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Mathematical calculation of COVID-19 disease in Pakistan by emergency response modeling based on complex Pythagorean fuzzy information

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Abstract. The new emerged infectious disease that is known the coronavirus disease (COVID-19), which is a high contagious viral infection that started in December 2019 in China city Wuhan and spread very fast to the rest of the world. This infection caused millions of infected cases globally and still poses an alarming situation for human lives. Pakistan in Asian countries is considered the third country with higher number of cases of coronavirus with more than 649824. Recently, some mathematical models have been constructed for better understanding the coronavirus infection. Mostly, these models are based on classical integer-order derivative using real numbers which cannot capture the fading memory. So at the current position it is a challenge for the world to understand and control the spreading of COVID-19. Therefore, the aim of our paper is to develop some novel techniques, namely complex Pythagorean fuzzy weighted averaging (abbreviated as CPFWA) operator, complex Pythagorean fuzzy ordered weighted averaging (abbreviated as CPFOWA) operator, complex Pythagorean fuzzy hybrid averaging (abbreviated as CPFHA) operator, induced complex Pythagorean fuzzy ordered weighted averaging (abbreviated as I-CPFOWA) operator and induced complex Pythagorean fuzzy hybrid averaging (abbreviated as I-CPFHA) operator to analysis the spreading of COVID-19. At the end of the paper, an illustrative the emergency situation of COVID-19 is given for demonstrating the effectiveness of the suggested approach along with a sensitivity analysis, showing the feasibility and reliability of its results.

Keywords: Complex Pythagorean set, CPFWA operator, CPFOWA operator, CPFHA operator, I-CPFOWA operator, I-CPFHA operator.

1. Introduction

The emerged infectious virus that is known as the coronavirus (COVID-19), which is transmittable virus reported at the city of Wuhan in China at the end of 2019. After studies and diagnosis these cases reported as novel coronavirus (COVID-19). Actually the word "coronavirus" comes from the Latin word "corona" which means a "crown, circle of light or nimbus". This lethal virus has infected the whole world and several people have been died due to this noxious virus. In Pakistan the first case reported in March 2020. This virus not only affected the common life of the people but also affected the healthy society. This virus affects directly to your lungs. It has some symptoms like pneumonia and influenza. Now it is

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a challenge for the world to spend the peaceful and normal life due the COVID-19. The COVID-19 is a viral and pandemic disease; therefore the world health organization (WHO) professed an emergency situation due their spreading. Commonly, the virus infects through respiratory droplets emitted and during close touch when people cough or sneeze. When you touch an infected person its virus can be transmitted to your face. This virus is most transferable, because it can be transmitted to your body before started the symptoms. Usually it can be appear within 5 days; however it can be extended in the human body from 2 to 14 days. The most common symptoms of the virus are cough, fever, Pneumonia, chest tightness and acute respiratory distress syndrome. The suitable and perfect treatment is not available still, but the primary care is Symptomatic, covering the mouth in the case of coughing, self-isolation from the affected people, supportive therapy, hand washing, keeping distance from other people and monitoring.

However, there are many upcoming events related with COVID-19, such as how many people can be affected tomorrow?, how many individuals people can be affected during peak time?, the appearance of the inflection point of affected people?, the current treatments, and technologies can be control COVID-19?, to understand these events what is the best mathematical models which can help us. To control and prevent the emergency situation of COVID-19, some scholars have been contributed.

1.1. Emergency response modeling

Yang and Wang [1] developed a mathematical model to show the transmitted of COVID-19. Togacar et al. [2] utilized the fuzzy color and stacking methods to develop the deducting approach for COVID-19. Tuite et al. [3] developed a mathematical model for the transmission of COVID-19, for the population of Canada. Ciufolini and Paolozzi [4] developed Gauss error function using a mathematical model of COVID-19, outbreak in Italy. For predicting the timeframe of COVID-19, Sohail and Nutini [5] developed a mathematical model. Khan and Atangana [6] utilize dynamic mathematical model and develop a fractional derivative for COVID-19. Some related research to COVID-19 and some othere desease can be found in [9–17]. The COVID-19 is recently identified so information on the prevalent is still insufficient, and the medical technologies such as clinical trials are still in a critical phase of investigation.

1.2. Fuzzy information and emergency response modeling

Due to the less information of COVID-19 it is hard to apply directly to the current mathematical models and questions need to be answered about how successful the ongoing emergency response is and whether health care services can be spent more scientifically in the future, and so forth. In emergency case, decision-makers should implement an appropriate emergency scheme to control the further spiraling of the crisis. Therefore it is important to develop a decision making to understand the human actions to deliver people with effective means of responding to emergency situations. In a real life it is always difficult to solve uncertain data due to their incompleteness. In decision-making process, the decision and verdict information and data is mostly and often incomplete, indeterminate and inconsistent information. Therefore Zadeh [18] introduced the notion of fuzzy set which is more powerful and controlling tool to process fuzzy information in our daily life problems. However, there are some problems in this approach, because it has only one element called membership function. To control the above limitations, Atanassov [19] developed the idea of intuitionistic fuzzy set by adding non-membership function, whose sum is less than or equal to one. Later on, several scholars developed some aggregation operators using IFNs. Xu [20], Xu and Yager [21] introduced several arithmetic and geometric aggregation operators using algebraic operational laws and applied then on group decision making. Wang and Liu [22, 23] introduced the notion of some Einstein operators, such as IFEWG operator, IFEOWG operator, IFEWA operator, IFEOWA operator, and applied them to also to group decision making.

But there are several cases where the decision maker may provide the degree of membership and non-membership of a particular attribute in such a way that their sum is greater than one. For example, suppose a man expresses his preferences towards the alternative in such a way that degree of their satisfaction is 0.6 and degree of rejection is 0.8. Clearly its sum is greater than one. Therefore, Yager [24] introduced the concept of Pythagorean fuzzy set where the square sum of membership and non-membership is less than or equal to one. Pythagorean fuzzy set is more powerful tool to solve uncertain problems. Like intuitionistic fuzzy operators, Pythagorean fuzzy operators also become an interesting area for research, after the advent of Pythagorean fuzzy sets theory. Rahman et al. [25–31] introduced several aggregation operators based on Pythagorean fuzzy numbers and applied them on group decision making problems.

Ashraf and Abdullah [7] and Ashraf, Abdullah and Almagrabi [8] developed some new mathematical models for COVID-19 using fuzzy information. Rahman [49] introduced some new mathematical models for COVID-19 based on interval-valved intuitionistic fuzzy information.

1.3. Complex fuzzy information

From the above study, it has been investigated that they have explored decision making problems under FS, IFS and PFS, which are able only to handle the ambiguity and fuzziness of the exists data. These models are unable to present the fractional ignorance of the data and its variations at a given phase of time. But in the sets of complex data, ambiguity and vagueness of the data occur concurrently with changes to the phase of the given data. Therefore, Remote et al. [32] presented the notion of complex fuzzy set (CFS), where the range of membership function can be extended from the [0, 1] to the unit disc. In CFS membership $H = \hbar e^{i\alpha}$ has two elements such as $\hbar$ and $e^{i\alpha}$, where the amplitude term $\hbar \in [0, 1]$ and the phase term $e^{i\alpha} \in [0, 2\pi]$. Later on, several scholars [33–35] worked on the field of CFSs. Alkouri and Salleh [36, 37] generalized the notion of CFS to complex intuitionistic fuzzy set (CIFS), by adding non-membership $Q = qe^{i\beta}$ with some conditions such as, $\hbar + q \leq 1$ and $\alpha + \beta \leq 2\pi$. Rani and Garg [38, 39] developed the idea of distance measures between two CIFSs and also develop some complex power operators. Garg and Rani [40, 41] presented the some generalized operators using CIFNs. Garg and Rani [42] introduced the notion of complex interval-valued intuitionistic fuzzy set (CIVIFS), which is the generalization of complex intuitionistic fuzzy set. Ullah et al. [43] introduced the notion of complex Pythagorean fuzzy set (CPFS), which is the generalization of FSs, CFSs, IFSs and CIFSs. In CPFS $H = \hbar e^{i\alpha}$ and $Q = qe^{i\beta}$ are the membership and non-membership with conditions such as $\hbar^2 + q^2 \leq 1$ and $\alpha^2 + \beta^2 \leq 4\pi^2$. Akram and Naz [44], Akram, Garg et al. [45] explored the graph theory and extended the Electre-1 and Topsis methods respectively. Mahmood et al [46] developed some new mathematical models for COVID-19 using fuzzy information. Akram et al. [47, 48] developed many aggregation operators using complex fuzzy information.

This paper having the following main approaches.

i) To control and reduce the spreading rate of the COVID-19 using emergency group decision-making.
ii) To develop some new operational laws base on complex fuzzy environment.
iii) To develop several operators namely, CPFWA operator, CPFOWA operator, CPFHA operator, I-CPFOWA operator, and I-CPFHA operator.
iv) To design an algorithm for emergency decision making for COVID-19.

The rest of the paper constructed as follows: In section 2, we present some basic information. In section 3, we present some properties of CPFS. In section 4, we develop several operators. In section 5, we present emergency decision-making. In section 6, we develop defferent techniques for emergency decision-making. In section 7, we construct a practice example. Section 8, havin limitation. In section 9, we develop the conclusion.

	Notations	Name	Abbreviation
$\hbar$	Membership	Complex fuzzy set	CFS
$\lambdabar$	Non-membership	Complex intuitionistic fuzzy set	CIFS
Λ	CPFNs	Complex Pythagorean fuzzyset	CPFS
s	Score function	Complex Pythagorean fuzzy number	CPFN
h	Accuracy degree	World health organization	WHO
Q	CFS	Health care system	HCS
I	CIFS	Risk communication	RC
$\mathfrak{M}$	CPFS	Lock down	LD
		Research need	RN

2. Preliminaries

Definition 1. [32] A Complex fuzzy set $\boldsymbol{Q}$ on a fixed set $\boldsymbol{Z}$ is the set of ordered pairs presented by the following equation:

$$Q = \left\{ y, \hbar_Q(y) e^{i\alpha_Q(y)} \, |y \in Z \right\}, \tag{1}$$

where $\hbar_Q : Q \to \{r : r \in C, |r| \prec 1\}$ is called the membership degree, in the complex plane it lies in a unit circle, $i = \sqrt{-1}$, $\hbar_Q(y) \in [0, 1]$ and $e^{i\alpha_Q(y)}$ is a real valued function.

Definition 2. [36] A complex intuitionistic fuzzy set $\boldsymbol{I}$ on a fixed set $\boldsymbol{Z}$ is the set of ordered pairs presented by the following equation:

$$I=\left\{\left\langle y,\hbar_I(y)\,e^{i\alpha_I(y)},\text{ƛ}_I(y)\,e^{i\beta_I(y)}\right\rangle\Big|\,y\in Z\right\},\tag{2}$$

where $i=\sqrt{-1}$, $\hbar_I(y),\text{ƛ}_I(y)\in[0,1]$ are called complex membership and non-membership function respectively, and $\alpha_I(y)$, $\beta_I(y)\in[0,2\pi]$ with conditions $0\leq\hbar_I(y)+\text{ƛ}_I(y)\leq 1$ and $0\leq\frac{\alpha_I(y)}{2\pi}+\frac{\beta_I(y)}{2\pi}\leq 1,\forall\,\mathrm{y}\in Z$.

Definition 3. [43] A Complex Pythagorean fuzzy set $\mathfrak{M}$ on a fixed set $\boldsymbol{Z}$ is the set of ordered pairs presented by the following equation:

$$\mathfrak{M}=\left\{\left\langle y,\hbar_{\mathfrak{M}}(y)\,e^{i\alpha_{\mathfrak{M}}(y)},\text{ƛ}_{\mathfrak{M}}(y)\,e^{i\beta_{\mathfrak{M}}(y)}\right\rangle\Big|\,y\in Z\right\},\tag{3}$$

where $i=\sqrt{-1}$, $\hbar_{\mathfrak{M}}(z),\text{ƛ}_{\mathfrak{M}}(z)\in[0,1]$ are called complex membership and non-membership function respectively, and $\alpha_{\mathfrak{M}}(z),\beta_{\mathfrak{M}}(z)\in[0,2\pi]$ with some conditions such as, $0\leq(\hbar_{\mathfrak{M}}(y))^2+(\text{ƛ}_{\mathfrak{M}}(y))^2\leq 1$ and $0\leq\left(\frac{\alpha_{\mathfrak{M}}(y)}{2\pi}\right)^2+\left(\frac{\beta_{\mathfrak{M}}(y)}{2\pi}\right)^2\leq 1,\forall\,\mathrm{y}\in Z$.

Definition 4. [43] If $\Lambda_j\,(j=1,2,3)$, then conditions hold:

i) $$\Lambda_1\oplus\Lambda_2=\begin{pmatrix}\sqrt{\hbar_{\Lambda_1}^2+\hbar_{\Lambda_2}^2-\hbar_{\Lambda_1}^2\hbar_{\Lambda_2}^2}\;e^{i2\pi\sqrt{\left(\frac{\alpha_{\Lambda_1}}{2\pi}\right)^2+\left(\frac{\alpha_{\Lambda_2}}{2\pi}\right)^2-\left(\frac{\alpha_{\Lambda_1}}{2\pi}\right)^2\left(\frac{\alpha_{\Lambda_2}}{2\pi}\right)^2}},\\ \left(\text{ƛ}_{\Lambda_1}\text{ƛ}_{\Lambda_2}\right)\;e^{\left(\frac{\beta_{\Lambda_1}}{2\pi}\right)\left(\frac{\beta_{\Lambda_2}}{2\pi}\right)}\end{pmatrix},$$

ii) $$\Lambda_1\otimes\Lambda_2=\begin{pmatrix}\left(\hbar_{\Lambda_1}\hbar_{\Lambda_2}\right)\;e^{\left(\frac{\alpha_{\Lambda_1}}{2\pi}\right)\left(\frac{\alpha_{\Lambda_2}}{2\pi}\right)},\\ \sqrt{\text{ƛ}_{\Lambda_1}^2+\text{ƛ}_{\Lambda_2}^2-\text{ƛ}_{\Lambda_1}^2\text{ƛ}_{\Lambda_2}^2}\;e^{i2\pi\sqrt{\left(\frac{\beta_{\Lambda_1}}{2\pi}\right)^2+\left(\frac{\beta_{\Lambda_2}}{2\pi}\right)^2-\left(\frac{\beta_{\Lambda_1}}{2\pi}\right)^2\left(\frac{\beta_{\Lambda_2}}{2\pi}\right)^2}}\end{pmatrix},$$

iii) $$\partial(\Lambda)=\left(\sqrt{1-\left(1-\hbar_{\Lambda}^2\right)^{\partial}}e^{i2\pi\sqrt{1-\left(1-\left(\frac{\alpha_\Lambda}{2\pi}\right)^2\right)^{\partial}}},\text{ƛ}_{\Lambda}^{\partial}e^{i2\pi\left(\frac{\beta_\Lambda}{2\pi}\right)^{\partial}}\right),$$

iv) $$\Lambda^{\partial}=\left(\hbar_{\Lambda}^{\partial}e^{i2\pi\left(\frac{\alpha_\Lambda}{2\pi}\right)^{\partial}},\sqrt{1-\left(1-\text{ƛ}_{\Lambda}^2\right)^{\partial}}e^{i2\pi\sqrt{1-\left(1-\left(\frac{\beta_\Lambda}{2\pi}\right)^2\right)^{\partial}}}\right).$$

Definition 5. [44] If $\Lambda=\left(\hbar_\Lambda e^{i\alpha_\Lambda},\text{ƛ}_\Lambda e^{i\beta_\Lambda}\right)$ be a CPFN, then its score function and accuracy degree can be defined as follows: $s(\Lambda)=\left(\hbar^2-\text{ƛ}^2\right)+\frac{1}{4\pi^2}\left(\alpha^2-\beta^2\right)$ and $h(\Lambda)=\left(\hbar^2+\text{ƛ}^2\right)+\frac{1}{4\pi^2}\left(\alpha^2+\beta^2\right)$ with $s(\Lambda)\in[-2,2]$ and $h(\Lambda)\in[0,2]$ respectively.

Definition 6. [44] If $\Lambda_1=\left(\hbar_{\Lambda_1}e^{i\alpha_{\Lambda_1}},\text{ƛ}_{\Lambda_1}e^{i\beta_{\Lambda_1}}\right)$ and $\Lambda_2=\left(\hbar_{\Lambda_2}e^{i\alpha_{\Lambda_2}},\text{ƛ}_{\Lambda_2}e^{i\beta_{\Lambda_2}}\right)$ are two CPFNs, then the following conditions hold:

i) If $s(\Lambda_1)\succ s(\Lambda_2)$, then $\Lambda_1\succ\Lambda_2$.
ii) If $s(\Lambda_1)=s(\Lambda_2)$ then we have three conditions
   a) If $h(\Lambda_1)=h(\Lambda_2)$, then $\Lambda_1=\Lambda_2$.
   b) If $h(\Lambda_1)\succ h(\Lambda_2)$, then $\Lambda_1\succ\Lambda_2$.

3. Some operations on complex Pythagorean fuzzy set

Definition 7. If $\Lambda_j=\left\{y,\hbar_{\Lambda_j}(y)\,e^{i\alpha_{\Lambda_j}(y)},\text{ƛ}_{\Lambda_j}(y)\,e^{i\beta_{\Lambda_j}(y)}\,|y\in Z\right\}(j=1,2)$ are two CPFNs, then some basic operations on Λ_1 and Λ_2 are defined as follows:

i) $\Lambda_1\cup\Lambda_2=\left\{y,\hbar_{\Lambda_1\cup\Lambda_2}(y)\,e^{i\alpha_{\Lambda_1\cup\Lambda_2}(y)},\text{ƛ}_{\Lambda_1\cup\Lambda_2}(y)\,e^{i\beta_{\Lambda_1\cup\Lambda_2}(y)}\,|y\in Z\right\}$, where $\hbar_{\Lambda_1\cup\Lambda_2}(y)=\max\left\{\hbar_{\Lambda_1}(y),\hbar_{\Lambda_2}(y)\right\}$, $\lambdabar_{\Lambda_1\cup\Lambda_2}(y)=\min\left\{\lambdabar_{\Lambda_1}(y),\lambdabar_{\Lambda_2}(y)\right\}$ and $\alpha_{\Lambda_1\cup\Lambda_2}(y)=\max\left\{\alpha_{\Lambda_1}(y),\alpha_{\Lambda_2}(y)\right\}$, $\beta_{\Lambda_1\cup\Lambda_2}(y)=\min\left\{\beta_{\Lambda_1}(y),\beta_{\Lambda_2}(y)\right\}$.

ii) $\Lambda_1\cap\Lambda_2=\left\{y,\hbar_{\Lambda_1\cap\Lambda_2}(y)e^{i\alpha_{\Lambda_1\cap\Lambda_2}(y)},\lambdabar_{\Lambda_1\cap\Lambda_2}(y)e^{i\beta_{\Lambda_1\cap\Lambda_2}(y)}\,|\,y\in Z\right\}$,
where $\hbar_{\Lambda_1\cap\Lambda_2}(y)=\min\left\{\hbar_{\Lambda_1}(y),\hbar_{\Lambda_2}(y)\right\}$, $\lambdabar_{\Lambda_1\cap\Lambda_2}(y)=\max\left\{\lambdabar_{\Lambda_1}(y),\lambdabar_{\Lambda_2}(y)\right\}$ and $\alpha_{\Lambda_1\cap\Lambda_2}(y)=\min\left\{\alpha_{\Lambda_1}(y),\alpha_{\Lambda_2}(y)\right\}$, $\beta_{\Lambda_1\cap\Lambda_2}(y)=\max\left\{\beta_{\Lambda_1}(y),\beta_{\Lambda_2}(y)\right\}$.

iii) $\Lambda_1^c=\left\{y\lambdabar_{\Lambda_1}(y)e^{i\beta_{\Lambda_1}(y)},\hbar_{\Lambda_1}(y)e^{i\alpha_{\Lambda_1}(y)}\,|\,y\in Z\right\}$.

Theorem 1. Let Λ_1 and Λ_2 are any two CPFNs and a real number $\partial\succ 0$, then the resulting values of: $\Lambda_1\oplus\Lambda_2$, $\Lambda_1\otimes\Lambda_2$, $(\Lambda_1)^\partial$ and $\partial(\Lambda_1)$ are also CPFN.

Proof. First we show that $\Lambda_1\oplus\Lambda_2$ is CPFN. Let $\Lambda_3=\Lambda_1+\Lambda_2=\left(\hbar_{\Lambda_3}e^{i\alpha_{\Lambda_3}},\lambdabar_{\Lambda_3}e^{i\beta_{\Lambda_3}}\right)$. Since $\Lambda_1=\left(\hbar_{\Lambda_1}e^{i\alpha_{\Lambda_1}},\lambdabar_{\Lambda_1}e^{i\beta_{\Lambda_1}}\right)$ and $\Lambda_2=\left(\hbar_{\Lambda_2}e^{i\alpha_{\Lambda_2}},\lambdabar_{\Lambda_2}e^{i\beta_{\Lambda_2}}\right)$ are two CPFNs. By Definition 3, we have $\hbar_{\Lambda_j},\lambdabar_{\Lambda_j}\in[0,1]\,(j=1,2)$ with some conditions such as, $0\le\left(\hbar_{\Lambda_j}\right)^2+\left(\lambdabar_{\Lambda_j}\right)^2\le 1$ and $\alpha_{\Lambda_j},\beta_{\Lambda_j}\in[0,2\pi]\,(j=1,2)$ with $0\le\left(\frac{\alpha_{\Lambda_j}}{2\pi}\right)^2+\left(\frac{\beta_{\Lambda_j}}{2\pi}\right)^2\le 1$. As $0\le\hbar_{\Lambda_j}\le 1$, this implies that $0\le\sqrt{1-\prod_{j=1}^{2}\left(1-\hbar_{\Lambda_j}^2\right)}\le 1$. Hence, $0\le\hbar_{\Lambda_3}\le 1$. On the other hand $0\le\lambdabar_{\Lambda_j}\le 1$, this implies that $0\le\prod_{j=1}^{2}\lambdabar_{\Lambda_j}\le 1$. This implies that $0\le\lambdabar_{\Lambda_3}\le 1$. Furthermore $\left(\hbar_{\Lambda_j}\right)^2+\left(\lambdabar_{\Lambda_j}\right)^2\le 1$, this implies that $\left(\lambdabar_{\Lambda_j}\right)^2\le 1-\left(\hbar_{\Lambda_j}\right)^2$. Thus we have, $\left(\sqrt{\prod_{j=1}^{2}\left(\lambdabar_{\Lambda_j}\right)^2}\right)^2\le\left(\sqrt{\prod_{j=1}^{2}1-\left(\hbar_{\Lambda_j}\right)^2}\right)^2$ and hence $\left(\hbar_{\Lambda_j}\right)^2+\left(\lambdabar_{\Lambda_j}\right)^2$

$$=\left(\sqrt{1-\prod_{j=1}^{2}\left(1-\hbar_{\Lambda_j}^2\right)}\right)^2+\left(\sqrt{\prod_{j=1}^{2}\left(\lambdabar_{\Lambda_j}\right)^2}\right)^2\le\left(\sqrt{1-\prod_{j=1}^{2}\left(1-\hbar_{\Lambda_j}^2\right)}\right)^2+\left(\sqrt{\prod_{j=1}^{2}\left(1-\hbar_{\Lambda_j}^2\right)}\right)^2$$

$$=1-\prod_{j=1}^{2}\left(1-\hbar_{\Lambda_j}^2\right)+\prod_{j=1}^{2}\left(1-\hbar_{\Lambda_j}^2\right)=1.$$

Now $\hbar_{\Lambda_j}\ge 0,\lambdabar_{\Lambda_j}\ge 0$, this implies that $\left(\hbar_{\Lambda_j}\right)^2\ge 0$, $\left(\lambdabar_{\Lambda_j}\right)^2\ge 0$. Hence, $\left(\hbar_{\Lambda_j}\right)^2+\left(\lambdabar_{\Lambda_j}\right)^2\ge 0$. Thus $0\le\left(\hbar_{\Lambda_j}\right)^2+\left(\lambdabar_{\Lambda_j}\right)^2\le 1$. On the same way we can prove that $\alpha_{\Lambda_j},\beta_{\Lambda_j}\in[0,2\pi]$ with $0\le\left(\frac{\alpha_{\Lambda_j}}{2\pi}\right)^2+\left(\frac{\beta_{\Lambda_j}}{2\pi}\right)^2\le 1$. Thus, we obtain $\Lambda_3=\Lambda_1+\Lambda_2$ is also CPFN.

Now we are going show that $(\Lambda_1)^\partial$ is CPFN, for this we have, $\Lambda_1=\left(\hbar_{\Lambda_1}e^{i\alpha_{\Lambda_1}},\lambdabar_{\Lambda_1}e^{i\beta_{\Lambda_1}}\right)$, then by Definition 3, we have $\hbar_{\Lambda_1},\lambdabar_{\Lambda_1}\in[0,1]$, $\alpha_{\Lambda_1},\beta_{\Lambda_1}\in[0,2\pi]$ with $0\le\left(\hbar_{\Lambda_1}\right)^2+\left(\lambdabar_{\Lambda_1}\right)^2\le 1$ and $0\le\left(\frac{\alpha_{\Lambda_1}}{2\pi}\right)^2+\left(\frac{\beta_{\Lambda_1}}{2\pi}\right)^2\le 1$. Since $\hbar_{\Lambda_1}\ge 0$, it implies that $\left(\hbar_{\Lambda_1}\right)^\partial\ge 0$, $\sqrt{1-\left(1-\hbar_{\Lambda_1}^2\right)^\partial}\ge 0$. On the other hand $\lambdabar_\Lambda\ge 0$, then $\left(\lambdabar_\Lambda\right)^\partial\ge 0$. Further $\sqrt{1-\left(1-\hbar_\Lambda^2\right)^\partial}\ge 0$ and $\left(\lambdabar_\Lambda^2\right)^\partial\ge 0$, then we have $\left(\sqrt{1-\left(1-\hbar_\Lambda^2\right)^\partial}\right)^2+\left(\sqrt{\left(\lambdabar_\Lambda^2\right)^\partial}\right)^2\le\left(\sqrt{1-\left(1-\hbar_\Lambda^2\right)^\partial}\right)^2+\left(\sqrt{\left(1-\hbar_\Lambda^2\right)^\partial}\right)^2=1-\left(1-\hbar_\Lambda^2\right)^\partial+\left(1-\hbar_\Lambda^2\right)^\partial=1$. On the same way we can prove that $\alpha_{\Lambda_1},\beta_{\Lambda_1}\in[0,2\pi]$ with $0\le\left(\frac{\alpha_{\Lambda_1}}{2\pi}\right)^2+\left(\frac{\beta_{\Lambda_1}}{2\pi}\right)^2\le 1$. Thus, $(\Lambda_1)^\partial$ is CPFN. On the same way we can prove the remaining parts.

Theorem 2. Let Λ_1, Λ_2 and Λ_3 are any three CPFNs, then the following hold:

1) $\Lambda_1\cup\Lambda_2\cup\Lambda_3=\Lambda_1\cup\Lambda_3\cup\Lambda_2$,
2) $\Lambda_1\cap\Lambda_2\cap\Lambda_3=\Lambda_1\cap\Lambda_3\cap\Lambda_2$,
3) $\Lambda_1\oplus\Lambda_2\oplus\Lambda_3=\Lambda_1\oplus\Lambda_3\oplus\Lambda_2$,
4) $\Lambda_1\otimes\Lambda_2\otimes\Lambda_3=\Lambda_1\otimes\Lambda_3\otimes\Lambda_2$.

Proof. the proof is easy, so it is omitted here

4. Some complex Pythagorean fuzzy aggregation operators

In this section, we introduce some new operators namely, CPFWA operator, CPFOWA operator, CPFHA operator, I-CPFOWA operator, and I-CPFHA operator with their properties.

Definition 8. Let $\Lambda_j=\left(\hbar_{\Lambda_j}e^{i\alpha_{\Lambda_j}},\lambdabar_{\Lambda_j}e^{i\beta_{\Lambda_j}}\right)$ $(j=1,...,n)$ be a collection of CPFVs, where

$\Phi = (\Phi_1, \Phi_2, ..., \Phi_n)^T$ be the weighted vector of Λ_j with conditions $\Phi_j \in [0, 1]$ and $\sum_{j=1}^{n} \Phi_j = 1$. Then the complex Pythagorean fuzzy weighted averaging (CPFWA) aggregation operator can be defined as:

$$\text{CPFWA}_{\Phi}(\Lambda_1, \Lambda_2, ...\Lambda_n)$$

$$= \left(\sqrt{1 - \prod_{j=1}^{n} \left(1 - \hbar^2_{\Lambda_j}\right)^{\Phi_j}} e^{i2\pi \sqrt{1-\prod_{j=1}^{n}\left(1-\left(\frac{\alpha_{\Lambda_j}}{2\pi}\right)^2\right)^{\Phi_j}}}, \prod_{j=1}^{n} \left(\lambdabar_{\Lambda_j}\right)^{\Phi_j} e^{i2\pi \prod_{j=1}^{n}\left(\frac{\beta_{\Lambda_j}}{2\pi}\right)^{\Phi_j}} \right), \tag{4}$$

Theorem 3. Let $\Lambda_j = \left(\hbar_{\Lambda_j} e^{i\alpha_{\Lambda_j}}, \lambdabar_{\Lambda_j} e^{i\beta_{\Lambda_j}}\right)$ $(j = 1, ..., n)$ be a collection of CPFVs, then their aggregated value by using the CPFWA operator is still CPFV, and

$$\text{CPFWA}_{\Phi}(\Lambda_1, \Lambda_2, ...\Lambda_n)$$

$$= \left(\sqrt{1 - \prod_{j=1}^{n} \left(1 - \hbar^2_{\Lambda_j}\right)^{\Phi_j}} e^{i2\pi \sqrt{1-\prod_{j=1}^{n}\left(1-\left(\frac{\alpha_{\Lambda_j}}{2\pi}\right)^2\right)^{\Phi_j}}}, \prod_{j=1}^{n} \left(\lambdabar_{\Lambda_j}\right)^{\Phi_j} e^{i2\pi \prod_{j=1}^{n}\left(\frac{\beta_{\Lambda_j}}{2\pi}\right)^{\Phi_j}} \right), \tag{5}$$

Proof. Since Λ_j is a CPFN and for any real number $\Phi_j \succ 0$. Then by Theorem 1, $\Phi_j \Lambda_j$ is also a CPFN. So the first part of Theorem is proved. By mathematical induction principle.

Step 1: For $\boldsymbol{n = 2}$, we have $\Lambda_1 = \left(\hbar_{\Lambda_1} e^{i\alpha_{\Lambda_1}}, \lambdabar_{\Lambda_1} e^{i\beta_{\Lambda_1}}\right)$ and $\Lambda_2 = \left(\hbar_{\Lambda_2} e^{i\alpha_{\Lambda_2}}, \lambdabar_{\Lambda_2} e^{i\beta_{\Lambda_2}}\right)$, so by the operational laws of CPFNs, we get and $\Phi_1 \Lambda_1 =$

$$\left(\sqrt{1 - \left(1 - \left(\hbar_{\Lambda_1}\right)^2\right)^{\Phi_1}} e^{i2\pi \sqrt{1-\left(1-\left(\frac{\alpha_{\Lambda_1}}{2\pi}\right)^2\right)^{\Phi_1}}}, \left(\lambdabar_{\Lambda_1}\right)^{\Phi_1} e^{i2\pi\left(\frac{\beta_{\Lambda_1}}{2\pi}\right)^{\Phi_1}} \right),$$

$$\Phi_2 \Lambda_2 = \left(\sqrt{1 - \left(1 - \left(\hbar_{\Lambda_2}\right)^2\right)^{\Phi_2}} e^{i2\pi \sqrt{1-\left(1-\left(\frac{\alpha_{\Lambda_2}}{2\pi}\right)^2\right)^{\Phi_2}}}, \left(\lambdabar_{\Lambda_2}\right)^{\Phi_2} e^{i2\pi\left(\frac{\beta_{\Lambda_2}}{2\pi}\right)^{\Phi_2}} \right).$$

Now by Definition 8, we have the following:

$$\text{CPFWA}_{\Phi}(\Lambda_1, \Lambda_2)$$

$$= \left(\sqrt{1 - \prod_{j=1}^{2} \left(1 - \left(\hbar_{\Lambda_j}\right)^2\right)^{\Phi_j}} e^{i2\pi \sqrt{1-\prod_{j=1}^{2}\left(1-\left(\frac{\alpha_{\Lambda_j}}{2\pi}\right)^2\right)^{\Phi_j}}}, \prod_{j=1}^{2} \left(\lambdabar_{\Lambda_j}\right)^{\Phi_j} e^{i2\pi \prod_{j=1}^{2}\left(\frac{\beta_{\Lambda_j}}{2\pi}\right)^{\Phi_j}} \right)$$

Hence, for $\boldsymbol{n = 2}$, the result in (5) is true.

Step 2: Suppose that (5) is true for $\boldsymbol{n = s}$, where s is any positive integer.

$\text{CPFWA}_{\Phi}(\Lambda_1, \Lambda_2, ..., \Lambda_s)$

$$= \left(\sqrt{1 - \prod_{j=1}^{s} \left(1 - \left(\hbar_{\Lambda_j}\right)^2\right)^{\Phi_j}} e^{i2\pi \sqrt{1 - \prod_{j=1}^{s} \left(1 - \left(\frac{\alpha_{\Lambda_j}}{2\pi}\right)^2\right)^{\Phi_j}}}, \prod_{j=1}^{s} \left(\lambdabar_{\Lambda_j}\right)^{\Phi_j} e^{i2\pi \prod_{j=1}^{s} \left(\frac{\beta_{\Lambda_j}}{2\pi}\right)^{\Phi_j}} \right).$$

Step 3: Assume that (5) true for $\boldsymbol{n = s}$, we show that it is true for $\boldsymbol{n = s+1}$, for this we have

$\text{CPFWA}_{\Phi}(\Lambda_1, \Lambda_2, ..., \Lambda_{s+1})$

$$= \left(\sqrt{1 - \prod_{j=1}^{s} \left(1 - \left(\hbar_{\Lambda_j}\right)^2\right)^{\Phi_j}} e^{i2\pi \sqrt{1 - \prod_{j=1}^{s} \left(1 - \left(\frac{\alpha_{\Lambda_j}}{2\pi}\right)^2\right)^{\Phi_j}}}, \prod_{j=1}^{s} \left(\lambdabar_{\Lambda_j}\right)^{\Phi_j} e^{i2\pi \prod_{j=1}^{s} \left(\frac{\beta_{\Lambda_j}}{2\pi}\right)^{\Phi_j}} \right) \oplus$$

$$\left(\sqrt{1 - \left(1 - \left(\hbar_{\Lambda_{s+1}}\right)^2\right)^{\Phi_{s+1}}} e^{i2\pi \sqrt{1 - \left(1 - \left(\frac{\alpha_{\Lambda_{s+1}}}{2\pi}\right)^2\right)^{\Phi_{s+1}}}}, \left(\lambdabar_{\Lambda_{s+1}}\right)^{\Phi_{s+1}} e^{i2\pi \left(\frac{\beta_{\Lambda_{s+1}}}{2\pi}\right)^{\Phi_{s+1}}} \right)$$

$$= \left(\sqrt{1 - \prod_{j=1}^{s+1} \left(1 - \left(\hbar_{\Lambda_j}\right)^2\right)^{\Phi_j}} e^{i2\pi \sqrt{1 - \prod_{j=1}^{s+1} \left(1 - \left(\frac{\alpha_{\Lambda_j}}{2\pi}\right)^2\right)^{\Phi_j}}}, \prod_{j=1}^{s+1} \left(\lambdabar_{\Lambda_j}\right)^{\Phi_j} e^{i2\pi \prod_{j=1}^{s+1} \left(\frac{\beta_{\Lambda_j}}{2\pi}\right)^{\Phi_j}} \right)$$

Thus (5) is true for $\boldsymbol{n = s+1}$. By mathematical induction (5) holds for all positive integer.

Property 1 (Idempotency). If Λ_j $(j = 1, ..., n)$ be a collection of CPFVs, and let Λ_O be another CPFV such tha $\Lambda_j = \Lambda_O$, then

$\text{CPFWA}_{\Phi}(\Lambda_1, \Lambda_2, ..., \Lambda_n)$

$$= \left(\sqrt{1 - \prod_{j=1}^{n} \left(1 - \left(\hbar_{\Lambda_O}\right)^2\right)^{\Phi_j}} e^{i2\pi \sqrt{1 - \prod_{j=1}^{n} \left(1 - \left(\frac{\alpha_{\Lambda_O}}{2\pi}\right)^2\right)^{\Phi_j}}}, \prod_{j=1}^{n} \left(\lambdabar_{\Lambda_O}\right)^{\Phi_j} e^{i2\pi \prod_{j=1}^{n} \left(\frac{\beta_{\Lambda_O}}{2\pi}\right)^{\Phi_j}} \right)$$

$$= \left(\hbar_{\Lambda_O} e^{i\alpha_{\Lambda_O}}, \lambdabar_{\Lambda_O} e^{i\beta_{\Lambda_O}}\right) = \Lambda_O$$

Property II (Boundedness). If $\Lambda_j = \left(\hbar_{\Lambda_j} e^{i\alpha_{\Lambda_j}}, \lambdabar_{\Lambda_j} e^{i\beta_{\Lambda_j}}\right)$ $(j = 1, ..., n)$ be a collection of CPFVs such as: $\Lambda_{\min} = \left(\hbar_{\min} e^{i\alpha_{\min}}, \lambdabar_{\min} e^{i\beta_{\min}}\right)$ and $\Lambda_{\max} = \left(\hbar_{\max} e^{i\alpha_{\max}}, \lambdabar_{\max} e^{i\beta_{\max}}\right)$, where $\hbar_{\min} = \min_j \{\hbar_j\}$, $\alpha_{\min} = \min_j \{\alpha_j\}$, $\lambdabar_{\min} = \min_j \{\lambdabar_j\}$, $\beta_{\min} = \min_j \{\beta_j\}$, $\hbar_{\max} = \min_j \{\hbar_j\}$, $\alpha_{\max} = \min_j \{\alpha_j\}$, $\lambdabar_{\max} = \min_j \{\lambdabar_j\}$ and $\beta_{\max} = \min_j \{\beta_j\}$, then

$$\Lambda_{\min} \le \text{CPFWA}_{\Phi}(\Lambda_1, \Lambda_2, ..., \Lambda_n) \le \Lambda_{\max}, \quad (6)$$

Proof. Since $\Lambda = \mathrm{CPFWA}(\Lambda_1, \Lambda_2, ..., \Lambda_n) = \left(\hbar_{\Lambda_j} e^{i\alpha_{\Lambda_j}}, ƛ_{\Lambda_j} e^{i\beta_{\Lambda_j}}\right)$, then for any Λ_j, we have

$$\sqrt{\left(\min_j \{\Lambda_{\min}\}\right)^2} \le \sqrt{\left(\Lambda_j\right)^2} \le \sqrt{\left(\max_j \{\Lambda_{\max}\}\right)^2}$$

$$\Leftrightarrow \sqrt{\prod_{j=1}^{n}\left(1-\left(\max_j \{\Lambda_{\max}\}\right)^2\right)^{\Phi_j}}$$

$$\le \sqrt{\prod_{j=1}^{n}\left(1-\left(\Lambda_j\right)^2\right)^{\Phi_j}}$$

$$\le \sqrt{\prod_{j=1}^{n}\left(1-\left(\min_j \{\Lambda_{\min}\}\right)^2\right)^{\Phi_j}}$$

$$\Leftrightarrow \sqrt{\left(\min_j \{\Lambda_{\min}\}\right)^2} \le \sqrt{1-\prod_{j=1}^{n}\left(1-\left(\Lambda_j\right)^2\right)^{\Phi_j}}$$

$$\le \sqrt{\left(\max_j \{\Lambda_{\max}\}\right)^2}$$

$$\Leftrightarrow \min_j \{\Lambda_{\min}\} \le \sqrt{1-\prod_{j=1}^{n}\left(1-\left(\Lambda_j\right)^2\right)^{\Phi_j}}$$

$$\le \max_j \{\Lambda_{\max}\}$$

$$\mathrm{CPFOWA}_\Phi(\Lambda_1, \Lambda_2, ...\Lambda_n)$$

$$= \left(\sqrt{1-\prod_{j=1}^{n}\left(1-\hbar^2_{\Lambda_{\varphi(j)}}\right)^{\Phi_j}}\, e^{i2\pi\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\frac{\alpha_{\Lambda_{\varphi(j)}}}{2\pi}\right)^2\right)^{\Phi_j}}}, \prod_{j=1}^{n}\left(ƛ_{\Lambda_{\varphi(j)}}\right)^{\Phi_j} e^{i2\pi\prod_{j=1}^{n}\left(\frac{\beta_{\Lambda_{\varphi(j)}}}{2\pi}\right)^{\Phi_j}}\right), \tag{9}$$

Thus $\min_j \{\hbar_j\} \le \hbar_j \le \max_j \{\hbar_j\}$. On the same way $\min_j \{\alpha_j\} \le \alpha_j \le \max_j \{\alpha_j\}$. Similarly

$$\min_j \{ƛ_j\} \le ƛ_j \le \max_j \{ƛ_j\} \Leftrightarrow \prod_{j=1}^{n}\left(\min_j \{ƛ_j\}\right)^{\Phi_j}$$

$$\le \prod_{j=1}^{n}\left(ƛ_j\right)^{\Phi_j} \le \prod_{j=1}^{n}\left(\max_j \{ƛ_j\}\right)^{\Phi_j}$$

$$\Leftrightarrow \left(\min_j \{ƛ_j\}\right)^{\sum_{j=1}^{n}\Phi_j} \le \prod_{j=1}^{n}\left(ƛ_j\right)^{\Phi_j}$$

$$\le \left(\max_j \{ƛ_j\}\right)^{\sum_{j=1}^{n}\Phi_j}$$

$$\Leftrightarrow \min_j \{ƛ_j\} \le \prod_{j=1}^{n}\left(ƛ_j\right)^{\Phi_j} \le \max_j \{ƛ_j\}$$

Thus $\min_j \{ƛ_j\} \le ƛ_j \le \max_j \{ƛ_j\}$. On the same way $\min_j \{\beta_j\} \le \beta_j \le \max_j \{\beta_j\}$. Then we have

$$\Lambda_{\min} \le \mathrm{CPFWA}_\Phi(\Lambda_1, \Lambda_2, \Lambda_3, ..., \Lambda_n) \le \Lambda_{\max}. \tag{7}$$

Property III (Monotonicity). If $\Lambda_j = \left(\hbar_{\Lambda_j} e^{i\alpha_{\Lambda_j}}, ƛ_{\Lambda_j} e^{i\beta_{\Lambda_j}}\right)$, $\Delta_j = \left(\hbar_{\Delta_j} e^{i\alpha_{\Delta_j}}, ƛ_{\Delta_j} e^{i\beta_{\Delta_j}}\right)$ are two families of CPFVs, with $\hbar_{\Lambda_j} \le \hbar_{\Delta_j}, \alpha_{\Lambda_j} \le \alpha_{\Delta_j}, ƛ_{\Lambda_j} \ge ƛ_{\Delta_j}, \beta_{\Lambda_j} \ge \beta_{\Delta_j}$. Then we have

$$\mathrm{CPFWA}_\Phi(\Lambda_1, \Lambda_2, ..., \Lambda_n) \le \mathrm{CPFWA}_\Phi(\Delta_1, \Delta_2, ..., \Delta_n). \tag{8}$$

Proof. Proof is similar as above, so here it is omitted.

Definition 9. The complex Pythagorean fuzzy ordered weighted averaging (CPFOWA) aggregation operator can be defined as:

where $(\varphi(1), \varphi(2), ...\varphi(n))$ is any permutation of $(1, 2, ..., n)$ with condition $\Lambda_{\varphi(j-1)} \ge \Lambda_{\varphi(j)}$ for all j. $\Phi = (\Phi_1, \Phi_2, ..., \Phi_n)^T$ be the weight vector of Λ_j with some conditions $\Phi_j \in [0, 1]$ and $\sum_{j=1}^{n} \Phi_j = 1$.

Theorem 4. Let $\Lambda_j = \left(\hbar_{\Lambda_j} e^{i\alpha_{\Lambda_j}}, ƛ_{\Lambda_j} e^{i\beta_{\Lambda_j}}\right)$ $(j = 1, 2, ..., n)$ be a collection of CPFVs, then their aggregated value by using the CPFOWA operator is still CPFV.

Proof. Proof is similar to Theorem 3, so here we omit.

Definition 10. The complex Pythagorean fuzzy hybrid averaging (CPFHA) aggregation operator can be defined as:

$$\mathrm{CPFHA}_{\varpi,\Phi}\left(\Lambda_1,\Lambda_2,\ldots\Lambda_n\right)$$

$$=\left(\sqrt{1-\prod_{j=1}^{n}\left(1-\hbar^2_{\dot{\Lambda}_{\varphi(j)}}\right)^{\Phi_j}}\,e^{i2\pi\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\frac{{}^{\alpha}\dot{\Lambda}_{\varphi(j)}}{2\pi}\right)^2\right)^{\Phi_j}}},\ \prod_{j=1}^{n}\left(\bar{\lambda}_{\dot{\Lambda}_{\varphi(j)}}\right)^{\Phi_j}e^{i2\pi\prod_{j=1}^{n}\left(\frac{{}^{\beta}\dot{\Lambda}_{\varphi(j)}}{2\pi}\right)^{\Phi_j}}\right), \tag{10}$$

where $\dot{\Lambda}_{\varphi(j)}$ be the greatest value and $\dot{\Lambda}_{\varphi(j)}=n\varpi_j\Lambda_j$. And $\boldsymbol{n}$ be the coefficient of balancing which play role for balances. If $\Phi=(\Phi_1,\Phi_2,\ldots,\Phi_n)^T$ goes to $\left(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\right)^T$, then $(n\varpi_1\Lambda_1,n\varpi_2\Lambda_2,\ldots,n\varpi_n\Lambda_n)^T$ goes to $(\Lambda_1,\Lambda_2,\ldots\Lambda_n)^T$. $(\varpi_1,\varpi_2,\ldots,\varpi_n)^T$ be the weighted vector and $\Phi=(\Phi_1,\Phi_2,\ldots,\Phi_n)^T$ be the associated vector both have the same conditions, such as belong to the closed interval and their sum should be equal to one.

Definition 11. The I-CPFOWA operator can be defined as follow:

$$\text{I - CPFOWA}_{\Phi}\left(\langle u_1,\Lambda_1\rangle,\langle u_2,\Lambda_2\rangle,\ldots,\langle u_n,\Lambda_n\rangle\right)$$

$$=\left(\sqrt{1-\prod_{j=1}^{n}\left(1-\hbar^2_{\Lambda_{\varphi(j)}}\right)^{\Phi_j}}\,e^{i2\pi\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\frac{{}^{\alpha}\Lambda_{\varphi(j)}}{2\pi}\right)^2\right)^{\Phi_j}}},\ \prod_{j=1}^{n}\left(\bar{\lambda}_{\Lambda_{\varphi(j)}}\right)^{\Phi_j}e^{i2\pi\prod_{j=1}^{n}\left(\frac{{}^{\beta}\Lambda_{\varphi(j)}}{2\pi}\right)^{\Phi_j}}\right), \tag{11}$$

where $\Phi=(\Phi_1,\Phi_2,\ldots,\Phi_n)^T$ be the weighted vector of Λ_j with conditions such as $\Phi_j\in[0,1]$ and $\sum_{j=1}^{n}\Phi_j=1$, $\Lambda_{\varphi(j)}$ be the value of CPFOWA pair $\left(u_j,\Lambda_j\right)$ having the jth largest u_j in $\left(u_j,\Lambda_j\right)$ is referred to as the order inducing variable and Λ_j as the complex Pythagorean fuzzy argument.

Theorem 5. Let $\left\langle u_j,\Lambda_j\right\rangle(j=1,2,\ldots,n)$ be a collection of 2-tuples, then their aggregated value by using the I-CPFOWA operator is still CPFV.

Proof. Proof is similar to Theorem 3, so here we omit.

Definition 12. The I-CPFHA operator can be defined as:

$$\text{I - CPFHA}_{\varpi,\Phi}\left(\langle u_1,\Lambda_1\rangle,\langle u_2,\Lambda_2\rangle,\ldots,\langle u_n,\Lambda_n\rangle\right)$$

$$=\left(\sqrt{1-\prod_{j=1}^{n}\left(1-\hbar^2_{\dot{\Lambda}_{\varphi(j)}}\right)^{\Phi_j}}\,e^{i2\pi\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\frac{{}^{\alpha}\dot{\Lambda}_{\varphi(j)}}{2\pi}\right)^2\right)^{\Phi_j}}},\ \prod_{j=1}^{n}\left(\bar{\lambda}_{\dot{\Lambda}_{\varphi(j)}}\right)^{\Phi_j}e^{i2\pi\prod_{j=1}^{n}\left(\frac{{}^{\beta}\dot{\Lambda}_{\varphi(j)}}{2\pi}\right)^{\Phi_j}}\right), \tag{12}$$

where $\dot{\Lambda}_{\varphi(j)}$ be the greatest value and $\dot{\Lambda}_{\varphi(j)}=n\varpi_j\Lambda_j$. And $\boldsymbol{n}$ be the coefficient of balancing which play role for balances. If $\Phi=(\Phi_1,\Phi_2,\ldots,\Phi_n)^T$ goes to $\left(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\right)^T$, then $(n\varpi_1\Lambda_1,n\varpi_2\Lambda_2,\ldots,n\varpi_n\Lambda_n)^T$ goes to $(\Lambda_1,\Lambda_2,\ldots\Lambda_n)^T$. $(\varpi_1,\varpi_2,\ldots,\varpi_n)^T$ be the weighted vector and $\Phi=(\Phi_1,\Phi_2,\ldots,\Phi_n)^T$ be their associated with $\sum_{j=1}^{n}\Phi_j=1$.

5. Application of the new techniques for emergency decision-making

In this section, we develop some new methods such as CPFWA operator, CPFOWA operator, CPFHA operator, I-CPFOWA operator and I-CPFHA operator to control the outbreak of COVID-19.

Table 1
Linguistic variables

	$Ç_1$	$Ç_2$	$Ç_3$	$Ç_4$	$Ç_5$	$Ç_6$	$Ç_7$	$Ç_8$
HCS	G	G	E	M	VVB	VB	E	VG
RC	M	VB	VG	G	G	E	B	VG
LD	B	G	E	M	VB	G	VVB	G
RN	G	G	M	VB	G	VG	E	VVB

Health care system (HCS), Risk communication (RC), Lock down (LD), Research need (RN).

Table 2
Linguistic variables and their corresponding CPF-numbers

Linguistic variables	Corresponding CPF-numbers
Very Very Bad (VVB)	$\left(0.2e^{i2\pi(0.2)}, 0.9e^{i2\pi(0.9)}\right)$
Very Bad (VB)	$\left(0.2e^{i2\pi(0.3)}, 0.9e^{i2\pi(0.8)}\right)$
Bad (B)	$\left(0.4e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.8)}\right)$
Medium (M)	$\left(0.6e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.5)}\right)$
Good (G)	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$
Very Good (VG)	$\left(0.8e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.2)}\right)$
Excellent (E)	$\left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)$

The spreading of coronavirus, is so fast, therefore the WHO collaborates with public health experts and laboratory organizations, clinical management. This problem is taken from [49], where the author used generalized interval-valed Einstein aggregation operators, but here we used complex algebraic aggregation operators.

Algorithm: Let $A = \{A_1, A_2, ..., A_n\}$ be a finite set of $\boldsymbol{n}$ alternatives and $\boldsymbol{Ç} = \left\{\boldsymbol{Ç}_1, \boldsymbol{Ç}_2, ..., \boldsymbol{Ç}_m\right\}$ be a finite set of $\boldsymbol{m}$ different criteria whose weight vector is $\Phi = (\Phi_1, \Phi_2, \Phi_3, ..., \Phi_m)^T$ such that $\Phi_j \in [0, 1]$ and $\sum_{j=1}^{m} \Phi_j = 1$. To determine the more suitable alternative for selection, the proposed approaches are used to develop a MAGDM problrm under the complex Pythagorean fuzzy environment. The major steps are the following:

Step 1: Collect information of the all experts about the alternatives under the different criteria.

Step 2: Using the proposed operators to calculate all the preference values.

Step 3: To determine the scores of all preference values.

Step 4: Based on the score function select the most suitable alternative.

6. Different techniques for emergency decision making

In this section, the experts provide their ideas in the terms of linguistic variables:

Based of Table 1, the health experts evaluated fuzzy linguistic information and their corresponding complex Pythagorean fuzzy numbers presented in Table 2.

6.1. Some algebraic aggregation operators

In this section, we develop some algebraic operators for emergency decision making to select the best alternative from all alternatives under consideration. The major steps of the proposed methods are the following.

Step 1: In this step, we provide the evauation of emergency measures to reduce the outbreak of coronavirus in linguistic terms. After that we convert the linguistic terms into CPFNs.

Step 2: Utilize CPFWA operators to calculate all the preference values.

Step 3: Calculate the scores of all preference values.

Step 4: Ranking the alternatives and select that having the highest score function.

6.2. Some induced aggregation operators

In this section, we develop some induced operators for emergency decision making. The major steps are the following.

Step 1: In this step, we convert the linguistic terms in the terms of I-CPFNs.

Step 2: Utilize I-CPFWA operator to calculate all the preference values.

Step 3: Calculate the score of preference values

Step 4: Select that alternative having the highest score function.

7. Example for the proposed methods

In this section, we present a real case such as people health emergency decision making for outbreak of coronavirus, reported in China.

Case study: In this section, we develop an application of the proposed methods for COVID-19, reported in China. The first case was reported December 2019, in Wuhan, China and appeared in the whole world in March 2020 as a prevalent and widespread disease. The Chinese government implemented the largest lockdown in human history in the beginning of 2020, due to this millions of people were distressed. The ratio of spreading of COVID-19, is very high, therefore the World health Organization (WHO) collaborates with public health experts and laboratory organizations, prevention and checking of diseases, clinical management and mathematical modeling. To show the validity and rationality of the new methods, we develop a real case that is COVID-19. Generally the following eight factors reduce the spreading of COVID-19.

Clinical Management ($\c{C}_1$): Currently there are no suitable treatments for COVID-19, however Clinical management requires timely adoption of appropriate creativities for infection avoidance and control for complication management, providing strategic organ care where necessary.

First-aid Training ($\c{C}_2$): The spread rate of COVID-19 is very high, so avoid and sidestep from those people having the symptoms of this disease. So it is recommended that attend a fully supervised online or practical first aid course persons to learn how to get out medical emergency.

Increased Personal Protective Equipment ($\c{C}_3$): The shortage of testing kits is a main factor, in this situation we have to increase the testing kits production. It is the responsibility of local government to provide some necessary equipment's such as face masks, gloves, ventilators and gowns in every region in the country, because the best way is personal hygiene.

Expert Technician ($\c{C}_4$): The spread rate of this disease is very high, therefore the expert technician are working on potential and suitable vaccines.

Banned intra-city Transportation ($\c{C}_5$): It is necessary for local government that takes step for banned intra-city transportation and stopped all flights for local people safety.

Table 3
Corresponding linguistic information

	HCS	RC	LD	RN
$\c{C}_1$	***G***	***M***	***B***	***G***
$\c{C}_2$	G	***VB***	***G***	***G***
$\c{C}_3$	***E***	***VG***	***E***	***M***
$\c{C}_4$	***M***	***G***	M	***VB***
$\c{C}_5$	***VVB***	***G***	***VB***	***G***
$\c{C}_6$	***VB***	***E***	***G***	***VG***
$\c{C}_7$	***E***	***B***	***VVB***	***E***
$\c{C}_8$	***VG***	***VG***	***G***	***VVB***

Global Uncertainty ($\c{C}_6$): Economic effect: The whole world depends on the Chinese growth. But due to the coronavirus transport sectors will damage the economic and also harm trade and consumption, so coronavirus directly affected the global market in the current environment.

Planning and cooperation country level ($\c{C}_7$): It is necessary for government to cooperation and collaboration with its provinces, to reduce the uncertainty of COVID-19 in has country.

Monitoring ($\c{C}_8$): It is necessary for government to appoint the experts to track the current situation and provide the positive suggestion on how to control the current situation.

7.1. By algebraic aggregation operators

Step 1: On the based of Table 1: The health experts weighed their information in the form fuzzy linguistic variables shown in Table 3:

Convert linguistic terms into fuzzy information:

Step 2: Using the CPFWA aggregation operators to calculate the all preference values, where $\Phi = (0.10,\ 0.10,\ 0.11,\ 0.12,\ 0.13,\ 0.14,\ 0.15,\ 0.15)^T$ be their weighted vector. Thus we get

$$\Im_1 = \left(0.757e^{i2\pi(0.754)}, 0.437e^{i2\pi(0.378)}\right),$$

$$\Im_2 = \left(0.750e^{i2\pi(0.747)}, 0.446e^{i2\pi(0.368)}\right)$$

$$\Im_3 = \left(0.696e^{i2\pi(0.691)}, 0.570e^{i2\pi(0.504)}\right),$$

$$\Im_4 = \left(0.738e^{i2\pi(0.735)}, 0.485e^{i2\pi(0.411)}\right)$$

Table 4
Complex Pythagorean fuzzy information

	HCS	RC	LD	RN
$\c{C}_1$	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$	$\left(0.6e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.5)}\right)$	$\left(0.4e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.8)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$
$\c{C}_2$	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$	$\left(0.2e^{i2\pi(0.3)}, 0.9e^{i2\pi(0.8)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$
$\c{C}_3$	$\left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.2)}\right)$	$\left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)$	$\left(0.6e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.5)}\right)$
$\c{C}_4$	$\left(0.6e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.5)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$	$\left(0.6e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.5)}\right)$	$\left(0.2e^{i2\pi(0.3)}, 0.9e^{i2\pi(0.8)}\right)$
$\c{C}_5$	$\left(0.2e^{i2\pi(0.2)}, 0.9e^{i2\pi(0.9)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$	$\left(0.2e^{i2\pi(0.3)}, 0.9e^{i2\pi(0.8)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$
$\c{C}_6$	$\left(0.2e^{i2\pi(0.3)}, 0.9e^{i2\pi(0.8)}\right)$	$\left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.2)}\right)$
$\c{C}_7$	$\left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)$	$\left(0.4e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.8)}\right)$	$\left(0.2e^{i2\pi(0.2)}, 0.9e^{i2\pi(0.9)}\right)$	$\left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)$
$\c{C}_8$	$\left(0.8e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.2)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.2)}\right)$	$\left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)$	$\left(0.2e^{i2\pi(0.2)}, 0.9e^{i2\pi(0.9)}\right)$

Table 5
Inducing complex Pythagorean fuzzy information

	HCS	RC	LD	RN
$\c{C}_1$	$\left\langle 0.9, \left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)\right\rangle$	$\left\langle 0.8, \left(0.6e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.5)}\right)\right\rangle$	$\left\langle 0.6, \left(0.4e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.8)}\right)\right\rangle$	$\left\langle 0.9, \left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)\right\rangle$
$\c{C}_2$	$\left\langle 0.8, \left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)\right\rangle$	$\left\langle 0.7, \left(0.2e^{i2\pi(0.3)}, 0.9e^{i2\pi(0.8)}\right)\right\rangle$	$\left\langle 0.7, \left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)\right\rangle$	$\left\langle 0.8, \left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)\right\rangle$
$\c{C}_3$	$\left\langle 0.6, \left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)\right\rangle$	$\left\langle 0.6, \left(0.8e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.2)}\right)\right\rangle$	$\left\langle 0.5, \left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)\right\rangle$	$\left\langle 0.7, \left(0.6e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.5)}\right)\right\rangle$
$\c{C}_4$	$\left\langle 0.5, \left(0.6e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.5)}\right)\right\rangle$	$\left\langle 0.5, \left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)\right\rangle$	$\left\langle 0.4, \left(0.6e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.5)}\right)\right\rangle$	$\left\langle 0.6, \left(0.2e^{i2\pi(0.3)}, 0.9e^{i2\pi(0.8)}\right)\right\rangle$
$\c{C}_5$	$\left\langle 0.7, \left(0.2e^{i2\pi(0.2)}, 0.9e^{i2\pi(0.9)}\right)\right\rangle$	$\left\langle 0.4, \left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)\right\rangle$	$\left\langle 0.8, \left(0.2e^{i2\pi(0.3)}, 0.9e^{i2\pi(0.8)}\right)\right\rangle$	$\left\langle 0.5, \left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)\right\rangle$
$\c{C}_6$	$\left\langle 0.3, \left(0.2e^{i2\pi(0.3)}, 0.9e^{i2\pi(0.8)}\right)\right\rangle$	$\left\langle 0.3, \left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)\right\rangle$	$\left\langle 0.9, \left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)\right\rangle$	$\left\langle 0.3, \left(0.8e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.2)}\right)\right\rangle$
$\c{C}_7$	$\left\langle 0.4, \left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)\right\rangle$	$\left\langle 0.2, \left(0.4e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.8)}\right)\right\rangle$	$\left\langle 0.2, \left(0.2e^{i2\pi(0.2)}, 0.9e^{i2\pi(0.9)}\right)\right\rangle$	$\left\langle 0.2, \left(0.9e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.2)}\right)\right\rangle$
$\c{C}_8$	$\left\langle 0.2, \left(0.8e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.2)}\right)\right\rangle$	$\left\langle 0.1, \left(0.8e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.2)}\right)\right\rangle$	$\left\langle 0.3, \left(0.8e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.4)}\right)\right\rangle$	$\left\langle 0.1, \left(0.2e^{i2\pi(0.2)}, 0.9e^{i2\pi(0.9)}\right)\right\rangle$

Step 3: In this step, we calculate the scores of all preference values as follows:

$$s(\Im_1) = \left((0.757)^2 - (0.437)^2\right) + \frac{1}{4\pi^2}\left((0.754)^2 - (0.378)^2\right) = 0.807$$

$$s(\Im_2) = \left((0.750)^2 - (0.446)^2\right) + \frac{1}{4\pi^2}\left((0.747)^2 - (0.368)^2\right) = 0.786$$

$$s(\Im_3) = \left((0.696)^2 - (0.570)^2\right) + \frac{1}{4\pi^2}\left((0.691)^2 - (0.504)^2\right) = 0.382$$

$$s(\Im_4) = \left((0.738)^2 - (0.485)^2\right) + \frac{1}{4\pi^2}\left((0.735)^2 - (0.411)^2\right) = 0.680$$

Step 4: Based on the score, we have $A_1 \succ A_2 \succ A_4 \succ A_3$. Thus the best alternative is A_1.

Table 6
Aggregated CPF information by CPFWA operator

Alternatives	Aggregated values
HCS	$\left(0.758e^{i2\pi(0.755)}, 0.435e^{i2\pi(0.375)}\right)$
RC	$\left(0.750e^{i2\pi(0.747)}, 0.446e^{i2\pi(0.368)}\right)$
LD	$\left(0.701e^{i2\pi(0.753)}, 0.558e^{i2\pi(0.496)}\right)$
RN	$\left(0.738e^{i2\pi(0.735)}, 0.485e^{i2\pi(0.411)}\right)$

Table 7
Aggregated CPF information by CPFOWA operator

Alternatives	Aggregated values
HCS	$\left(0.764e^{i2\pi(0.756)}, 0.434e^{i2\pi(0.382)}\right)$
RC	$\left(0.761e^{i2\pi(0.746)}, 0.432e^{i2\pi(0.367)}\right)$
LD	$\left(0.702e^{i2\pi(0.752)}, 0.559e^{i2\pi(0.494)}\right)$
RN	$\left(0.740e^{i2\pi(0.744)}, 0.484e^{i2\pi(0.413)}\right)$

7.2. By induced aggregation operators

Step 1: On the based of Table 1 the health experts weighed their information in the form of inducing fuzzy linguistic variables shown in Table 5:

Step 2: Using the I-CPFWA qggregation operators to calculate all the preference values, where $\Phi = (0.10,\ 0.10,\ 0.11,\ 0.12,\ 0.13,\ 0.14,\ 0.15,\ 0.15)^T$ be their weighted vector, then we have

$$\Im_1 = \left(0.758e^{i2\pi(0.755)}, 0.435e^{i2\pi(0.375)}\right),$$
$$\Im_2 = \left(0.750e^{i2\pi(0.747)}, 0.446e^{i2\pi(0.368)}\right)$$
$$\Im_3 = \left(0.701e^{i2\pi(0.753)}, 0.558e^{i2\pi(0.496)}\right),$$
$$\Im_4 = \left(0.738e^{i2\pi(0.735)}, 0.485e^{i2\pi(0.411)}\right)$$

Step 3: In this step, we calculate the scores of all preference values as follows:

$$s(\Im_1) = 0.814,\ s(\Im_2) = 0.786,$$
$$s(\Im_3) = 0.501,\ s(\Im_4) = 0.680$$

Table 8
Aggregated CPF information by CPFHA operator

Alternatives	Aggregated values
HCS	$\left(0.774e^{i2\pi(0.764)}, 0.421e^{i2\pi(0.379)}\right)$
RC	$\left(0.771e^{i2\pi(0.756)}, 0.442e^{i2\pi(0.377)}\right)$
LD	$\left(0.712e^{i2\pi(0.753)}, 0.560e^{i2\pi(0.492)}\right)$
RN	$\left(0.741e^{i2\pi(0.754)}, 0.485e^{i2\pi(0.423)}\right)$

Table 9
Aggregated CPF information by I-CPFOWA operator

Alternatives	Aggregated values
HCS	$\left(0.758e^{i2\pi(0.755)}, 0.435e^{i2\pi(0.375)}\right)$
RC	$\left(0.750e^{i2\pi(0.747)}, 0.446e^{i2\pi(0.368)}\right)$
LD	$\left(0.701e^{i2\pi(0.753)}, 0.558e^{i2\pi(0.496)}\right)$
RN	$\left(0.738e^{i2\pi(0.735)}, 0.485e^{i2\pi(0.411)}\right)$

Table 10
Aggregated CPF information by I-CPFHA operator

Alternatives	Aggregated values
HCS	$\left(0.756e^{i2\pi(0.756)}, 0.432e^{i2\pi(0.365)}\right)$
RC	$\left(0.751e^{i2\pi(0.757)}, 0.447e^{i2\pi(0.378)}\right)$
LD	$\left(0.721e^{i2\pi(0.743)}, 0.568e^{i2\pi(0.492)}\right)$
RN	$\left(0.748e^{i2\pi(0.725)}, 0.495e^{i2\pi(0.402)}\right)$

Table 11
Score function of the proposed methods

	CPFWA	CPFOWA	CPFHA	I-CPFOWA	I-CPFHA
HCS	**0.807**	**0.820**	**0.861**	**0.814**	**0.823**
RC	**0.786**	**0.814**	**0.828**	**0.786**	**0.794**
LD	**0.382**	**0.501**	**0.518**	**0.501**	**0.507**
RN	**0.680**	**0.696**	**0.703**	**0.680**	**0.678**

Step 4: Based on the score, we have $A_1 \succ A_2 \succ A_4 \succ A_3$. Thus the best alternative is A_1.

Here we, aggregated information of all the proposed methods as follows:

Table 12
Comparative analysis of the proposed methods

Methods	Scores	Ranking
CPFWA	$s(\Im_1) \succ s(\Im_2) \succ s(\Im_4) \succ s(\Im_3)$	$A_1 \succ A_2 \succ A_4 \succ A_3$
CPFOWA	$s(\Im_1) \succ s(\Im_2) \succ s(\Im_4) \succ s(\Im_3)$	$A_1 \succ A_2 \succ A_4 \succ A_3$
CPFHA	$s(\Im_1) \succ s(\Im_2) \succ s(\Im_4) \succ s(\Im_3)$	$A_1 \succ A_2 \succ A_4 \succ A_3$
I-CPFWA	$s(\Im_1) \succ s(\Im_2) \succ s(\Im_4) \succ s(\Im_3)$	$A_1 \succ A_2 \succ A_4 \succ A_3$
I-CPFHA	$s(\Im_1) \succ s(\Im_2) \succ s(\Im_4) \succ s(\Im_3)$	$A_1 \succ A_2 \succ A_4 \succ A_3$

Coronavirus information in Pakistan

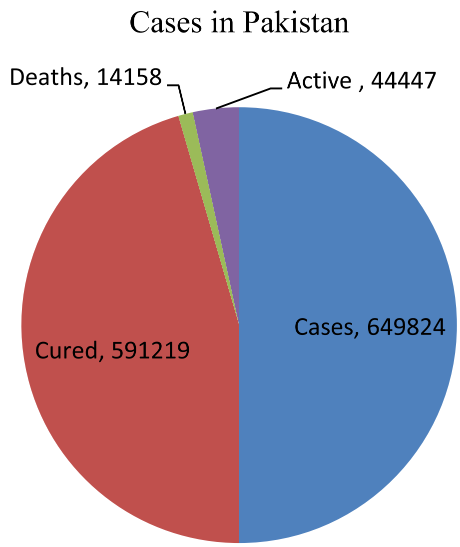

Fig. 1.

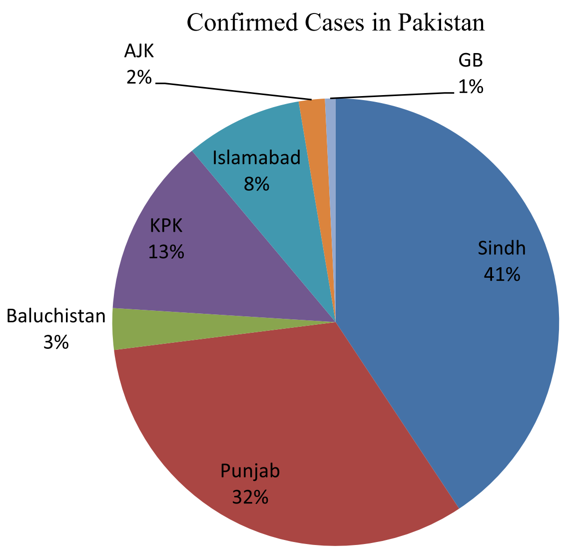

Fig. 2.

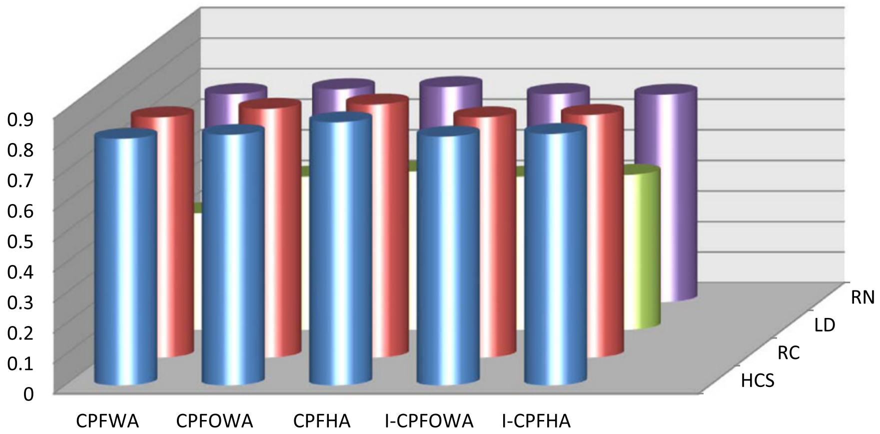

Fig. 3.

Table 13
Stated the sensitivity analysis

Model	Uncertainty	Falsity	Hesitation	Periodicity	2-D information	Square in power
FSs	Yes	No	No	No	No	No
IFSs	Yes	Yes	Yes	No	No	No
PFSs	Yes	Yes	Yes	No	No	No
CFSs	Yes	No	No	Yes	Yes	No
CIFSs	Yes	Yes	Yes	Yes	Yes	No
CPFSs	Yes	Yes	Yes	Yes	Yes	Yes

8. Limitations

Complex Pythagorean fuzzy set is the generalization of the existing studies such as fuzzy set, intuitionistic fuzzy set, Pythagorean fuzzy set, complex fuzzy set, and complex intuitionistic fuzzy set by considering much more information related to an object during the process and to handle the two-dimensional information in a single set.

However, these novel methods have some limitations such as: If $\mathfrak{M} = \left(\hbar e^{i\alpha}, \bar{\lambda} e^{i\beta}\right)$ be complex Pythagorean fuzzy numbers, where $i = \sqrt{-1}$, $\hbar$, $\bar{\lambda} \in [0, 1]$ are called complex membership and non-membership function respectively, and α, $\beta \in [0, 2\pi]$ with some conditions such as, $0 \leq \hbar^2 + \bar{\lambda}^2 \leq 1$ and $0 \leq \left(\frac{\alpha}{2\pi}\right)^2 + \left(\frac{\beta}{2\pi}\right)^2 \leq 1$. Therefore throughout this paper we consider those complex Pythagorean fuzzy numbers which satisfy the above restriction.

8.1. Comparative analysis

Complex Pythagorean fuzzy set is the generalization of the existing studies such as fuzzy set [18], intuitionistic fuzzy set [19], Pythagorean fuzzy set [24], complex fuzzy set [32], and complex intuitionistic fuzzy set [36, 37] by considering much more information related to an object during the process and to handle the two-dimensional information in a single set. For instance fuzzy set (with only crisp membership degrees with amplitude term only), intuitionistic fuzzy set and Pythagorean fuzzy set (with a real-valued membership and non-membership degrees and only considered amplitude term), complex fuzzy set (contains only complex-valued membership degree), complex intuitionistic fuzzy set contains information (both the membership and non-membership degrees are complex valued with condition their sum less than or equal to one), but complex Pythagorean fuzzy set contains information (both the membership and non-membership degrees are complex valued with condition sum of their squares less than or equal to one). Thus, the proposed aggregation operators under CPFSs environment are more generalizations than the existing operators.

Mahmood et al. [46], Akram et al. [47, 48] developed some aggregation operators using complex fuzzy information, but these papers having no concept of inducing aggregation operators and averaging aggregation operators which are more powerful tools for decision making problems. Thus in this paper we introduce the notion of some novel aggregation operators such as CPFWA operator, CPFOWA operator, CPFHA operator, I-CPFOWA operator, I-CPFHA operator. Hence our proposed methods are more flexible and effectiveness as compared to their existing methods.

The proposed methods not only perform for CPF data rather they can be successfully applied to one dimensional phenomena for Pythagorean fuzzy information and intuitionistic fuzzy information by taking their phase terms equal to zero. So, the proposed methods are more flexible to overcome the limitations of the existing methods.

9. Conclusion

COVID-19 is an infectious disease that can be spread through touch and is believed to be spread throughout the entire population through direct contact between people. Outbreak prevention measures aimed at reducing the number of mixed populations can slow the peak and reduce the final extent of the epidemic. So at the current position it is a challenge for the world to control the spreading of COVID-19. Therefore in this paper we have developed a complex Pythagorean fuzzy model to control and analysis the infection of COVID-19. For this, we presented some methods such as CPFWA operator, CPFOWA operator, CPFHA operator, I-CPFOWA operator and I-CPFHA operator. At the end of the paper, we have developed an illustrative emergency situation of COVID-19 to show the effectiveness along with a sensitivity analysis.

In further research, it is necessary to give the applications of these operators to the other domains such as, inducing variables, pattern recognition, Confidence levels, Hamacher operators, Power operators, Symmetric operator, Logarithmic operators, Dombi operators, Linguistic terms, trigonometric operation, ranking method for normal intuitionistic sets, etc.

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