\sigma^{MP_1 * MP_2}_{mp_2}(p''_1, q''_1) = \sigma_{mp_2}(p''_1) \wedge \sigma'^{(}_{mp_2} q''_1)$ for all $(p''_1, q''_1) \in (V''_1 \times_M V''_2)$ and
$\mu'^{MP_1 * MP_2}_{mp_1}((p''_1, q''_1)(p''_2, q''_2)) = \{\sigma_{mp_1}(p''_1) \vee$
$\sigma_{mp_1}(q''_1)$ if $p''_1 = p''_2;\ q''_1 q''_2 \in E''_2,$
$\sigma_{mp_1}(p''_1) \vee \mu'_{mp_1}(q''_1 q''_2)$ if $q''_1 = q''_2;\ p''_1 p''_2 \in E_{1''}$
$\mu'^{MP_1 * MP_2}_{mp_2}((p''_1, q''_1)(p''_2, q''_2)) = \{\sigma_{mp_2}(p''_1) \wedge$
$\sigma_{mp_2}(q''_1)$ if $p''_1 = p''_2; q''_1 q''_2 \in E''_2$
$\sigma_{mp_2}(p''_1) \wedge \mu'_{mp_2}(q''_1 q''_2)$ if $q''_1 = q''_2;\ p''_1 p''_2 \in E''_1\}$

Definition 2.12. [12] In a SVNG, $N_G = (A, B)$, the domination number is defined by the minimum cardinality among all the minimal dominating set of N_G and it is denoted by $\gamma_{SVN}(N_G)$.

Definition 2.13. [20] Let $A = (\mathbb{T}, \mathbb{I}, \mathbb{F})$ be a SVNG, then the score function S is defined as follows

$$S(A) = \frac{2 + T - I - F}{3}$$

Let $\mathbb{G}_1 = (\mathcal{V}_1, \mathcal{E}_1)$ and $\mathbb{G}_2 = (\mathcal{V}_2, \mathcal{E}_2)$ be two SVNG's, while applying any operation '*' on the SVNG's $\mathbb{G}_1$ and $\mathbb{G}_2$ such that the required SVNG $\mathbb{G}_1 * \mathbb{G}_2$ contains the vertex set $(\mathcal{V}_1 * \mathcal{V}_2)$, cartesian product of $\mathcal{V}_1$ and $\mathcal{V}_2$. Selecting any such operation, which satifies the above condition will be applied on any two SVNG's and it can be constructed into a network. In this paper, we present such different operations on SVNG like lexicographic, symmetric difference, maximal product, and residue product are presented with appropriate examples. We have modeled a real-life problem for selecting the best optimal network using SVNG and those operators are used to find the most efficient one.

The operations on single-valued Neutrosophic Graphs (SVNG) such as Lexicographic, Symmetric difference, Residue product and Max product are studied from [2, 6, 8].

3. Domination on operations of single-valued Neutrosophic graphs

3.1. Lexicographic product of two single-valued Neutrosophic graphs

3.1.1. Definition

Let $LP_1 = (M_1, N_1)$ and $LP_2 = (M_2, N_2)$ be two single-valued Neutrosophic networks of the graphs $G_{LP_1} = (V_1, E_1)$ and $G_{LP_2} = (V_2, E_2)$ respectively. The Lexicographic product graph is denoted as $\mathbb{LP}_1 \cdot \mathbb{LP}_2$ is the pair (M, N) of single-valued Neutrosophic graph such that

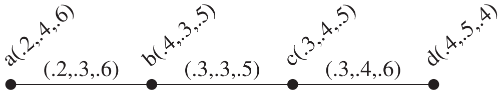

Fig. 1. SVNG $\mathbb{G}_1$.

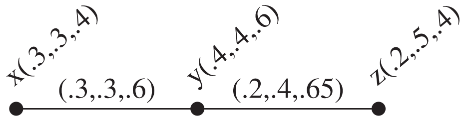

Fig. 2. SVNG $\mathbb{G}_2$.

i)

$$T_M(x_1, x_2) = \min\left(T_{M_1}(x_1), T_{M_2}(x_2)\right)$$
$$I_M(x_1, x_2) = \min\left(I_{M_1}(x_1), I_{M_2}(x_2)\right)$$
$$F_M(x_1, x_2) = \max\left(F_{M_1}(x_1), F_{M_2}(x_2)\right)$$
$$\forall (x_1, x_2) \in M_1 \times M_2.$$

ii)

$$T_N((x, x_2)(x, y_2))$$
$$= \min\left(T_{M_1}(x), T_{M_2}(x_2y_2)\right)$$
$$I_N((x, x_2)(x, y_2))$$
$$= \min\left(I_{M_1}(x), I_{N_2(x_2y_2)}\right)$$
$$F_N((x, x_2)(x, y_2))$$
$$= \max\left(F_{M_1}(x), F_{N_2}(x_2y_2)\right)$$
$$\forall x \in M_1, \quad x_2y_2 \in N_2.$$

iii)

$$T_N((x_1, x_2)(y_1, y_2))$$
$$= \min\left(T_{N_1}(x_1y_1), T_{N_2}(x_2y_2)\right)$$
$$I_N((x_1, x_2)(y_1, y_2))$$
$$= \min\left(I_{N_1}(x_1y_1), I_{N_2}(x_2y_2)\right)$$
$$F_N((x_1, x_2)(y_1, y_2))$$
$$= \max\left(F_{N_1}(x_1y_1), F_{N_2}(x_2y_2)\right),$$
$$\forall x_1y_1 \in N_1 \text{ and } x_2y_2 \in N_2.$$

3.1.2. Example

Let SVNG $\mathbb{G}_1$ and SVNG $\mathbb{G}_2$ be two Lexicographic SVNN shown in Fig 1 and Fig 2 of the graphs $G_{LP_1} = (V_1, E_1)$ and $G_{LP_2} = (V_2, E_2)$ respectively. The Lexicographic product of single-valued Neutrosophic network $\mathbb{LP}_1 \cdot \mathbb{LP}_2$ is shown in Fig. 3.

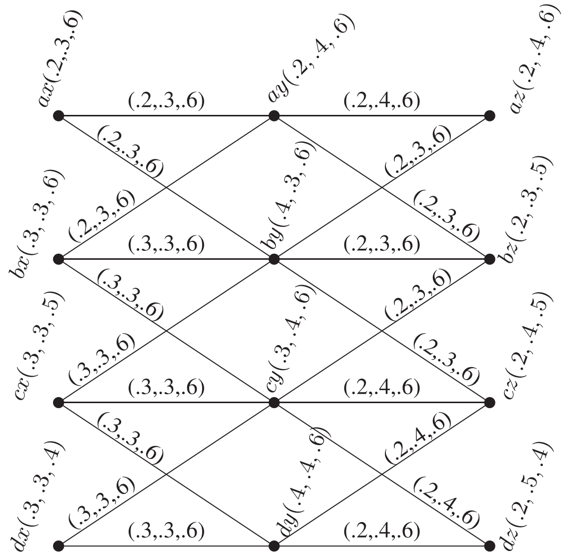

Fig. 3. $\mathbb{LP}_1 \cdot \mathbb{LP}_2$.

To analyze the optimal network from the constructed network, we define an efficient score function to find the minimum domination number of the weighted SVNG network. The score function defined by us is more efficient than the existing score function defined in 2.13 since, Indeterminacy value (I) does not depend on both Truth (T) and Falsity (F) value because I is not a complement of T and F and the values of T, I, F are independent of each other. Even though the value of indeterminacy is uncertain, we assume it by taking 0.5 as both chances of truth and falsity which makes our work the significant advantage of defining efficient networks.

Hence, we define the Edge score function (ESF) and Vertex score function (VSF) of a single-valued Neutrosophic graph to find the minimum weight of the spanning tree as follows:

$$ESF = \frac{2 + T_\mu(x, y) - (0.5)I_\mu(x, y) - F_\mu(x, y)}{3}$$

$$VSF = \frac{2 + T_\sigma(x) - (0.5)I_\sigma(x) - F_\sigma(x)}{3}$$

The weighted $\mathbb{LP}_1 \cdot \mathbb{LP}_2$ represented in Fig 4, and it's minimal dominating sets are as follows;

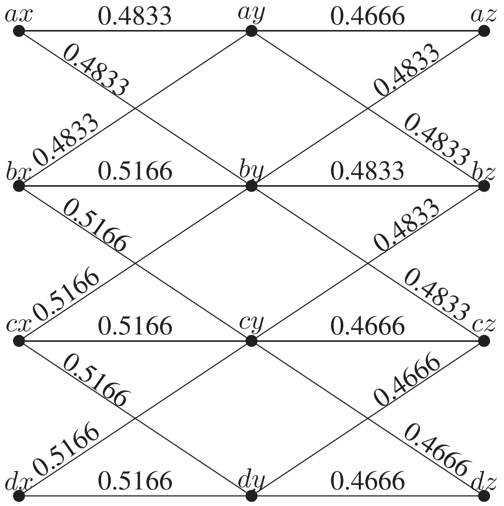

Fig. 4. $\mathbb{LP}_1 \cdot \mathbb{LP}_2$ with minimum score function.

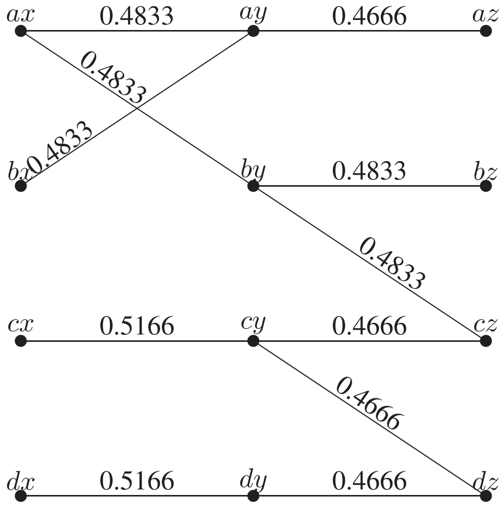

Fig. 5. Minimum Spanning Tree of $\mathbb{LP}_1 \cdot \mathbb{LP}_2$.

$$S_1 = \{ay, cz, dy\}$$

$$S_2 = \{ay, bz, dy\}$$

$$S_3 = \{ay, bx, dy\}$$

$$S_4 = \{ay, cz, dy\}$$

$$S_5 = \{by, cy, dy, ay\}.$$

The vertex cardinality of the members of above dominating sets are $ay = 0.4666$, $cz = 0.5$, $dy = 0.5333$, $bz = 0.52$, $bx = 0.5166$, $by = 0.55$, $cy = 0.5$.

The domination number of the dominating sets S_1, S_2, S_3, S_4, S_5 are

$$S_1(\mathbb{LP}_1 \cdot \mathbb{LP}_2) = 0.4666 + 0.5 + 0.5333 = 1.4999$$

$$S_2(\mathbb{LP}_1 \cdot \mathbb{LP}_2) = 0.4666 + 0.5166 + 0.5333 = 1.5165$$

$$S_3(\mathbb{LP}_1 \cdot \mathbb{LP}_2) = 0.4666 + 0.55 + 0.5333 = 1.5495$$

$$S_4(\mathbb{LP}_1 \cdot \mathbb{LP}_2) = 0.4666 + 0.5 + 0.5333 = 1.4999$$

$$S_5(\mathbb{LP}_1 \cdot \mathbb{LP}_2) = 0.55 + 0.5 + 0.5333 + 0.4666 = 2.049$$

The domination number of $\mathbb{LP}_1 \cdot \mathbb{LP}_2$ is 1.4999 which is obtained from the dominating sets S_1 and S_4.

The minimal spanning tree of the weighted network $\mathbb{LP}_1 \cdot \mathbb{LP}_2$ is found using the Kruskal's algorithm is shown in Fig 5 and hence, its minimum weight of spanning tree is 5.1996.

3.2. Symmetric difference of two single-valued Neutrosophic graphs

Let $SD_1 = (\sigma_1, \mu_1)$ and $SD_2 = (\sigma_2, \mu_2)$ be two SVNGs of the graphs $G_{SD_1} = (V_1, E_1)$ & $G_{SD_2} = (V_2, E_2)$ respectively. Then the symmetric difference of SD_1 & SD_2 is defined and denoted as

$$\mathbb{SD}_1 \oplus \mathbb{SD}_2 = (\sigma_1 \oplus \sigma_2, \mu_1 \oplus \mu_2) \text{ is defined as follows}$$

i) $\forall\, (x, y) \in V_1 \times V_2$.

$$T_{\sigma_{SD_1}} \oplus T_{\sigma_{SD_2}}(x, y) = T_{\sigma_{SD_1}}(x) \wedge T_{\sigma_{SD_2}}(y),$$

$$I_{\sigma_{SD_1}} \oplus I_{\sigma_{SD_2}}(x, y) = I_{\sigma_{SD_1}}(x) \wedge I_{\sigma_{SD_2}}(y) \quad \text{and}$$

$$F_{\sigma_{SD_1}} \oplus F_{\sigma_{SD_2}}(x, y) = T_{\sigma_{SD_1}}(x) \vee F_{\sigma_{SD_2}}(y);$$

ii) $\forall\, x \in V_1$ and $(y, z) \in E_2$,

$$(T_{\mu_{SD_1}} \oplus T_{\mu_{SD_2}})((x, y), (x, z)) = T_{\sigma_{SD_1}}(x) \wedge T_{\mu_{SD_2}}(y, z);$$

$$(I_{\mu_{SD_1}} \oplus I_{\mu_{SD_2}})((x, y), (x, z)) = I_{\sigma_{SD_1}}(x) \wedge I_{\mu_{SD_2}}(y, z);$$

$(F_{\mu_{SD_1}} \oplus F_{\mu_{SD_2}})((x, y), (x, z)) = F_{\sigma_{SD_1}}(x) \vee F_{\mu_{SD_2}}(y, z);$

iii) $\forall\, x \in V_2$ and $(y, z) \in E_1$,

$$(T_{\mu_{SD_1}} \oplus T_{\mu_{SD_2}})((y, x), (z, x)) = T_{\mu_{SD_1}}(y, z) \wedge T_{\sigma_{SD_2}}(x);$$

$$(I_{\mu_{SD_1}} \oplus I_{\mu_{SD_2}})((y, x), (z, x)) = I_{\mu_{SD_1}}(y, z) \wedge I_{\sigma_{SD_2}}(x);$$

$$(F_{\mu_{SD_1}} \oplus F_{\mu_{SD_2}})((y, x), (z, x)) = F_{\mu_{SD_1}}(y, z) \vee F_{\sigma_{SD_2}}(x);$$

iv) $\forall\, (x, y) \notin E_1$ and $(z, w) \in E_2$,

$$(T_{\mu_{SD_1}} \oplus T_{\mu_{SD_2}})((x, z), (y, w)) = \min\{T_{\sigma_{SD_1}}(x), T_{\sigma_{SD_1}}(y), T_{\mu_{SD_2}}(z, w)\};$$

$$(I_{\mu_{SD_1}} \oplus I_{\mu_{SD_2}})((x, z), (y, w)) = \min\{I_{\sigma_{SD_1}}(x), I_{\sigma_{SD_1}}(y), I_{\mu_{SD_2}}(z, w)\};$$

$$(F_{\mu_{SD_1}} \oplus F_{\mu_{SD_2}})((x, z), (y, w)) = \max\{F_{\sigma_{SD_1}}(x), F_{\sigma_{SD_1}}(y), F_{\mu_{SD_2}}(z, w)\};$$

v) $\forall\, (x, y) \in E_1$ and $(z, w) \notin E_2$,

a) $$(T_{\mu_{SD_1}} \oplus T_{\mu_{SD_2}})((x, z), (y, w)) = \min\{T_{SD\mu_1}(x, y), T_{\sigma_{SD_2}}(z), T_{\sigma_{SD_2}}(w)\};$$

$$(I_{SD\mu_1} \oplus I_{\mu_{SD_2}})((x, z), (y, w)) = \min\{I_{SD\mu_1}(x, y), I_{\sigma_{SD_2}}(z), I_{\sigma_{SD_2}}(w)\};$$

$$(F_{SD\mu_1} \oplus F_{\mu_{SD_2}})((x, z), (y, w)) = \max\{F_{SD\mu_1}(x, y), F_{\sigma_{SD_2}}(z), F_{\sigma_{SD_2}}(w)\}.$$

3.2.1. Example

Let SVNG $\mathbb{G}_1$ and SVNG $\mathbb{G}_2$ be two Symmetric difference SVNN shown in Fig 1 and Fig 2 of the graphs $G_{SD_1} = (V_1, E_1)$ and $G_{SD_2} = (V_2, E_2)$ respectively. The Symmetric difference of single-valued Neutrosophic network $\mathbb{SD}_1 \oplus \mathbb{SD}_2$ is shown in Fig. 6.

The weighted $\mathbb{SD}_1 \oplus \mathbb{SD}_2$ is represented in Fig 7, and its corresponding dominating sets are as follows;

$$S_1 = \{ay, cz, dy\}$$

$$S_2 = \{bx, by, dy\}$$

$$S_3 = \{cx, cy, ay\}$$

$$S_4 = \{ay, cz, dx\}.$$

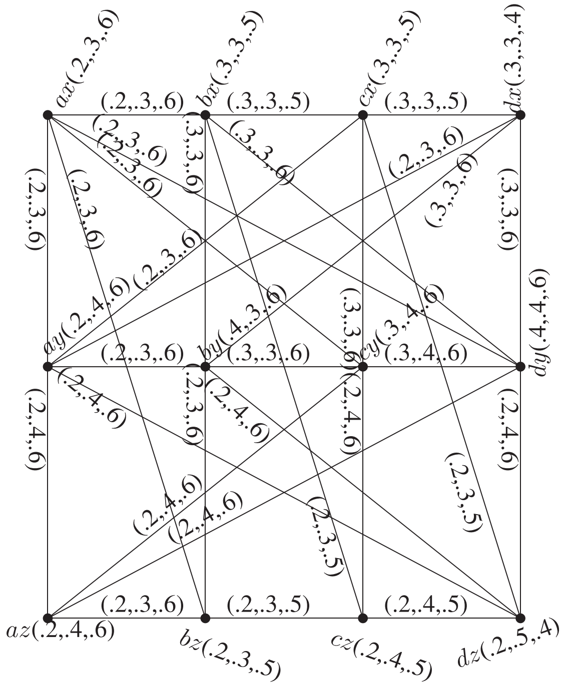

Fig. 6. $\mathbb{SD}_1 \oplus \mathbb{SD}_2$.

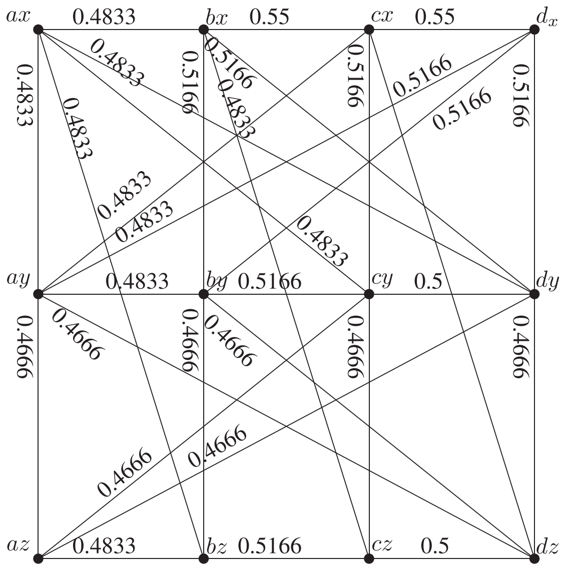

Fig. 7. $\mathbb{SD}_1 \oplus \mathbb{SD}_2$ with minimum score function.

The domination number of the dominating sets S_1, S_2, S_3, S_4 are

$$S_1(\mathbb{SD}_1 \oplus \mathbb{SD}_2) = 0.4666 + 0.5 + 0.5333 = 1.4999$$

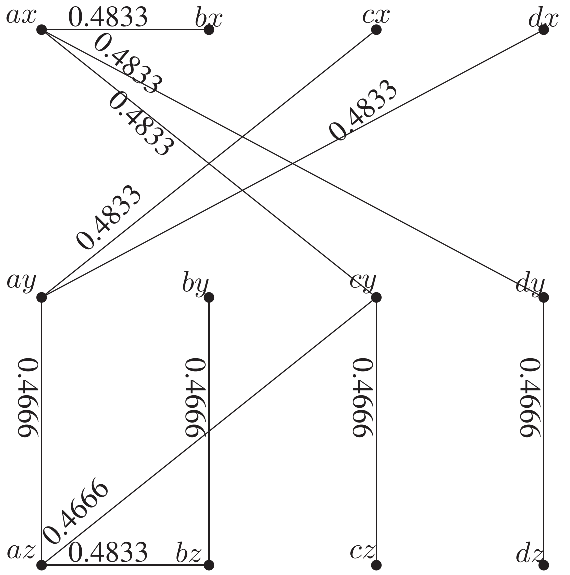

Fig. 8. Minimum Spanning Tree of $\mathbb{SD}_1 \oplus \mathbb{SD}_2$.

$$S_2(\mathbb{SD}_1 \oplus \mathbb{SD}_2)$$
$$= 0.55 + 0.55 + 0.5333 = 1.6333$$
$$S_3(\mathbb{SD}_1 \oplus \mathbb{SD}_2)$$
$$= 0.4666 + 0.55 + 0.5 = 1.5166$$
$$S_4(\mathbb{SD}_1 \oplus \mathbb{SD}_2)$$
$$= 0.4666 + 0.5 + 0.5833 = 1.5166.$$

The domination number of $\mathbb{SD}_1 \oplus \mathbb{SD}_2$ is 1.4999 which is obtained from the dominating set S_1.

The minimal spanning tree of the weighted network $\mathbb{SD}_1 \oplus \mathbb{SD}_2$ is shown in Fig 8 and hence, its weight of minimum spanning tree is 4.6498.

3.3. Residue product of two single-valued Neutrosophic graphs

Let $RP_1 = (\sigma_1, \mu_1)$ and $RP_2 = (\sigma_2, \mu_2)$ be two single-valued Neutrosophic networks of the graphs $G_{RP_1} = (V_1, E_1)$ and $G_{RP_2} = (V_2, E_2)$ respectively. Then the Residue product $\mathbb{RP}_1 \cdot \mathbb{RP}_2 = (\sigma_1 \cdot \sigma_2, \mu_1 \cdot \mu_2)$ is defined as

i) $\forall\, (x, y) \in V_1 \times V_2$,

$$T_{\sigma_1} \cdot T_{\sigma_2}(x, y) = T_{\sigma_1}(x) \wedge T_{\sigma_2}(y),$$
$$I_{\sigma_1} \cdot I_{\sigma_2}(x, y) = I_{\sigma_1}(x) \wedge I_{\sigma_2}(y) \text{ and}$$
$$F_{\sigma_1} \cdot F_{\sigma_2}(x, y) = F_{\sigma_1}(x) \vee F_{\sigma_2}(y),$$

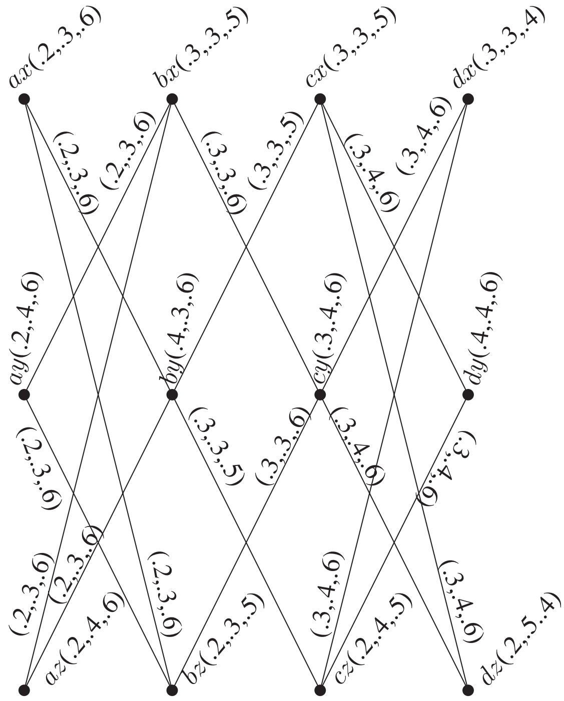

Fig. 9. $\mathbb{RP}_1 \cdot \mathbb{RP}_2$.

ii) $\forall\, (x, y) \in E_1$ and $z \neq w \in V_2$,

$$(T_{\mu_1} \cdot T_{\mu_2})((x, z), (y, w)) = T_{\mu_1}(x, y);$$
$$(I_{\mu_1} \cdot I_{\mu_2})((x, z), (y, w)) = I_{\mu_1}(x, y) \text{ and}$$
$$(F_{\mu_1} \cdot F_{\mu_2})((x, z), (y, w)) = F_{\mu_1}(x, y).$$

3.3.1. Example

Let SVNG $\mathbb{G}_1$ and SVNG $\mathbb{G}_2$ be two Residue product SVNN shown in Fig 1 and Fig 2 of the graphs $G_{RP_1} = (V_1, E_1)$ and $G_{RP_2} = (V_2, E_2)$ respectively. The Residue product of single-valued Neutrosophic network $\mathbb{RP}_1 \cdot \mathbb{RP}_2$ is shown in Fig 9.

The weighted $\mathbb{RP}_1 \cdot \mathbb{RP}_2$ represented in Fig 10, and it's corresponding dominating sets are as follows;

$$S_1 = \{bx, cx, bz, cz\}$$
$$S_2 = \{by, cy, dy, bz\}$$
$$S_3 = \{ay, by, cy, dy\}$$
$$S_4 = \{cx, az, bz, cz\}.$$

The domination number of the dominating sets S_1, S_2, S_3, S_4 are

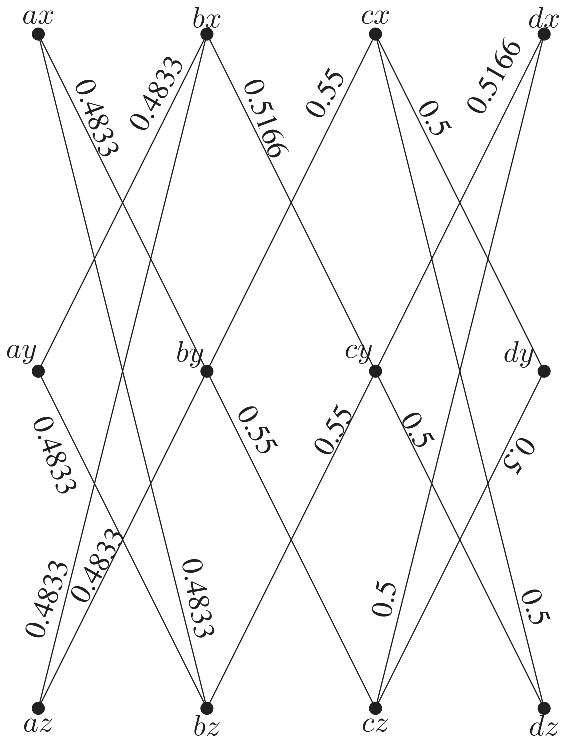

Fig. 10. $\mathbb{RP}_1 \cdot \mathbb{RP}_2$ with minimum score function.

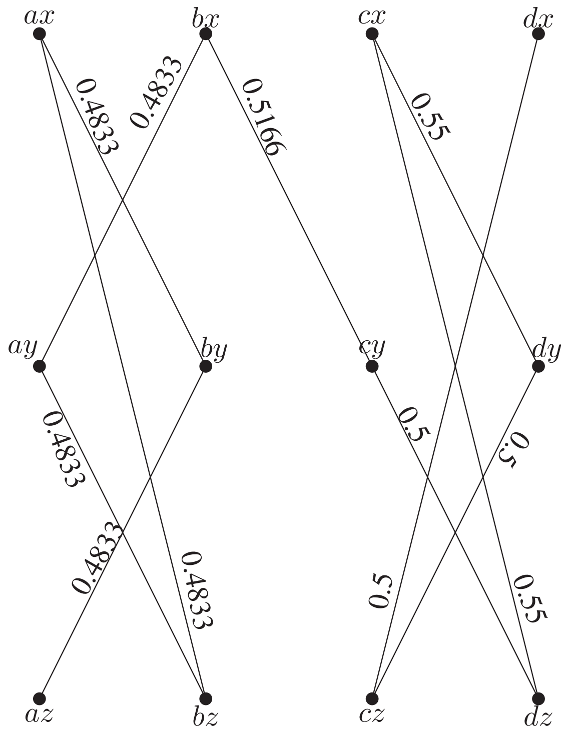

Fig. 11. Minimal Spanning Tree of $\mathbb{RP}_1 \cdot \mathbb{RP}_2$.

$$
\begin{aligned}
S_1(\mathbb{RP}_1 \cdot \mathbb{RP}_2) &= 0.55 + 0.55 + 0.5166 + 0.5 \\
&= 2.1166 \\
S_2(\mathbb{RP}_1 \cdot \mathbb{RP}_2) &= 0.55 + 0.5 + 0.5333 + 0.5166 \\
&= 2.0999 \\
S_3(\mathbb{RP}_1 \cdot \mathbb{RP}_2) &= 0.4833 + 0.55 + 0.5 + 0.5333 \\
&= 2.0666 \\
S_4(\mathbb{RP}_1 \cdot \mathbb{RP}_2) &= 0.55 + 0.4666 + 0.5166 + 0.5 \\
&= 2.0332.
\end{aligned}
$$

The domination number of $\mathbb{RP}_1 \cdot \mathbb{RP}_2$ is 2.0332 which is obtained from the dominating set S_4.

The minimal spanning tree of the weighted network $\mathbb{RP}_1 \cdot \mathbb{RP}_2$ is shown in Fig 11 and hence, its minimum weight of the spanning tree is 5.4333.

3.4. Max product of two single-valued Neutrosophic graphs

Let $MP_1 = (\sigma_{mp_1}, \mu_{mp_1})$ and $MP_2 = (\sigma_{mp_2}, \mu_{mp_2})$ be two single-valued Neutrosophic networks of the graphs $G_{MP_1} = (V_{mp_1}, E_{mp_1})$ and $G_{MP_2} = (V_{mp_2}, E_{mp_2})$ respectively. Then the maximal product of the graphs MP_1 and MP_2 is denoted by

$$\mathbb{MP}_1 * \mathbb{MP}_2 = \left(\sigma_{mp_1} * \sigma_{mp_2}, \mu_{mp_1} * \mu_{mp_2}\right)$$

and defined as:

i) $\forall\, (x, y) \in V_{mp_1} \times V_{mp_2}$,

$$(T_{\sigma_{mp_1}} * T_{\sigma_{mp_2}})(x, y) = T_{\sigma_{mp_1}}(x) \vee T_{\sigma_{mp_2}}(y),$$

$$(I_{\sigma_{mp_1}} * I_{\sigma_{mp_2}})(x, y) = I_{\sigma_{mp_1}}(x) \vee I_{\sigma_{mp_2}}(y),$$

and

$$(F_{\sigma_{mp_1}} * F_{\sigma_{mp_2}})(x, y) = T_{\sigma_{mp_1}}(x) \wedge F_{\sigma_{mp_2}}(y);$$

ii) $\forall\, x \in V_{mp_1}$ and $(y, z) \in E_{mp_2}$,

a) $\left(T_{\mu_{mp_1}} * T_{\mu_{mp_2}}\right)((x, y), (x, z)) = T_{\sigma_{mp_1}}(x) \vee T_{\mu_{mp_2}}(y, z)$;

b) $\left(I_{\mu_{mp_1}} * I_{\mu_{mp_2}}\right)((x, y), (x, z)) = I_{\sigma_{mp_1}}(x) \vee I_{\mu_{mp_2}}(y, z)$;

c) $\left(F_{\mu_{mp_1}} * F_{\mu_{mp_2}}\right)((x, y), (x, z)) = F_{\sigma_{mp_1}}(x) \wedge F_{\mu_{mp_2}}(y, z)$;

iii) $\forall\, x \in V_{mp_2}$ and $(y, z) \in E_{mp_1}$,

a) $\left(T_{\mu_{mp_1}} * T_{\mu_{mp_2}}\right)((y, x), (z, x)) = T_{\mu_{mp_1}}(y, z) \vee T_{\sigma_{mp_2}}(x)$;

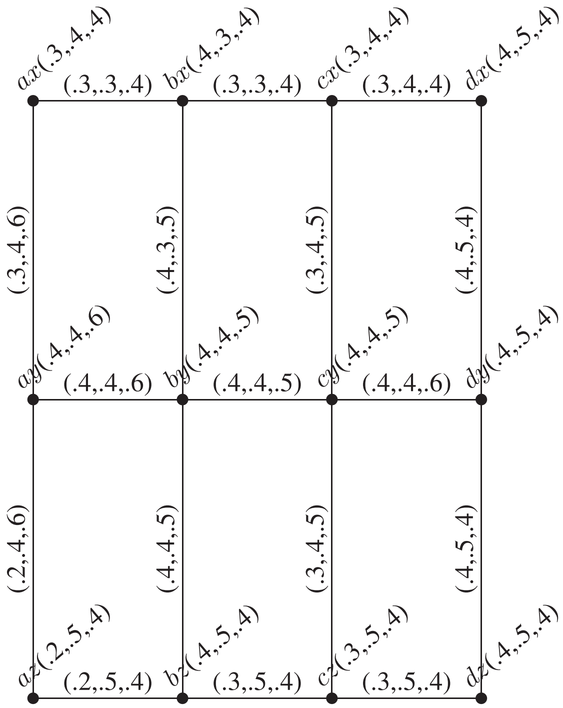

Fig. 12. $\mathbb{MP}_1 * \mathbb{MP}_2$.

b) $\left(I_{\mu_{mp_1}} * I_{\mu_{mp_2}}\right)((y, x), (z, x)) = I_{\mu_{mp_1}}(y, z) \vee I_{\sigma_{mp_2}}(x)$;

c) $\left(F_{\mu_{mp_1}} * F_{\mu_{mp_2}}\right)((y, x), (z, x)) = F_{\mu_{mp_1}}(z, y) \wedge F_{\sigma_{mp_2}}(x)$;

3.4.1. Example

Let SVNG $\mathbb{G}_1$ and SVNG $\mathbb{G}_2$ be two Max product SVNN shown in Fig 1 and Fig 2 of the graphs $G_{MP_1} = (V_1, E_1)$ and $G_{MP_2} = (V_2, E_2)$ respectively. The Max product of single-valued Neutrosophic network $\mathbb{MP}_1 * \mathbb{MP}_2$ is shown in Fig 12.

The maximal product of $\mathbb{MP}_1 * \mathbb{MP}_2$ is given as follows.

The weighted $\mathbb{MP}_1 * \mathbb{MP}_2$ is represented in Fig 13, and it's corresponding dominating sets are as follows;

$$S_1 = \{bx, dy, az, cz\}$$

$$S_2 = \{ax, cx, ay, dz\}$$

$$S_3 = \{cx, ay, dy, cz\}$$

$$S_4 = \{ay, dy, bx, bz\}$$

The domination number of the dominating sets S_1, S_2, S_3, S_4 are

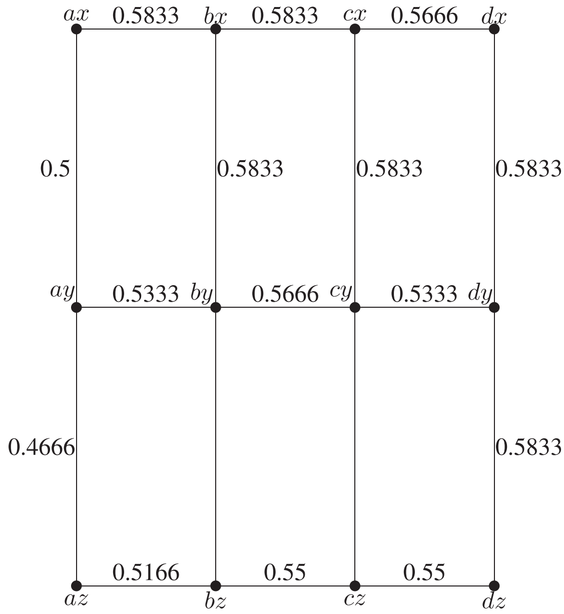

Fig. 13. $\mathbb{MP}_1 * \mathbb{MP}_2$ with minimal score function.

$$S_1(\mathbb{MP}_1 * \mathbb{MP}_2) = 0.6166 + 0.5833 + 0.5166 + 0.55 = 2.2665$$

$$S_2(\mathbb{MP}_1 * \mathbb{MP}_2) = 0.5666 + 0.5666 + 0.5333 + 0.5833 = 2.2498$$

$$S_3(\mathbb{MP}_1 * \mathbb{MP}_2) = 0.5666 + 0.5333 + 0.5833 + 0.55 = 2.2332$$

$$S_4(\mathbb{MP}_1 * \mathbb{MP}_2) = 0.5333 + 0.5833 + 0.6166 + 0.5833 = 2.3165.$$

The domination number of $\mathbb{MP}_1 * \mathbb{MP}_2$ is 2.2332 which is obtained from the dominating set S_3.

The minimal spanning tree of the weighted network $\mathbb{MP}_1 * \mathbb{MP}_2$ is shown in Fig 14 and hence, its minimum weight of spanning tree is 5.8996.

4. Application

4.1. An application of symmetric difference network

Technology salespeople fulfil responsibilities throughout their workday to help consumers find the technology that can benefit them the most. Technology sales are the result of connecting customers with

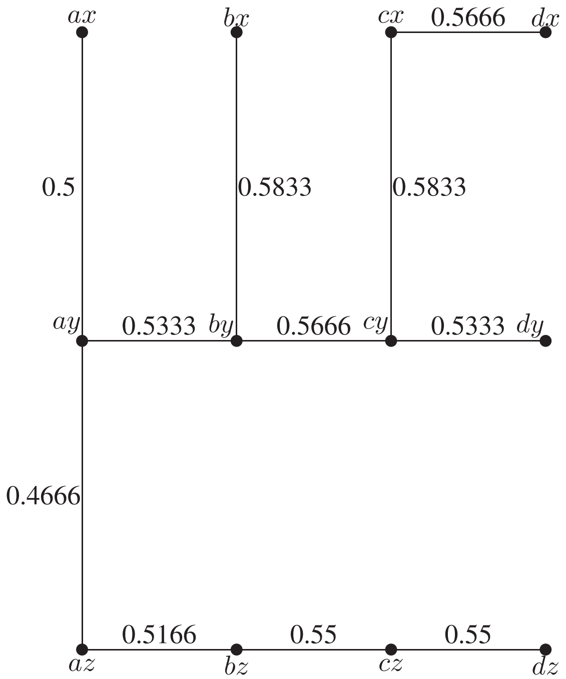

Fig. 14. Minimal Spanning Tree of $\mathbb{MP}_1 * \mathbb{MP}_2$.

technology that can provide a solution to a specific problem.

Technology sales professionals face a unique set of challenges, such as needing a deep understanding of the complex products they sell and possessing the people's skills needed to build trust as well as sales abilities to close deals with prospects.

A sales training program is designed to help sales professionals achieve sales success for themselves or for their organizations. Most sales training programs help to develop the sales skills and techniques needed to approach leads, create new sales opportunities, close deals, and build rapport with clients and customers.

Sales team members have the right combination of technical knowledge and practical sales know how to simultaneously do well. For this reason, sales training designed specifically for technology companies is important. Especially whether selling a new technology or in a highly competitive market, these training can help the technology sales team develop the sales skills needed to serve more, reach decision makers and take deals off the line to maximize revenue.

Let us consider a group of experts who will train the group of trainees to develop their sales skills. Assume that Network $\mathbb{SD}_1$ as trainees has a concern for persons (nodes) whom they have a flow arising from their knowledge or skills. Nodes $a(.2, .4, .6)$, $b(.4, .3, .5)$, $c(.3, .4, .5)$, $d(.4, .5, .4)$ are represented as Trainee 1, Trainee 2, Trainee 3 and Trainee 4 respectively.

Let us assume that Network $\mathbb{SD}_2$ consists of expects x(.3,.3,.4), y(.4,.4,.6), z(.2,.5,.4), whose role is to train the trainees with their skills so that each trainee can attain a new skill when trained by the experts.

The role of each expert in training is different from one another. So, when a skill is trained by an expert to a trainee a new skill is developed by them and also their existing skill will make the sales training more effective in technology. A trainee therefore is trained by experts and does attain other skills expect their own core competency so that the trainee can have a cleaver focus on what they can do the trust to attain and wider the scope to capture high-value opportunities in sales technology.

The experts of Network $\mathbb{SD}_2$ plays a different role in sales training. Each expert (or) trainer is well-developed with special sales training. For example, expert 'x(.3,.3,.4)' is good at training inside and field sales for the trainees. Expert 'y(.4,.4,.6)' is used in service sales training skills and expert 'z(.2,.5,.4)' is prone to sales management skills. These experts combine their sales training to develop the trainees for the letter sales development to achieve the business objectives through effective management.

For example, $ax(.2, .3, .6)$ be the sales executive of the Network developed by the expert 'x(.3,.3,.4)' with training in inside sales for a trainee who is good at effective communication when 'ax' is trained they are built into a better sales executive with their existing skill $ay(.2, .4., .6)$ as insurance sales officer with a skill of better communication and training of service sales expert and $az(.2, .4, .6)$ the account manager who is trained by the sales management expert $by(.2, .3, .65)$ is attained by the trainee 'b(.4,.3,.5)' with good networking skills who is trained by the expert with service sales [Sales Development Representative] and so on the expertise in each field are developed by the experts to the trainees in sales technology.

The roles of each node are different from one another, when these nodes are connected into a Symmetric difference, the above Network $\mathbb{SD}_1 \oplus \mathbb{SD}_2$ is obtained from the two small networks $\mathbb{SD}_1$ and $\mathbb{SD}_2$ respectively.

Symmetric difference Network of the sales technology allows effective management of business and pursue network. Organizational structure in the first place.

Flexibility is one of the main reasons for trained employees to engage in a network organization by outsourcing work. This allows them to complete the tasks in a minimal duration of time without facing major problems.

The neutrosophic network nodes are linked to one another for the flow of information in less time to other nodes. The truth-membership degree of each node indicates the better-skilled person trained in the organization. The indeterminacy-membership degree of each node demonstrates how much the person-âŁ™s skill is uncertain. The falsity-membership degree of each node tells the fewer skills gained by the person. The flow of information from one node to another the node in the network takes place in effective time management. The truth-membership degree, the indeterminacy-membership degree and the falsity-memb-ership degree of each link is given by effective time management of the node in collab-oration. From the above single-valued Neutrosophic network models, we find the Optimal network whose minimal spanning tree make the network more flexi-ble with the minimum possible weights with effective score function are found and thus the optimal network with minimum optimal value increase in profits of the organizations.

The limitations of the study is, an effective opti-mum network is obtained from the each constructed network with a minimum weight of spanning tree using score function. The score function defined in our study gives an Optimal value from which the effective optimal network is chosen from the vari-ous operations applied on single valued Neutrosophic graphs. This study can also be extended to different operations applied on graphs.

4.2. *Optimal network algorithm*

Step-1: Constructed a set of finite networks say $N = N_1, N_2, \cdots, N_r$ using the distinct operations on network with vertex set $V = V_1 \times V_2$.

Step-2: Find the value of score function of each nodes and links of the constructed networks $N_1, N_2, \cdots, N_r$.

Step-3: Find the minimal dominating set and dominating number of each constructed networks $N_1, N_2, \cdots, N_r$.

Step-4: Let the domination number of the constructed network $N_1, N_2, \cdots, N_r$ be $D_{N_1}, D_{N_2}, \cdots, D_{N_r}$ respectively.

Step-5: Discover the minimal spanning trees of the constructed networks and Let it be $T_{ST_1}, T_{ST_2}, \cdots, T_{ST_R}$ of the networks $N_1, N_2, \cdots, N_r$ respectively and find the mini-mum weights of $T_{ST_1}, T_{ST_2}, \cdots, T_{ST_R}$ using score function.

Step-6: Let the minimum weight of $T_{ST_1}, T_{ST_2}, \cdots, T_{ST_R}$ be $W_{ST_1}, W_{ST_2}, \cdots, W_{ST_R}$.

Step-7: Compute the optimal value for each con-structed network $N_1, N_2, \cdots, N_r$, where the optimal value is defined as the minimum value of the sum of the domination number and the minimum weight of the spanning tree.

$$O_{N_i} = D_{N_i} + W_{ST_i}$$
$$OptimalValue,\, O_{V_N} = \min_i\{O_{N_i}\}, i = 1, 2, \cdots, r.$$

Step-8: Among these values which network gives the optimal value is said to be the optimal network.

From section-3 we arrived at the following;

The domination number of $\mathbb{LP}_1 \cdot \mathbb{LP}_2$ is 1.4999.

The domination number of $\mathbb{SD}_1 \oplus \mathbb{SD}_2$ is 1.4999.

The domination number of $\mathbb{RP}_1 \cdot \mathbb{RP}_2$ is 2.0332.

The domination number of $\mathbb{MP}_1 * \mathbb{MP}_2$ is 2.2332.

Minimum weight of spanning tree $\mathbb{LP}_1 \cdot \mathbb{LP}_2 =$ 5.1996.

Minimum weight of spanning tree $\mathbb{SD}_1 \oplus \mathbb{SD}_2 =$ 4.6498.

Minimum weight of spannin,g tree $\mathbb{RP}_1 \cdot \mathbb{RP}_2 =$ 5.4333.

Minimum weight of spanning tree $\mathbb{MP}_1 * \mathbb{MP}_2 =$ 5.8996.

Using the Optimal network algorithm, symmet-ric difference $\mathbb{SD}_1 \oplus \mathbb{SD}_2$ network has the minimum domination number and its weight of the spanning tree is minimum, which gives the best optimal net-work of all the other networks constructed here.

5. Conclusion

The single-valued Neutrosophic models give more precision, flexibility, and compatibility to the system as compared to the classical, fuzzy, intuitionistic fuzzy and Neutrosophic models. In this paper, the authors arrive at some operations such as Lexicographic, Symmetric difference, Residue product and Max product on single-Valued Neutrosophic graphs. Also, investigated some of their properties to find their efficiency and discussed the real-world application of the Symmetric difference network with a minimum spanning tree algorithm which is generated to achieve the minimum efficient productivity to complete the tasks in a social network. In the future, the study will be extended to other operations along with strategies to achive the efficiency of the constructed network.

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