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Correction

Comment on "Improvement of the distance between intuitionistic fuzzy sets and its applications"

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Abstract. Here, necessary corrections on the proof the Theorem 1 of Xu (J Intell Fuzzy Syst 33(3): 1563-1575, 2017) are stated in brief. Throughout, we use the same notations and equation numbers as in Xu.

Keywords: Intuitionistic fuzzy sets, distance measure, Euclidean distance

Intuitionistic fuzzy sets(IFSs) were proposed by Atanassov [1] as a generalization of the fuzzy sets. As the most interesting topics in IFSs theory, distance measures are involved in fuzzy decision making, patter recognition, fuzzy reasoning, etc, [2–6].

In 2017, a measuring distance between intuitionistic fuzzy sets, proposed by Xu [5], was successfully applied into pattern recognition problems and medical diagnosis. However, there is a small mistake about the proof of the Theorem 1 in Xu [5]. In order to show the detailed correction instructions, the definitions involved in the paper [5] are as follows.

Definition 1. A metric distance D in a non-empty set X is a real value function $D: X \times X \to [0, +\infty)$, which satisfies the following conditions, $\forall x, y, z \in X$:
($MD1$) $D(x, y) = 0$ if and only if $x = y$;
($MD2$) $D(x, y) = D(y, x)$;
($MD3$) $D(x, y) + D(y, z) \geq D(x, z)$.

Definition 2. [7] Let D be a mapping: $\mathrm{IFSs}(X) \times \mathrm{IFSs}(X) \to [0, 1]$. For $\forall A, B, C \in \mathrm{IFSs}(X)$, $D(A, B)$ is a distance measure between IFSs A and B, if D satisfies the following properties:
($DP1$) $0 \leq D(A, B) \leq 1$;
($DP2$) $D(A, B) = 0$ if and only if $A = B$;
($DP3$) $D(A, B) = D(B, A)$;
($DP4$) If $A \subseteq B \subseteq C$, then $D(A, C) \geq D(A, B)$, $D(A, C) \geq D(B, C)$.

Definition 3. [5] Let $A=\{\langle x, \mu_A(x), \upsilon_A(x)\rangle | x \in X\}$ be a IFSs in $X=\{x\}$, then the assignments of the hesitancy degree $\pi_A(x)$ to membership degree $\mu_A(x)$ and non-membership degree $\nu_A(x)$ are defined as

$$\begin{aligned} Assign_A^{\pi\mu}(x) &= [\pi_A(x) + 2\mu_A(x)]/2, \\ Assign_A^{\pi\nu}(x) &= [\pi_A(x) + 2\nu_A(x)]/2. \end{aligned} \tag{1}$$

We take the four parts $\mu_A(x)$, $\nu_A(x)$, $Assign_A^{\pi\mu}(x)$ and $Assign_A^{\pi\nu}(x)$ into account the distances between IFSs, thereby a new distance measure, denoted as D_{IFSs}, is defined.

Definition 4. [5] Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$, $B = \{\langle x, \mu_B(x), \upsilon_B(x)\rangle | x \in X\}$ be two IFSs in $X=\{x\}$, then the distance measure between A and B is defined as

$$D_{\mathrm{IFSs}}(A, B) = \frac{1}{2}\sqrt{(\Delta_\mu^{AB})^2 + (\Delta_\nu^{AB})^2 + (\Delta_{\pi\mu}^{AB})^2 + (\Delta_{\pi\nu}^{AB})^2}, \tag{2}$$

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where, Δ_{μ}^{AB}=μ_A-μ_B, Δ_{ν}^{AB}=ν_A-ν_B, $\Delta_{\pi\mu}^{AB}$ = $Assign_A^{\pi\mu}$- $Assign_B^{\pi\mu}$ and $\Delta_{\pi\nu}^{AB}$=$Assign_A^{\pi\nu}$ -$Assign_B^{\pi\nu}$.

Theorem 1. [5] *Let* $A=\{\langle x, \mu_A(x), \upsilon_A(x)\rangle | x \in X\}$, $B=\{\langle x, \mu_B(x), \upsilon_B(x)\rangle | x \in X\}$ *be two IFSs in* $X=\{x\}$, *then* $D_{\text{IFSs}}(A, B)$ *is a distance measure satisfying the Definition 1 and Definition 2.*

In the paper [5], the proof of the third step is as follows.

3) For $\forall A, B, C \in \text{IFSs}(X)$, we have

$$(\Delta_{\mu}^{AC})^2 = (\Delta_{\mu}^{AB} + \Delta_{\mu}^{BC})^2 \leq (\Delta_{\mu}^{AB})^2 + (\Delta_{\mu}^{BC})^2;$$
$$(\Delta_{\nu}^{AC})^2 = (\Delta_{\nu}^{AB} + \Delta_{\nu}^{BC})^2 \leq (\Delta_{\nu}^{AB})^2 + (\Delta_{\nu}^{BC})^2;$$
$$(\Delta_{\pi\mu}^{AC})^2 = (\Delta_{\pi\mu}^{AB} + \Delta_{\pi\mu}^{BC})^2 \leq (\Delta_{\pi\mu}^{AB})^2 + (\Delta_{\pi\mu}^{BC})^2;$$
$$(\Delta_{\pi\nu}^{AC})^2 = (\Delta_{\pi\nu}^{AB} + \Delta_{\pi\nu}^{BC})^2 \leq (\Delta_{\pi\nu}^{AB})^2 + (\Delta_{\pi\nu}^{BC})^2. \tag{3}$$

Thus, $D_{\text{IFSs}}(A, C) \leq D_{\text{IFSs}}(A, B) + D_{\text{IFSs}}(B, C)$, which indicates that D_{IFSs} satisfies $(MD3)$.

However, the conclusion $D_{\text{IFSs}}(A, C) \leq D_{\text{IFSs}}(A, B) + D_{\text{IFSs}}(B, C)$ is derived from the formula (3), which is a wrong logical reasoning. Where, it should be noted that the formula (3) is correct. In fact, from the formula (3) and the property of inequality, we can obtained

$$[(\Delta_{\mu}^{AC})^2 + (\Delta_{\nu}^{AC})^2 + (\Delta_{\pi\mu}^{AC})^2 + (\Delta_{\pi\nu}^{AC})^2] \leq \{[(\Delta_{\mu}^{AB})^2 + (\Delta_{\nu}^{AB})^2 + (\Delta_{\pi\mu}^{AB})^2 + (\Delta_{\pi\nu}^{AB})^2] + [(\Delta_{\mu}^{BC})^2 + (\Delta_{\nu}^{BC})^2 + (\Delta_{\pi\mu}^{BC})^2 + (\Delta_{\pi\nu}^{BC})^2]\}. \tag{4}$$

While, from the Definition 4, we have

$$4[D_{\text{IFSs}}(A,C)]^2=(\Delta_{\mu}^{AC})^2+(\Delta_{\nu}^{AC})^2+(\Delta_{\pi\mu}^{AC})^2+(\Delta_{\pi\nu}^{AC})^2,$$
$$4[D_{\text{IFSs}}(A,B)]^2=(\Delta_{\mu}^{AB})^2+(\Delta_{\nu}^{AB})^2+(\Delta_{\pi\mu}^{AB})^2+(\Delta_{\pi\nu}^{AB})^2,$$
$$4[D_{\text{IFSs}}(B,C)]^2=(\Delta_{\mu}^{BC})^2+(\Delta_{\nu}^{BC})^2+(\Delta_{\pi\mu}^{BC})^2+(\Delta_{\pi\nu}^{BC})^2.$$

Therefore,

$$[D_{\text{IFSs}}(A, C)]^2 \leq [D_{\text{IFSs}}(A, B)]^2 + [D_{\text{IFSs}}(B, C)]^2. \tag{5}$$

However, based on the formula (5), it is not obtained

$$D_{\text{IFSs}}(A, C) \leq D_{\text{IFSs}}(A, B) + D_{\text{IFSs}}(B, C).$$

This shows that the proof of the paper [5] is incorrect.

The proper proof of the third step is as follows.

3) According to the Definition 4, the distance measure $2D_{\text{IFSs}}(A, B)$ can be viewed as the Euclidean distance between two points $(\mu_A, \nu_A, Assign_A^{\pi\mu}, Assign_A^{\pi\nu})$ and $(\mu_B, \nu_B, Assign_B^{\pi\mu}, Assign_B^{\pi\nu})$ in a four-dimensional real number space. For $\forall A, B, C \in \text{IFSs}(X)$, there are three real points.

$$A : (\mu_A, \nu_A, Assign_A^{\pi\mu}, Assign_A^{\pi\nu}),$$
$$B : (\mu_B, \nu_B, Assign_B^{\pi\mu}, Assign_B^{\pi\nu}),$$
$$C : (\mu_C, \nu_C, Assign_C^{\pi\mu}, Assign_C^{\pi\nu}).$$

Since the distance measure $2D_{\text{IFSs}}(A, B)$ is a metric distance, based on the third condition $(MD3)$ of the Definition 1 (The properties of triangular inequalities for Euclidean distance), we have

$$2D_{\text{IFSs}}(A, C) \leq 2D_{\text{IFSs}}(A, B) + 2D_{\text{IFSs}}(B, C),$$

which yields

$$D_{\text{IFSs}}(A, C) \leq D_{\text{IFSs}}(A, B) + D_{\text{IFSs}}(B, C).$$

The result shows that D_{IFSs} satisfies $(MD3)$.

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