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An Uncertain Multiple-Criteria Choice Method on Grounds of T-Spherical Fuzzy Data-Driven Correlation Measures

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Abstract. T-spherical fuzzy (T-SF) sets furnish a constructive and flexible manner to manifest higher-order fuzzy information in realistic decision-making contexts. The objective of this research article is to deliver an original multiple-criteria choice method that utilizes a correlation-focused approach toward computational intelligence in uncertain decision-making activities with T-spherical fuzziness. This study introduces the notion of T-SF data-driven correlation measures that are predicated on two types of the square root function and the maximum function. The purpose of these measures is to exhibit the overall desirability of choice options across all performance criteria using T-SF comprehensive correlation indices within T-SF decision environments. This study executes an application for location selection and demonstrates the effectiveness and suitability of the developed techniques in T-SF uncertain conditions. The comparative analysis and outcomes substantiate the justifiability and the strengths of the propounded methodology in pragmatic situations under T-SF uncertainties.
Key words: T-spherical fuzzy (T-SF) set, multiple-criteria choice method, correlation measure, T-SF comprehensive correlation index, location selection.

1. Introduction

Multiple-criteria choice modelling under uncertainty forms part of the intelligent decision support system and can be applied to explore an innovative advancement of intelligent decision-making approaches and models (Fernández-Martínez and Sánchez-Lozano, 2021; Jing *et al.*, 2021; Menekse and Camgoz-Akdag, 2022; Riaz *et al.*, 2021). Numerous multiple-criteria assessment models have flourished to evaluate predetermined choice options ascertained from (conflicting) performance criteria for finding the most suitable option (Al-Quran, 2021; Erdogan *et al.*, 2021; Kovač *et al.*, 2021; Naeem *et al.*, 2022).

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However, it is often troublesome and difficult to manipulate indistinct determinations and blurred assessments for quantifying performance ratings of the choice options in decision analysis processes within involuted and multiplex real-life environments (Al-Quran, 2021; Alsalem *et al.*, 2021; Liu *et al.*, 2021b; Menekse and Camgoz-Akdag, 2022; Oztaysi *et al.*, 2022). When there is intricate uncertain information in the assessment and evaluation processes of choice options, the current decision-making approaches may be challenging to ascertain the performance ratings of choice options on performance criteria, which can result in an unreliable and unacceptable evaluation outcome concerning the most desirable scheme (Chinram *et al.*, 2020; Cihat Onat, 2022; Jing *et al.*, 2021; Liu *et al.*, 2021a; Naeem *et al.*, 2022).

To overcome these types of difficulties, fuzzy sets are capable of providing a supportable representation of imprecise information both beneficially and efficiently (Kovač *et al.*, 2021; Liu *et al.*, 2021a; Wang, 2021; Wang *et al.*, 2021). In numerous realistic fields, fuzzy set theory has been generally accepted and recognized to conduct information modelling issues under uncertainty (Liu *et al.*, 2021b; Wang *et al.*, 2021). Nevertheless, ordinary fuzzy sets possess merely one membership function, which may be inadequate to fully expound the extent of uncertainty in the human cognition of things (Olugu *et al.*, 2021; Wang, 2021). As a result, several high-order fuzzy sets, such as uncertain sets involving intuitionistic, Pythagorean, q-rung orthopair, picture, spherical, and T-spherical fuzziness, have been successively advanced to appropriately manifest human subjective uncertainties in practice (Chen, 2022a, 2022b; Liu *et al.*, 2021c). In particular, the idea of T-spherical fuzzy (T-SF) sets, incipiently presented by Mahmood *et al.* (2019), can help bring the theoretical development and revolutionary implications according to its strengths of broadening the uncertain space via four parameters of impreciseness, thus composing favourable, neutral (so-called abstinence), unfavourable, and refusal evaluations (Alsalem *et al.*, 2021; Chen, 2022c; Wang and Zhang, 2022; Yang and Pang, 2022).

1.1. *T-SF Theory in Uncertain Decision Contexts*

T-SF sets generalize two uncertain sets on the grounds of the picture fuzzy configuration and the spherical fuzzy (SF) configuration. Picture fuzzy sets and SF sets were advocated by Cuong (2014) and Kahraman and Kutlu Gündoğdu (2018), respectively, and they are high-order mathematical constructions that are more general than ordinary fuzzy sets. Nonetheless, their membership functions are special types of membership functions of the T-SF structure. An illustration in Fig. 1 manifests some general variants of fuzzy sets involving four parameters. Herein, these parameters externalize four-dimensional membership functions consisting of a positive component (μ) for favourable evaluations, neutral component (η) for abstinence, negative component (ν) for unfavourable evaluations, and refusal component (γ) for refusal evaluations. The sum of μ, η, ν, and γ is equal to 1, which behaves as a prerequisite for the picture fuzzy configuration. The sum of μ^2, η^2, ν^2, and γ^2 is equal to 1, which indicates a prerequisite for the SF configuration. A positive integer q is placed where $q \in Z^+$. The sum of μ^q, η^q, ν^q, and γ^q is equal to 1, which demonstrates a prerequisite for the T-SF configuration. When $q = 1$ and $q = 2$,

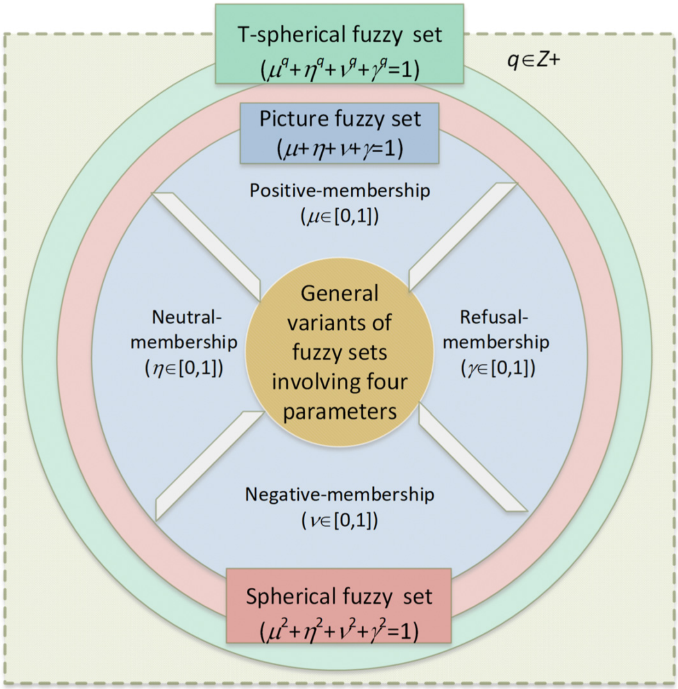


Fig. 1. General variants of fuzzy sets involving four parameters.

the T-SF configuration transforms into the picture fuzzy configuration and the SF configuration, respectively, which provides substance to the generalization of T-SF theory (Chen, 2022a, 2022b). Moreover, in the event that $\eta = 0$, the T-SF configuration transforms into the intuitionistic, Pythagorean, and q-rung orthopair fuzzy configurations when $q = 1, 2$, and $q \in Z^+$. By expounding the membership functions in a much wider range, T-SF sets can give expression to ambiguity and hesitation contained in human opinions in an efficacious manner (Mahnaz *et al.*, 2022; Nasir *et al.*, 2021; Wang, 2021). Moreover, the parameters μ, η, ν, and γ are adequate and appropriate for managing human determinations and assessments and elucidating complicated uncertainties within a changeable and unpredictable decision-making environment.

As of the advancement of T-SF theory in uncertain decision circumstances, a variety of valuable multiple-criteria assessment approaches and evaluation techniques have been constructed for facilitating intelligent decision support and aiding. By way of illustration, Abid *et al.* (2022) presented improved T-SF similarity measures to suggest an approach to decision-making and pattern recognition. Akram *et al.* (2022) analysed and addressed threats on social media platforms by employing an uncertain set of the complex cubic T-SF model and put forward a risk-assessing method for cyber-security and social media. By way of the interval-valued complex T-SF relation, Alothaim *et al.* (2022) identified Hasse diagrams in conformity with T-spherical partial orders to assess cybersecurity. Alsalem *et al.* (2021) expanded an opinion score-based technique and a fuzzy zero-inconsistency approach to T-SF contexts for implementing distribution decisions of the COVID-19 vaccine. Chen (2022a) instituted new notions of a superiority identifier and a guide index and propounded a T-SF regime prioritization procedure. Chen (2022b) advanced T-SF point operations to derive T-SF informational lower and upper estimations and propounded a point operator-driven method to treat complex assessment and evaluation tasks. By advocating a fresh distance measure with the Minkowski type, Chen (2022c) constructed Gaussian preference functions for conducting an evolved T-SF regime analysis. Nasir *et al.* (2021) investigated complex T-SF relations for depicting a global market's time-related interdependence in international trades. Ullah *et al.* (2021) advanced a new Dijkstra algorithm within the environment of T-SF graphs for addressing the shortest path issue. Wang *et al.* (2022) launched similarity measures and relations in interval-valued T-SF contexts and investigated an approach to medical diagnostic issues. To execute image segmentation, Xian *et al.* (2021) based on bias correction to establish a spatial T-SF C-means model.

Over and above that, Akram and Martino (2022) delivered T-SF soft rough average aggregation operations and further put forward a proficient group decision-making approach. To attain considerable accuracy in expounding fuzziness and indeterminate data, Al-Quran (2021) brought about weighted (geometric) averaging operators within T-spherical hesitant fuzzy environments for decision aiding. Chen *et al.* (2021) unfolded generalized and group-generalized T-SF geometric aggregation operations (including (ordered) weighted and hybrid geometric operations) to support multiple-criteria assessments. Next, in the circumstances of probabilistic T-spherical hesitant ambiguity, Gurmani *et al.* (2022) initiated aggregation operators and advanced an extended approach for boundary approximation region comparison in treating group decision issues. In interval-valued T-SF circumstances, Hussain *et al.* (2022a) utilized Frank aggregation operators to propose a method of assessing business proposals. Hussain *et al.* (2022b) exploited Aczel-Alsina t-norms and t-conorms to evolve Aczel-Alsina weighted average and geometric operation in T-SF settings for resolving decision-making issues. Karaaslan and Al-Husseinawi (2022) presented arithmetic and geometric averaging operations in hesitant T-spherical Dombi fuzzy settings for group decision-making. Khan *et al.* (2022) employed power-weighted averaging and geometric operations in complex T-SF settings to suggest a performance measurement method under uncertainties. Liu *et al.* (2021c) explored Maclaurin symmetric (weighted) mean operators for normal T-SF numbers and utilized such operators for multiple-criteria decision assistance. Mahnaz *et al.* (2022) put forward T-SF Frank aggregation operators and utilized them to decide on an unknown preference structure. Wang (2021) came up with T-SF rough numbers for consideration to deliver interaction power Heronian mean operations to carry out collective decision analysis. Wang and Zhang (2022) propounded an interaction power Heronian aggregation method to handle T-SF decision information for decision aiding. Yang and Pang (2022) exploited T-SF entropy and symmetric T-SF cross-entropy measures for weight assessing and advocated T-SF Dombi Bonferroni mean operations for tackling multiple attribute decisions. Yang *et al.* (2021) launched T-SF cloud weighted Heronian mean operators to fuse evaluation information for digital transformation solutions. Zedam *et al.* (2022) advocated complex T-SF Hamacher weighted averaging and geometric operations and delivered an approach to cleaner production evaluation. Zeng *et al.* (2021) explored linguistic Muirhead mean operators to form an intricate decision involving complex T-spherical dual hesitant uncertainties.

Table 1 summarizes a recent review of multiple-criteria assessment and related literature, including specific fuzzy models in the T-SF and extended T-SF setting, the main proposed methods, and the core concepts (or techniques) of these studies. The aforementioned literature manipulates uncertain information in the T-SF configuration from various perspectives to support multiple-criteria assessment tasks. These studies also confirm that handling uncertain information in decision-making environments with the T-SF con-

Table 1
State-of-the-art review of multiple-criteria assessment approaches in T-SF contexts.

Reference	Fuzzy model	Main proposed method	Core concept (or technique)
Abid *et al.* (2022)	T-SF set	Approach to decision-making and pattern recognition	Similarity measure Improved T-SF similarity measure
Akram and Martino (2022)	T-SF soft rough set	Group decision-making approach	T-SF soft rough average aggregation operation Parameterized fuzzy modelling
Akram *et al.* (2022)	Complex cubic T-SF set	Risk-assessing method for cyber-security and social media	Cartesian product Complex cubic T-SF relation Threat-solving for a social media platform
Alothaim *et al.* (2022)	Interval-valued complex T-SF set	Method of assessing cybersecurity	Interval-valued complex T-SF relation Hasse diagram of interval-valued complex T-spherical partial orders
Al-Quran (2021)	T-spherical hesitant fuzzy set	Multiple attribute decision-making method	Operational laws of T-spherical hesitant fuzzy information Weighted (geometric) averaging operation
Alsalem *et al.* (2021)	T-SF set	Fuzzy decision by opinion score method	Fuzzy-weighted zero-inconsistency approach Distribution decisions of COVID-19 vaccine
Chen (2022a)	T-SF set	T-SF regime I and II methods	Superiority identifier Guide index
Chen (2022b)	T-SF set	Point operator-driven approach	T-SF point operation for upper and lower estimations Continuous ordered weighted average operation
Chen (2022c)	T-SF set	T-SF regime methodology	Gaussian preference function Minkowski-type distance measure Joint generalized index
Chen *et al.* (2021)	T-SF set	Generalized and group-generalized T-SF aggregation method	(Group-)generalized T-SF geometric aggregation operation Weighted, ordered weighted, and hybrid geometric operations
Gurmani *et al.* (2022)	T-spherical hesitant fuzzy set	Border approximation area comparison approach	T-spherical hesitant fuzzy structure with probability Aggregation method in probabilistic T-spherical hesitant fuzzy settings

(*continued on next page*)

Table 1
(*continued*)

Reference	Fuzzy model	Main proposed method	Core concept (or technique)
Hussain *et al.* (2022a)	Interval-valued T-SF set	Method of assessing business proposals	Frank aggregation operation Interval-valued T-SF Frank weighted averaging and geometric operations
Hussain *et al.* (2022b)	T-SF set	T-SF Aczel-Alsina aggregation method	Aczel-Alsina t-(co)norm T-SF Aczel-Alsina weighted average geometric operation
Karaaslan and Al-Husseinawi (2022)	Hesitant T-SF set	Hesitant T-SF Dombi operation-based method	Aggregation approach by way of Dombi operation Hesitant T-spherical Dombi fuzzy aggregation operation
Khan *et al.* (2022)	Complex T-SF set	Performance measurement method	Power aggregation operation Complex T-SF power-weighted averaging and geometric operation
Liu *et al.* (2021c)	Normal T-SF number	Normal T-spherical fuzzy aggregation method	Maclaurin symmetric (weighted) mean operation
Mahnaz *et al.* (2022)	T-SF set	T-SF Frank aggregation method	Frank t-(co)norm Frank aggregation operation T-SF entropy measure
Nasir *et al.* (2021)	Complex T-SF set	Complex T-SF relation method	Time-related interdependence of global markets Interdependence of international trade
Ullah *et al.* (2021)	T-SF set	Shortest path problem-solving method	Dijkstra algorithm Shortest path in T-SF network
Wang (2021)	T-SF rough number	Interactive power Heronian mean operator approach	Interaction operational law Heronian mean operation Power average operation
Wang and Zhang (2022)	T-SF set	Interaction power Heronian aggregation method	T-SF interaction power Heronian mean operation Power averaging operation
Wang *et al.* (2022)	Interval-valued T-SF set	Approach to medical diagnosis	Interval-valued T-SF relation Similarity measure Information measure
Xian *et al.* (2021)	T-SF set	Spatial T-SF C-means method	T-spherical fuzzification technology T-SF C-means model with bias correction
Yang and Pang (2022)	T-SF set	Multiple attribute decision-making method	T-SF Dombi Bonferroni mean operation T-SF entropy measure Symmetric T-SF cross-entropy
Yang *et al.* (2021)	T-SF set	Assessment index system for digital transformation solutions	T-SF cloud T-SF cloud (weighted) Heronian mean operations
Zedam *et al.* (2022)	Complex T-SF set	Cleaner production evaluation method	Complex T-SF Hamacher weighted averaging operation Complex T-SF Hamacher weighted geometric operation
Zeng *et al.* (2021)	Complex T-spherical dual hesitant uncertain linguistic set	Muirhead mean-based approach to enterprise informatization level evaluation	Linguistic Muirhead mean operation Uncertain linguistic weighted (dual) Muirhead mean operations in complex T-spherical dual hesitant settings

figuration is a correct and effective way to build a multiple-criteria evaluation method framework.

In particular, based on Table 1, it can be easily observed that many researchers discussed the modularization of multiple-criteria choice methods in the context of T-SF sets with aggregation operations or averaging (i.e. mean) operations, such as Akram and Martino (2022), Al-Quran (2021), Chen *et al.* (2021), Gurmani *et al.* (2022), Hussain *et al.* (2022a, 2022b), Karaaslan and Al-Husseinawi (2022), Khan *et al.* (2022), Liu *et al.* (2021c), Mahnaz *et al.* (2022), Wang (2021), Wang and Zhang (2022), Yang and Pang (2022), Yang *et al.* (2021), Zedam *et al.* (2022), and Zeng *et al.* (2021). That is, many of the above works of literature focus on models of aggregating or averaging operations, which belong to a measurement of the central tendency of a finite set of T-SF information. Nonetheless, they are still unable to reflect the relationship or correlation between T-SF characteristics performed by two available alternatives from the statistical point of view. Moreover, such models and methods may ignore the interrelationships between the two T-SF sets, and cannot precisely measure the degree of relationship or correlation between the two T-SF sets.

1.2. *Research Gap and Motivations*

With the establishment of T-SF theory, the correlation coefficients for T-SF information attempt a solid grounding of multiple-criteria evaluation issues in the fields of decision analysis (Guleria and Bajaj, 2021; Ullah *et al.*, 2020a). A correlation coefficient is one of the most commonly-used statistical notions to estimate linear relationships between quantitative objects (Özlü and Karaaslan, 2022; Riaz *et al.*, 2021), and it is often used in statistical analysis or machine learning. Correlation coefficients in statistics can be negative or positive contingent upon the direction of two objects' relationship and their values lie between -1 and 1. To expand the applicability of correlation coefficients, an extended definition can be carried out under SF and T-SF conditions (Guleria and Bajaj, 2021; Mahmood *et al.*, 2021). However, in intricate uncertain circumstances, extracting a proper correlation coefficient between two T-SF sets (or SF sets) is nontrivial.

Ullah *et al.* (2020b) indicated that the correlation coefficients in the intuitionistic fuzzy framework and the picture fuzzy framework do not apply to some practical issues. Because of this, they propounded an innovative notion of correlation coefficients in T-SF settings that range from 0 to 1; moreover, they discussed the fitness of this new measurement in T-SF contexts. Due to its generality, Ullah *et al.* (2020b) brought forward a clustering algorithm and a multiple attribute evaluation algorithm in T-SF uncertain conditions. In what follows, Guleria and Bajaj (2021) propounded the notion concerning correlation coefficients between T-SF sets and explored their useful properties to analyse the practicality in uncertain real-world conditions. With two applications in pattern recognition and medically diagnostic cases, Guleria and Bajaj gave substance to the effectuality of their evolved correlation coefficients. Riaz *et al.* (2021) exploited the statistical notions of covariances and variances to evolve a new correlation coefficient for hybrid SF and m-polar fuzzy information. Mahmood *et al.* (2021) initiated SF cosine similarity measures and (weighted) correlation co-efficient of SF sets for tackling pattern recognition and medical diagnostic issues. Fan *et al.* (2022) exploited an approach via correlation coefficients and standard deviations to generate the attribute weights and then initiated a T-SF complex proportional assessment method. Liu and Wang (2022) employed an inter-criteria correlation approach to generate objective weights and then combined the subjective weights using a minimum total deviation method for supporting decision analysis. In a T-SF framework, Özlü and Karaaslan (2022) coped with T-spherical type-2 hesitant fuzzy uncertain data to investigate an extended version of correlation coefficients. The aforementioned literature shows the usefulness and practical value of correlation coefficients in managing T-SF uncertain assessment issues with multiple-criteria analysis.

Published findings in support of the advantage of correlation coefficients under SF and T-SF conditions have focused on the usefulness of managing uncertainty contained in compounded and complicated problems efficaciously. However, there are some motivational considerations in advocating the widespread development of correlation coefficients with the help of apposite multiple-criteria analysis in T-SF settings.

(1) Few studies have focused on advancing efficient and easy-to-use T-SF correlation measures for differentiating the prioritization relations of available choice options, which is the foremost motivation of this research.
(2) Relatively less exploration of correlation-focused measurements as a concept to directly exploit T-SF correlation coefficients when dealing with intricately uncertain information is the second motivation for this research.
(3) In the existing T-SF literature predicated on correlation coefficients, the anchored comparisons relative to the universal T-SF set and the null T-SF set were not incorporated into the specification of T-SF correlation-focused measurements, which serves as the third motivation of this research.
(4) Comparing T-SF characteristics with universal T-SF sets and null T-SF sets based on existing T-SF correlation measures should be helpful for promoting the construction of an effective and beneficial multiple-criteria selection model, which is the last motivation of our research.

1.3. *Research Objective and Contributions*

The foremost purpose of this research is to construct a practical multiple-criteria choice method by virtue of a correlation-focused approach for facilitating computational intelligence in an uncertain decision analysis involving T-spherical fuzziness. This paper provides novel concepts of T-SF data-driven correlation measures for T-SF performance ratings based on statistical notions of weighted correlation coefficients in T-SF settings. An efficacious algorithmic procedure based on T-SF data-driven correlation measures and an advanced multiple-criteria choice model is propounded to prioritize available choice options for ascertaining the overall desirability of the performance criteria. The initiated approach is to use T-SF weighted informational energies and correlation functions to exactly establish the T-SF weighted correlation coefficients predicated on the “square root function” type and the “maximum function” type. This approach can model empirical data involving imprecision and ambiguity, which facilitates managing T-SF performance ratings in a befitting and effectual manner. Next, by aiming to receive the overall desirability across the criteria, this paper contributes the T-SF comprehensive correlation indices supported by two types of the square root function and the maximum function to identify the relative prioritization of choice options and decide on the most appropriate scheme. Furthermore, a real problem about location selection is demonstrated to illustrate befitting applications of the propounded methodology for verification. Depending on the investigation outcomes, the evolved methodology proves to be efficacious compared with other approaches.

This study makes some interesting contributions to intelligent decision-making practice. The principal contributions of this study are as follows:

(1) Through the development of new notions grounded in T-SF correlation coefficients, the evolved T-SF data-driven correlation measures mark a new phase in the advancement of current multiple-criteria choice methods.
(2) Based on the square root or maximum functions, a practical measurement of T-SF weighted correlation coefficients is presented to serve as a basis for multiple-criteria choice modelling.
(3) Considering anchored comparisons relative to the universal and null T-SF sets, this study delivers advantageous T-SF comprehensive correlation indices for prioritizing competing choice options.
(4) This research provides a practical application contribution in delineating a convenient-to-use procedural algorithm to facilitate intelligent decision support in uncertain circumstances. By exploiting realistic applications and comparisons, propounded techniques are considerably more robust and flexible as multiple-criteria tools than comparative approaches.

1.4. *Paper Organization*

In the present work, Section 2 depicts several fundamental notions concerned with T-SF theory. Section 3 advocates some beneficial T-SF data-driven correlation measures and then propounds an efficacious multiple-criteria choice method for treating intricate decision information involving T-spherical fuzziness. Section 4 exploits the initiated techniques to manipulate a location selection issue for a construction company and then puts into effect a comparative study with other approaches. In the end, Section 5 finishes this research work with the main results, limitations, and future research avenues.

2. Preliminary Definitions

This part presents an introductory description of T-SF sets and clarifies the relationships among picture fuzzy, SF, and T-SF sets. Throughout the article, the symbols μ, η, ν, and γ will denote four components of positive-, neutral- (i.e. so-called abstinence-membership), negative-, and refusal-membership, respectively, of a part or aspect in an initial universe to a fuzzy configuration.

Definition 1 (Cuong, 2014; Kahraman and Kutlu Gündoğdu, 2018; Mahmood *et al.*, 2019). The symbol U signifies a universal set that is a finite nonempty set. Place three mappings $\mu_T, \eta_T, \nu_T : U \to [0, 1]$. Let $T = \{\langle u, (\mu_T(u), \eta_T(u), \nu_T(u))\rangle \mid u \in U\}$ and q represent a generalized form of fuzzy sets and a positive integer, respectively; T is named:

1. A picture fuzzy set in U if $0 \leqslant \mu_T(u) + \eta_T(u) + \nu_T(u) \leqslant 1$ for each u;
2. An SF set in U if $0 \leqslant (\mu_T(u))^2 + (\eta_T(u))^2 + (\nu_T(u))^2 \leqslant 1$ for each u;
3. A T-SF set in U if $0 \leqslant (\mu_T(u))^q + (\eta_T(u))^q + (\nu_T(u))^q \leqslant 1$ for each u.

Definition 2 (Garg *et al.*, 2018; Ullah *et al.*, 2018). Place a T-SF set T taking a single positive-integer parameter q in the universal set U. Let $t(u)$ expound a triplet composed of $\mu_T(u)$, $\eta_T(u)$, and $\nu_T(u)$, namely, $t(u) = (\mu_T(u), \eta_T(u), \nu_T(u))$. The triplet $t(u)$ signifies a picture fuzzy number, an SF number, and a T-SF number when $q = 1$, $q = 2$, and $q \in Z^+$, respectively, wherein Z^+ represents a collection of positive integers.

Definition 3 (Ullah *et al.*, 2018; Mahmood *et al.*, 2019). Consider a T-SF number $t(u) = (\mu_T(u), \eta_T(u), \nu_T(u))$ contained in the T-SF set T. The degrees of refusal-membership $\gamma_T(u)$ having relevance for $t(u)$ are exactly delineated by $1 - \mu_T(u) - \eta_T(u) - \nu_T(u)$, $\sqrt{1 - (\mu_T(u))^2 - (\eta_T(u))^2 - (\nu_T(u))^2}$, and $\sqrt[q]{1 - (\mu_T(u))^q - (\eta_T(u))^q - (\nu_T(u))^q}$ when $q = 1$, $q = 2$, and $q \in Z^+$, respectively.

Definition 4 (*Modified from* Güner and Aygün (2022)). Let $T\text{-}SF(U)$ depict a collection of all T-SF sets delineated in a universal set U. Place $T_+ \in T\text{-}SF(U)$ and $T_- \in T\text{-}SF(U)$, where $T_+ = \{\langle u, (\mu_{T_+}(u), \eta_{T_+}(u), \nu_{T_+}(u))\rangle \mid u \in U\}$ and $T_- = \{\langle u, (\mu_{T_-}(u), \eta_{T_-}(u), \nu_{T_-}(u))\rangle \mid u \in U\}$.

1. T_+ is named a universal T-SF set if $T_+ = \{\langle u, (1, 0, 0)\rangle \mid u \in U\}$;
2. T_- is named a null T-SF set if $T_- = \{\langle u, (0, 0, 1)\rangle | u \in U\}$.

Definition 5 (Garg *et al.*, 2018; Liu *et al.*, 2019; Mahmood *et al.*, 2019). Concerning two T-SF sets $T_1 \in T\text{-}SF(U)$ and $T_2 \in T\text{-}SF(U)$ in the universal set U, it is recognized that $T_1 = \{\langle u, (\mu_{T_1}(u), \eta_{T_1}(u), \nu_{T_1}(u))\rangle \mid u \in U\}$ and $T_2 = \{\langle u, (\mu_{T_2}(u), \eta_{T_2}(u), \nu_{T_2}(u))\rangle \mid u \in U\}$. Certain fundamental set operations are precisely stated in this manner:

1. $T_1 \subseteq T_2$ if $\mu_{T_1}(u) \leqslant \mu_{T_2}(u)$, $\eta_{T_1}(u) \leqslant \eta_{T_2}(u)$, and $\nu_{T_1}(u) \geqslant \nu_{T_2}(u)$ for each u;
2. $T_1 = T_2$ if and only if $T_1 \subseteq T_2$ and $T_2 \subseteq T_1$;
3. $T_1 \cup T_2 = \{\langle u, (\max\{\mu_{T_1}(u), \mu_{T_2}(u)\}, \min\{\eta_{T_1}(u), \eta_{T_2}(u)\}, \min\{\nu_{T_1}(u), \nu_{T_2}(u)\})\rangle \mid u \in U\}$;
4. $T_1 \cap T_2 = \{\langle u, (\min\{\mu_{T_1}(u), \mu_{T_2}(u)\}, \min\{\eta_{T_1}(u), \eta_{T_2}(u)\}, \max\{\nu_{T_1}(u), \nu_{T_2}(u)\})\rangle \mid u \in U\}$;
5. The complement of T_1: $(T_1)^c = \{\langle u, (\nu_{T_1}(u), \eta_{T_1}(u), \mu_{T_1}(u))\rangle \mid u \in U\}$.

Definition 6 (Ju *et al.*, 2021). Give consideration to any three T-SF numbers $t_1(u) = (\mu_{T_1}(u), \eta_{T_1}(u), \nu_{T_1}(u))$, $t_2(u) = (\mu_{T_2}(u), \eta_{T_2}(u), \nu_{T_2}(u))$, and $t(u) = (\mu_T(u), \eta_T(u), \nu_T(u))$ associated with an element u in U. Place a real number $\alpha > 0$. Several operational laws for T-SF numbers are portrayed in this fashion:

1. $t_1(u) \oplus t_2(u)$
$$= \Big(\big[1-\big(1-(\mu_{T_1}(u))^q\big)\cdot\big(1-(\mu_{T_2}(u))^q\big)\big]^{1/q},\ \big[\big(1-(\mu_{T_1}(u))^q\big)\cdot\big(1-(\mu_{T_2}(u))^q\big) - \big(1-(\mu_{T_1}(u))^q-(\eta_{T_1}(u))^q\big)\cdot\big(1-(\mu_{T_2}(u))^q-(\eta_{T_2}(u))^q\big)\big]^{1/q},\ \big[\big(1-(\mu_{T_1}(u))^q-(\eta_{T_1}(u))^q\big)\cdot\big(1-(\mu_{T_2}(u))^q-(\eta_{T_2}(u))^q\big) - \big(1-(\mu_{T_1}(u))^q-(\eta_{T_1}(u))^q-(\nu_{T_1}(u))^q\big)\cdot\big(1-(\mu_{T_2}(u))^q - (\eta_{T_2}(u))^q-(\nu_{T_2}(u))^q\big)\big]^{1/q}\Big);$$
2. $t_1(u) \otimes t_2(u)$
$$= \Big(\big[\big(1-(\eta_{T_1}(u))^q-(\nu_{T_1}(u))^q\big)\cdot\big(1-(\eta_{T_2}(u))^q-(\nu_{T_2}(u))^q\big)-\big(1-(\mu_{T_1}(u))^q - (\eta_{T_1}(u))^q-(\nu_{T_1}(u))^q\big)\cdot\big(1-(\mu_{T_2}(u))^q-(\eta_{T_2}(u))^q-(\nu_{T_2}(u))^q\big)\big]^{1/q},\ \big[\big(1-(\nu_{T_1}(u))^q\big)\cdot\big(1-(\nu_{T_2}(u))^q\big)-\big(1-(\eta_{T_1}(u))^q-(\nu_{T_1}(u))^q\big) \cdot\big(1-(\eta_{T_2}(u))^q-(\nu_{T_2}(u))^q\big)\big]^{1/q},\big[1-\big(1-(\nu_{T_1}(u))^q\big)\cdot\big(1-(\nu_{T_2}(u))^q\big)\big]^{1/q}\Big);$$
3. $\alpha \odot t(u)$
$$= \Big(\big[1-\big(1-(\mu_T(u))^q\big)^\alpha\big]^{1/q},\big[\big(1-(\mu_T(u))^q\big)^\alpha - \big(1-(\mu_T(u))^q-(\eta_T(u))^q\big)^\alpha\big]^{1/q},\big[\big(1-(\mu_T(u))^q-(\eta_T(u))^q\big)^\alpha - \big(1-(\mu_T(u))^q-(\eta_T(u))^q-(\nu_T(u))^q\big)^\alpha\big]^{1/q}\Big);$$
4. $\big(t(u)\big)^\alpha$
$$= \Big(\big[\big(1-(\eta_T(u))^q-(\nu_T(u))^q\big)^\alpha - \big(1-(\mu_T(u))^q-(\eta_T(u))^q-(\nu_T(u))^q\big)^\alpha\big]^{1/q},\ \big[\big(1-(\nu_T(u))^q\big)^\alpha-\big(1-(\eta_T(u))^q-(\nu_T(u))^q\big)^\alpha\big]^{1/q},\ \big[1-\big(1-(\nu_T(u))^q\big)^\alpha\big]^{1/q}\Big).$$

3. Developed Methodology

The purpose of this section is to use effectual T-SF data-driven correlation measures and establish a novel multiple-criteria choice method for manipulating an intricate decision-making issue involving T-spherical fuzziness.

3.1. *Problem Description*

This subsection concerns the formulation regarding a selection problem raised for multiple-criteria assessments and resolutions.

Making allowance for a multiple-criteria choice issue, let $A = \{a_1, a_2, \dots, a_m\}$ and $C = \{c_1, c_2, \dots, c_n\}$ set forth two limited sets of choice options and performance criteria, respectively, in which the cardinal numbers $m, n \geqslant 2$. In connection to each performance criterion $c_j \in C$, place the normalized (standardized) weight $w_j \in [0, 1]$ with the conditioning of weight normalization, i.e. $\sum_{j=1}^{n} w_j = 1$. The set C is compartmentalized into the collection of positive (performance) criteria C_{Po} and the collection of negative (performance) criteria C_{Ne}. Herein, $C_{Po} \cap C_{Ne} = \varnothing$ and $C_{Po} \cup C_{Ne} = C$. Positive criteria (such as profit and productivity) refer to the performance attribute $c_j \in C_{Po}$ with a positive quality of being desirable from the decision-maker's viewpoint. More specifically, their higher levels are more favourable from the decision-maker's position. Negative criteria (such as cost and loss) refer to the performance attribute $c_j \in C_{Ne}$ with a negative quality of being desirable in line with the decision-maker's attitude, which indicates that their lower levels are more favourable from the decision-maker's position.

Multiple-criteria choice models portray decision-makers' considered evaluations as T-SF numbers of their assessments of the choice options' prominent features. On grounds of previous experience, knowledge, technical expertise, and appraisal perceptions, the performance ratings related to each choice option about a specific criterion are established after that the decision-maker has established the performance criteria for evaluating the choice options available. Let a T-SF number $t_{ij} = (\mu_{ij}, \eta_{ij}, \nu_{ij})$ involving a positive-integer exponent q signify a performance rating concerning an alternative $a_i \in A$ having relevance for a specified criterion $c_j \in C$ $(= C_{Po} \cup C_{Ne})$, where the prerequisite $0 \leqslant (\mu_{ij})^q + (\eta_{ij})^q + (\nu_{ij})^q \leqslant 1$ must be fulfilled. In what follows, the degree of refusal-membership is calculated as $\gamma_{ij} = \sqrt[q]{1 - (\mu_{ij})^q - (\eta_{ij})^q - (\nu_{ij})^q}$. By collecting the T-SF performance rating t_{ij} of a_i across all criteria in C, the T-SF characteristic T_i is formed using this fashion:

$$T_i = \{\langle c_j, t_{ij}\rangle \mid c_j \in C\} = \{\langle c_j, (\mu_{ij}, \eta_{ij}, \nu_{ij})\rangle \mid c_j \in C(= C_{Po} \cup C_{Ne})\}. \tag{1}$$

3.2. *T-SF Data-Driven Correlation Measures*

This subsection undertakes several moves to delineate relevant notions of the evolved correlation measures in the T-SF setting and then investigates their valuable features.

Definition 7. Place the best choice option a_+ and the worst choice option a_- in a multiple-criteria choice problem. In view of the collections C_{Po} (involving positive criteria) and C_{Ne} (involving negative criteria), the T-SF characteristics T_+ and T_- possessed by a_+ and a_-, respectively, are represented by way of the concepts of universal T-SF sets and null T-SF sets in this fashion:

1. $T_+ = \{\langle c_j, t_{+j}\rangle \mid c_j \in C\} = \{\langle c_j, (\mu_{+j}, \eta_{+j}, \nu_{+j})\rangle \mid c_j \in C\} = \{\langle c_j, (1, 0, 0)\rangle \mid c_j \in C_{Po}, \langle c_j, (0, 0, 1)\rangle \mid c_j \in C_{Ne}\}$;
2. $T_- = \{\langle c_j, t_{-j}\rangle \mid c_j \in C\} = \{\langle c_j, (\mu_{-j}, \eta_{-j}, \nu_{-j})\rangle \mid c_j \in C\} = \{\langle c_j, (0, 0, 1)\rangle \mid c_j \in C_{Po}, \langle c_j, (1, 0, 0)\rangle \mid c_j \in C_{Ne}\}$.

Definition 8. Considering the normalized (standardized) weight w_j and the T-SF characteristic T_i, let T_i^W state the T-SF weighted characteristic that contains the T-SF weighted performance rating $t_{ij}^w = (\mu_{ij}^w, \eta_{ij}^w, \nu_{ij}^w)$. Herein, $t_{ij}^w = (n \cdot w_j) \odot t_{ij}$, where the number of criteria n epitomizes a role of a balancing coefficient. T_i^W and t_{ij}^w are elucidated along these lines:

$$T_i^W = \{\langle c_j, t_{ij}^w \rangle \mid c_j \in C\} = \{\langle c_j, (\mu_{ij}^w, \eta_{ij}^w, \nu_{ij}^w)\rangle \mid c_j \in C\}, \tag{2}$$

$$\begin{aligned} t_{ij}^w = (n \cdot w_j) \odot t_{ij} = \Big(&\big[1 - \big(1 - (\mu_{ij})^q\big)^{n\cdot w_j}\big]^{1/q}, \\ &\big[\big(1 - (\mu_{ij})^q\big)^{n\cdot w_j} - \big(1 - (\mu_{ij})^q - (\eta_{ij})^q\big)^{n\cdot w_j}\big]^{1/q}, \\ &\big[\big(1 - (\mu_{ij})^q - (\eta_{ij})^q\big)^{n\cdot w_j} - \big(1 - (\mu_{ij})^q - (\eta_{ij})^q - (\nu_{ij})^q\big)^{n\cdot w_j}\big]^{1/q}\Big). \end{aligned} \tag{3}$$

Theorem 1. *Consider the T-SF characteristic T_i containing the T-SF performance rating t_{ij}. When $w_j = 1/n$ for each performance criterion c_j, the T-SF weighted performance rating $t_{ij}^w = t_{ij}$, and the T-SF weighted characteristic $T_i^W = T_i$.*

Proof. With the assistance of Definition 7, it is obtained that $t_{ij}^w = (n \cdot w_j) \odot t_{ij} = [n \cdot (1/n)] \odot t_{ij} = t_{ij}$, which bring about $T_i^W = T_i$ straightforwardly. The theorem is proved. □

Theorem 2. *In consideration of the best choice option a_+ and the worst choice option a_-, their corresponding T-SF weighted characteristics $T_+^W = T_+$ and $T_-^W = T_-$ regardless of the values of the weight w_j for all performance criteria in C.*

Proof. The T-SF weighted performance rating t_{+j}^w connected with the best choice option a_+ on a positive criterion $c_j \in C_{Po}$ is derived by: $t_{+j}^w = (n \cdot w_j) \odot t_{+j} = ([1 - (1 - 1^q)^{n\cdot w_j}]^{1/q}, [(1 - 1^q)^{n\cdot w_j} - (1 - 1^q - 0^q)^{n\cdot w_j}]^{1/q}, [(1 - 1^q - 0^q)^{n\cdot w_j} - (1 - 1^q - 0^q - 0^q)^{n\cdot w_j}]^{1/q}) = (1, 0, 0)$. Next, in what follows, the t_{+j}^w of a_+ on a negative criterion $c_j \in C_{Ne}$ is calculated like this: $t_{+j}^w = ([1 - (1 - 0^q)^{n\cdot w_j}]^{1/q}, [(1 - 0^q)^{n\cdot w_j} - (1 - 0^q - 0^q)^{n\cdot w_j}]^{1/q}, [(1 - 0^q - 0^q)^{n\cdot w_j} - (1 - 0^q - 0^q - 1^q)^{n\cdot w_j}]^{1/q}) = (0, 0, 1)$. Therefore, $T_+^W = \{\langle c_j, (1, 0, 0)\rangle \mid c_j \in C_{Po}, \langle c_j, (0, 0, 1)\rangle \mid c_j \in C_{Ne}\rangle\} = T_+$. Analogously, it can be acquired that $T_-^W = T_-$. The theorem is proved. □

Ullah *et al.* (2020a) conquered the non-appositeness limitation of correlation measurements in intuitionistic fuzzy settings or picture fuzzy settings to advance new correlation coefficients within T-SF environments. They put forward the notions of informational energies and correlation functions to exploit new correlation coefficients for T-SF information. By the same token, Guleria and Bajaj (2021) advocated the identical delineation of statistical correlation measurements in T-SF uncertain conditions. In the light of the correlation measures propounded by Guleria and Bajaj (2021) and Ullah *et al.* (2020b), this paper incorporates the T-SF weighted characteristics T_i^W, T_+^W, and T_-^W into the elucidation of correlation-focused measurements and evolves useful T-SF data-driven correlation measures for facilitating the constitution of an efficacious multiple-criteria choice model.

Definition 9. In consideration of the T-SF weighted characteristic $T_i^W = \{\langle c_j, (\mu_{ij}^w, \eta_{ij}^w, \nu_{ij}^w)\rangle \mid c_j \in C\}$ (with the refusal-membership $\gamma_{ij}^w = \sqrt[q]{1 - (\mu_{ij}^w)^q - (\eta_{ij}^w)^q - (\nu_{ij}^w)^q}$), its

T-SF weighted informational energy is expounded such that:

$$IE\big(T_i^W\big)=\sum_{j=1}^{n}\big[\big(\big(\mu_{ij}^w\big)^q\big)^2+\big(\big(\eta_{ij}^w\big)^q\big)^2+\big(\big(\nu_{ij}^w\big)^q\big)^2+\big(\big(\gamma_{ij}^w\big)^q\big)^2\big]. \tag{4}$$

Theorem 3. *The T-SF weighted informational energies $IE(T_i^W)$, $IE(T_+^W)$, and $IE(T_-^W)$ satisfy the following favourable features*:

1. $0\leqslant IE(T_i^W)\leqslant n$;
2. $IE(T_+^W)=n$;
3. $IE(T_-^W)=n$.

Proof. Supported by the axiomatic condition of T-SF sets, it is recognized that $(\mu_{ij}^w)^q+(\eta_{ij}^w)^q+(\nu_{ij}^w)^q+(\gamma_{ij}^w)^q=1$, which readily gives rise to $0\leqslant((\mu_{ij}^w)^q)^2+((\eta_{ij}^w)^q)^2+((\nu_{ij}^w)^q)^2+((\gamma_{ij}^w)^q)^2\leqslant 1$. In consequence, the outcome $0\leqslant IE(T_i^W)=\sum_{j=1}^n[((\mu_{ij}^w)^q)^2+((\eta_{ij}^w)^q)^2+((\nu_{ij}^w)^q)^2+((\gamma_{ij}^w)^q)^2]\leqslant n$ can be effortlessly confirmed. Next, in conformity with Theorem 2, it is acquainted with $T_+^W=T_+$ and $T_-^W=T_-$, which bring about $((\mu_{+j}^w)^q)^2+((\eta_{+j}^w)^q)^2+((\nu_{+j}^w)^q)^2+((\gamma_{+j}^w)^q)^2=1$ and $((\mu_{-j}^w)^q)^2+((\eta_{-j}^w)^q)^2+((\nu_{-j}^w)^q)^2+((\gamma_{-j}^w)^q)^2=1$, respectively. Under the circumstances, one can corroborate the consequences of $IE(T_+^W)=\sum_{j=1}^n 1=n$ and $IE(T_-^W)=\sum_{j=1}^n 1=n$. The theorem is proved. □

Definition 10. Given the T-SF weighted characteristics T_i^W, T_+^W, and T_-^W, the respective T-SF weighted correlation functions of T_i^W relative to T_+^W and T_-^W are elucidated by:

$$\begin{aligned}&CF\big(T_i^W,T_+^W\big)\\&\quad=\sum_{j=1}^{n}\big[\big(\mu_{ij}^w\big)^q\cdot\big(\mu_{+j}^w\big)^q+\big(\eta_{ij}^w\big)^q\cdot\big(\eta_{+j}^w\big)^q+\big(\nu_{ij}^w\big)^q\cdot\big(\nu_{+j}^w\big)^q+\big(\gamma_{ij}^w\big)^q\cdot\big(\gamma_{+j}^w\big)^q\big]\\&\quad=\sum_{c_j\in C_{Po}}\big(\mu_{ij}^w\big)^q+\sum_{c_j\in C_{Ne}}\big(\nu_{ij}^w\big)^q,\end{aligned} \tag{5}$$

$$\begin{aligned}&CF\big(T_i^W,T_-^W\big)\\&\quad=\sum_{j=1}^{n}\big[\big(\mu_{ij}^w\big)^q\cdot\big(\mu_{-j}^w\big)^q+\big(\eta_{ij}^w\big)^q\cdot\big(\eta_{-j}^w\big)^q+\big(\nu_{ij}^w\big)^q\cdot\big(\nu_{-j}^w\big)^q+\big(\gamma_{ij}^w\big)^q\cdot\big(\gamma_{-j}^w\big)^q\big]\\&\quad=\sum_{c_j\in C_{Po}}\big(\nu_{ij}^w\big)^q+\sum_{c_j\in C_{Ne}}\big(\mu_{ij}^w\big)^q.\end{aligned} \tag{6}$$

Theorem 4. *The T-SF weighted correlation functions $CF(T_i^W,T_+^W)$ and $CF(T_i^W,T_-^W)$ fulfill the following favourable features*:

1. $0\leqslant CF(T_i^W,T_+^W)\leqslant n$ *and* $0\leqslant CF(T_i^W,T_-^W)\leqslant n$;

2. $CF(T_i^W, T_+^W) = CF(T_+^W, T_i^W)$ *and* $CF(T_i^W, T_-^W) = CF(T_-^W, T_i^W)$;
3. $CF(T_+^W, T_-^W) = CF(T_-^W, T_+^W) = 0$;
4. $CF(T_i^W, T_i^W) = IE(T_i^W)$;
5. $CF(T_+^W, T_+^W) = n$ *and* $CF(T_-^W, T_-^W) = n$.

Proof. Firstly, let n_{Po} and n_{Ne} represent the numbers of criteria in C_{Po} and C_{Ne}, respectively, where $n_{Po} + n_{Ne} = n$. It is apparent that $0 \leqslant \sum_{c_j \in C_{Po}} (\mu_{ij}^w)^q \leqslant n_{Po}$, $0 \leqslant \sum_{c_j \in C_{Ne}} (\nu_{ij}^w)^q \leqslant n_{Ne}$, $0 \leqslant \sum_{c_j \in C_{Po}} (\nu_{ij}^w)^q \leqslant n_{Po}$, and $0 \leqslant \sum_{c_j \in C_{Ne}} (\mu_{ij}^w)^q \leqslant n_{Ne}$. Thus, $0 \leqslant CF(T_i^W, T_+^W) \leqslant n_{Po} + n_{Ne} = n$, and $0 \leqslant CF(T_i^W, T_-^W) \leqslant n_{Po} + n_{Ne} = n$. The properties in part 1 are confirmed. The commutative properties in part 2 are straightforward. Next, it demonstrates the correctness of $CF(T_+^W, T_-^W) = \sum_{c_j \in C_{Po}} (0)^q + \sum_{c_j \in C_{Ne}} (0)^q = 0$, which corroborates the property in part 3. In what follows, it can be effortlessly deduced that $CF(T_i^W, T_i^W) = IE(T_i^W)$ for the reason that $CF(T_i^W, T_i^W) = \sum_{j=1}^n [(\mu_{ij}^w)^q \cdot (\mu_{ij}^w)^q + (\eta_{ij}^w)^q \cdot (\eta_{ij}^w)^q + (\nu_{ij}^w)^q \cdot (\nu_{ij}^w)^q + (\gamma_{ij}^w)^q \cdot (\gamma_{ij}^w)^q] = \sum_{j=1}^n [((\mu_{ij}^w)^q)^2 + ((\eta_{ij}^w)^q)^2 + ((\nu_{ij}^w)^q)^2 + ((\gamma_{ij}^w)^q)^2] = IE(T_i^W)$; accordingly, it is manifested that $CF(T_+^W, T_+^W) = IE(T_+^W) = n$ and $CF(T_-^W, T_-^W) = IE(T_-^W) = n$, which demonstrates the truth of the properties in parts 4 and 5. The theorem is proved. □

Definition 11. Making allowance for T_i^W, T_+^W, and T_-^W, the respective T-SF weighted correlation coefficients of T_i^W relative to T_+^W and T_-^W based on the "square root function" type are delineated along these lines:

$$CC_{\surd}(T_i^W, T_+^W) = \frac{CF(T_i^W, T_+^W)}{\sqrt{IE(T_i^W) \cdot IE(T_+^W)}} = \frac{\sum_{c_j \in C_{Po}} (\mu_{ij}^w)^q + \sum_{c_j \in C_{Ne}} (\nu_{ij}^w)^q}{\sqrt{n \cdot \sum_{c_j \in C} \big[((\mu_{ij}^w)^q)^2 + ((\eta_{ij}^w)^q)^2 + ((\nu_{ij}^w)^q)^2 + ((\gamma_{ij}^w)^q)^2\big]}}, \tag{7}$$

$$CC_{\surd}(T_i^W, T_-^W) = \frac{CF(T_i^W, T_-^W)}{\sqrt{IE(T_i^W) \cdot IE(T_-^W)}} = \frac{\sum_{c_j \in C_{Po}} (\nu_{ij}^w)^q + \sum_{c_j \in C_{Ne}} (\mu_{ij}^w)^q}{\sqrt{n \cdot \sum_{c_j \in C} \big[((\mu_{ij}^w)^q)^2 + ((\eta_{ij}^w)^q)^2 + ((\nu_{ij}^w)^q)^2 + ((\gamma_{ij}^w)^q)^2\big]}}. \tag{8}$$

Theorem 5. *Through the utility of the "square root function" type, the T-SF weighted correlation coefficients* $CC_{\surd}(T_i^W, T_+^W)$ *and* $CC_{\surd}(T_i^W, T_-^W)$ *fulfill some favourable features*:

1. $0 \leqslant CC_{\surd}(T_i^W, T_+^W) \leqslant 1$ *and* $0 \leqslant CC_{\surd}(T_i^W, T_-^W) \leqslant 1$;
2. $CC_{\surd}(T_i^W, T_+^W) = CC_{\surd}(T_+^W, T_i^W)$ *and* $CC_{\surd}(T_i^W, T_-^W) = CC_{\surd}(T_-^W, T_i^W)$;

3. $CC_{\surd}(T_{+}^{W}, T_{-}^{W}) = CC_{\surd}(T_{-}^{W}, T_{+}^{W}) = 0$;
4. $CC_{\surd}(T_{i}^{W}, T_{+}^{W}) = 1$ *and* $CC_{\surd}(T_{i}^{W}, T_{-}^{W}) = 1$ *if and only if* $T_{i}^{W} = T_{+}^{W}$ *and* $T_{i}^{W} = T_{-}^{W}$*, respectively*;
5. $CC_{\surd}(T_{i}^{W}, T_{+}^{W}) = 0$ *and* $CC_{\surd}(T_{i}^{W}, T_{-}^{W}) = 0$ *if* $T_{i}^{W} = T_{-}^{W}$ *and* $T_{i}^{W} = T_{+}^{W}$*, respectively.*

Proof. Following Definition 9, the T-SF weighted informational energies of T_{i}^{W} and T_{+}^{W} are given in this fashion: $\sum_{j=1}^{n}[((\mu_{ij}^{w})^{q})^{2} + ((\eta_{ij}^{w})^{q})^{2} + ((\nu_{ij}^{w})^{q})^{2} + ((\gamma_{ij}^{w})^{q})^{2}]$ and $\sum_{j=1}^{n}[((\mu_{+j}^{w})^{q})^{2} + ((\eta_{+j}^{w})^{q})^{2} + ((\nu_{+j}^{w})^{q})^{2} + ((\gamma_{+j}^{w})^{q})^{2}]$, respectively. The Cauchy–Schwarz inequality is regarded as one of the most celebrated inequalities in mathematics. Its connotative meaning refers to $(\iota_1\beta_1 + \iota_2\beta_2 + \cdots + \iota_n\beta_n)^2 \leqslant ((\iota_1)^2 + (\iota_2)^2 + \cdots + (\iota_n)^2) \cdot ((\beta_1)^2 + (\beta_2)^2 + \cdots + (\beta_n)^2)$ for the real number sequences $(\iota_1, \iota_2, \ldots, \iota_n)$ and $(\beta_1, \beta_2, \ldots, \beta_n)$. Through the utility of the Cauchy–Schwarz inequality, the subsequent consequence can be yielded:

$$\begin{aligned}
&\big(CF\big(T_{i}^{W}, T_{+}^{W}\big)\big)^{2} \\
&\quad= \Bigg(\sum_{j=1}^{n}\big[\big(\mu_{ij}^{w}\big)^{q}\cdot\big(\mu_{+j}^{w}\big)^{q} + \big(\eta_{ij}^{w}\big)^{q}\cdot\big(\eta_{+j}^{w}\big)^{q} + \big(\nu_{ij}^{w}\big)^{q}\cdot\big(\nu_{+j}^{w}\big)^{q} + \big(\gamma_{ij}^{w}\big)^{q}\cdot\big(\gamma_{+j}^{w}\big)^{q}\big]\Bigg)^{2} \\
&\quad\leqslant \big[\big(\big(\mu_{i1}^{w}\big)^{q}\big)^{2} + \big(\big(\eta_{i1}^{w}\big)^{q}\big)^{2} + \big(\big(\nu_{i1}^{w}\big)^{q}\big)^{2} + \big(\big(\gamma_{i1}^{w}\big)^{q}\big)^{2} + \big(\big(\mu_{i2}^{w}\big)^{q}\big)^{2} + \big(\big(\eta_{i2}^{w}\big)^{q}\big)^{2} \\
&\qquad+ \big(\big(\nu_{i2}^{w}\big)^{q}\big)^{2} + \big(\big(\gamma_{i2}^{w}\big)^{q}\big)^{2} + \cdots + \big(\big(\mu_{in}^{w}\big)^{q}\big)^{2} + \big(\big(\eta_{in}^{w}\big)^{q}\big)^{2} + \big(\big(\nu_{in}^{w}\big)^{q}\big)^{2} \\
&\qquad+ \big(\big(\gamma_{in}^{w}\big)^{q}\big)^{2}\big]\cdot\big[\big(\big(\mu_{+1}^{w}\big)^{q}\big)^{2} + \big(\big(\eta_{+1}^{w}\big)^{q}\big)^{2} + \big(\big(\nu_{+1}^{w}\big)^{q}\big)^{2} + \big(\big(\gamma_{+1}^{w}\big)^{q}\big)^{2} \\
&\qquad+ \big(\big(\mu_{+2}^{w}\big)^{q}\big)^{2} + \big(\big(\eta_{+2}^{w}\big)^{q}\big)^{2} + \big(\big(\nu_{+2}^{w}\big)^{q}\big)^{2} + \big(\big(\gamma_{+2}^{w}\big)^{q}\big)^{2} + \cdots + \big(\big(\mu_{+n}^{w}\big)^{q}\big)^{2} \\
&\qquad+ \big(\big(\eta_{+n}^{w}\big)^{q}\big)^{2} + \big(\big(\nu_{+n}^{w}\big)^{q}\big)^{2} + \big(\big(\gamma_{+n}^{w}\big)^{q}\big)^{2}\big] \\
&\quad= \sum_{j=1}^{n}\big[\big(\big(\mu_{ij}^{w}\big)^{q}\big)^{2} + \big(\big(\eta_{ij}^{w}\big)^{q}\big)^{2} + \big(\big(\nu_{ij}^{w}\big)^{q}\big)^{2} + \big(\big(\gamma_{ij}^{w}\big)^{q}\big)^{2}\big] \\
&\qquad\cdot\sum_{j=1}^{n}\big[\big(\big(\mu_{+j}^{w}\big)^{q}\big)^{2} + \big(\big(\eta_{+j}^{w}\big)^{q}\big)^{2} + \big(\big(\nu_{+j}^{w}\big)^{q}\big)^{2} + \big(\big(\gamma_{+j}^{w}\big)^{q}\big)^{2}\big] \\
&\quad= IE\big(T_{i}^{W}\big)\cdot IE\big(T_{+}^{W}\big).
\end{aligned}$$

Using this as a basis, we draw the inference that $CF(T_{i}^{W}, T_{+}^{W}) \leqslant \sqrt{IE(T_{i}^{W}) \cdot IE(T_{+}^{W})}$; thus, $CF(T_{i}^{W}, T_{+}^{W})/\sqrt{IE(T_{i}^{W}) \cdot IE(T_{+}^{W})} \leqslant 1$. Because $CF(T_{i}^{W}, T_{+}^{W}), IE(T_{i}^{W}), IE(T_{+}^{W}) \geqslant 0$, it can be generated that $0 \leqslant CC_{\surd}(T_{i}^{W}, T_{+}^{W}) \leqslant 1$. By the same token, the correctness of $0 \leqslant CC_{\surd}(T_{i}^{W}, T_{-}^{W}) \leqslant 1$ can be proven; thus, the properties in part 1 are validated. The commutative properties in part 2 are straightforward. The property in part 3 is trivially known because $CF(T_{+}^{W}, T_{-}^{W}) = CF(T_{-}^{W}, T_{+}^{W}) = 0$. Regarding the necessity in part 4, the presupposition $CC_{\surd}(T_{i}^{W}, T_{+}^{W}) = 1$ indicates that $(CF(T_{i}^{W}, T_{+}^{W}))^2 = IE(T_{i}^{W}) \cdot IE(T_{+}^{W}) = IE(T_{i}^{W}) \cdot n$, which follows that $CF(T_{i}^{W}, T_{+}^{W}) =$

$CF(T_+^W, T_+^W) = IE(T_+^W) = n$ must be fulfilled. Thus, $T_i^W = T_+^W$. Concerning the sufficiency in part 4, the prerequisite $T_i^W = T_+^W$ brings about $CC_{\surd}(T_+^W, T_+^W) = CF(T_+^W, T_+^W)/\sqrt{IE(T_+^W) \cdot IE(T_+^W)} = IE(T_+^W)/IE(T_+^W) = 1$. Accordingly, it is received that $CC_{\surd}(T_i^W, T_+^W) = 1$ if and only if $T_i^W = T_+^W$. Analogously, one has $CC_{\surd}(T_i^W, T_-^W) = 1$ if and only if $T_i^W = T_-^W$. Therefore, the properties in part 4 are verified. In part 5, the prerequisite $T_i^W = T_-^W$ gives rise to $\mu_{ij}^w = 0$ and $\nu_{ij}^w = 0$ for $c_j \in C_{Po}$ and $c_j \in C_{Ne}$, respectively. By virtue of Definition 10, one obtains $CF(T_i^W, T_+^W) = \sum_{c_j \in C_{Po}} (\mu_{ij}^w)^q + \sum_{c_j \in C_{Ne}} (\nu_{ij}^w)^q = 0$, which leads to the conclusion that $CC_{\surd}(T_i^W, T_+^W) = 0/\sqrt{IE(T_i^W) \cdot IE(T_+^W)} = 0$. In contrast, the prerequisite $T_i^W = T_+^W$ brings about $\nu_{ij}^w = 0$ and $\mu_{ij}^w = 0$ for $c_j \in C_{Po}$ and $c_j \in C_{Ne}$, respectively. In the light of Definition 10, one receives $CF(T_i^W, T_-^W) = \sum_{c_j \in C_{Po}} (\nu_{ij}^w)^q + \sum_{c_j \in C_{Ne}} (\mu_{ij}^w)^q = 0$, which lets us deduce $CC_{\surd}(T_i^W, T_-^W) = 0/\sqrt{IE(T_i^W) \cdot IE(T_-^W)} = 0$. Thus, one can corroborate that $CC_{\surd}(T_i^W, T_+^W) = 0$ if $T_i^W = T_-^W$; moreover, $CC_{\surd}(T_i^W, T_-^W) = 0$ if $T_i^W = T_+^W$. The theorem is proved. □

Definition 12. Making allowance for T_i^W, T_+^W, and T_-^W, the respective T-SF weighted correlation coefficients of T_i^W relative to T_+^W and T_-^W based on the "maximum function" type are delineated along these lines:

$$\begin{aligned} CC_{\wedge}\big(T_i^W, T_+^W\big) &= \frac{CF(T_i^W, T_+^W)}{\max\{IE(T_i^W), IE(T_+^W)\}} \\ &= \frac{\sum_{c_j \in C_{Po}} (\mu_{ij}^w)^q + \sum_{c_j \in C_{Ne}} (\nu_{ij}^w)^q}{\max\big\{\sum_{c_j \in C}[((\mu_{ij}^w)^q)^2 + ((\eta_{ij}^w)^q)^2 + ((\nu_{ij}^w)^q)^2 + ((\gamma_{ij}^w)^q)^2], n\big\}} \\ &= \frac{\sum_{c_j \in C_{Po}} (\mu_{ij}^w)^q + \sum_{c_j \in C_{Ne}} (\nu_{ij}^w)^q}{n}, \end{aligned} \tag{9}$$

$$\begin{aligned} CC_{\wedge}\big(T_i^W, T_-^W\big) &= \frac{CF(T_i^W, T_-^W)}{\max\{IE(T_i^W), IE(T_-^W)\}} \\ &= \frac{\sum_{c_j \in C_{Po}} (\nu_{ij}^w)^q + \sum_{c_j \in C_{Ne}} (\mu_{ij}^w)^q}{\max\big\{\sum_{c_j \in C}[((\mu_{ij}^w)^q)^2 + ((\eta_{ij}^w)^q)^2 + ((\nu_{ij}^w)^q)^2 + ((\gamma_{ij}^w)^q)^2], n\big\}} \\ &= \frac{\sum_{c_j \in C_{Po}} (\nu_{ij}^w)^q + \sum_{c_j \in C_{Ne}} (\mu_{ij}^w)^q}{n}. \end{aligned} \tag{10}$$

Theorem 6. *Through the utility of the "maximum function" type, the T-SF weighted correlation coefficients $CC_{\wedge}(T_i^W, T_+^W)$ and $CC_{\wedge}(T_i^W, T_-^W)$ fulfill some favourable features*:

1. $0 \leqslant CC_{\wedge}(T_i^W, T_+^W) \leqslant 1$ *and* $0 \leqslant CC_{\wedge}(T_i^W, T_-^W) \leqslant 1$;
2. $CC_{\wedge}(T_i^W, T_+^W) = CC_{\wedge}(T_+^W, T_i^W)$ *and* $CC_{\wedge}(T_i^W, T_-^W) = CC_{\wedge}(T_-^W, T_i^W)$;
3. $CC_{\wedge}(T_+^W, T_-^W\big) = CC_{\wedge}(T_-^W, T_+^W\big) = 0$;
4. $CC_{\wedge}(T_i^W, T_+^W) = 1$ *and* $CC_{\wedge}(T_i^W, T_-^W) = 1$ *if and only if* $T_i^W = T_+^W$ *and* $T_i^W = T_-^W$, *respectively*;

5. $CC_{\wedge}(T_i^W, T_+^W) = 0$ *and* $CC_{\wedge}(T_i^W, T_-^W) = 0$ *if* $T_i^W = T_-^W$ *and* $T_i^W = T_+^W$, *respectively.*

Proof. Firstly, the proofs of parts 2, 3, and 5 are like the proving processes in parts 2, 3, and 5 of Theorem 5. In part 1, as analogous to the proof in Theorem 5, it is recognized that $(CF(T_i^W, T_+^W))^2 \leqslant IE(T_i^W) \cdot IE(T_+^W)$, which gives substance to $CF(T_i^W, T_+^W) \leqslant \max\{IE(T_i^W), IE(T_+^W)\}$. Accordingly, $CF(T_i^W, T_+^W)/\max\{IE(T_i^W), IE(T_+^W)\} \leqslant 1$. Because $CF(T_i^W, T_+^W), IE(T_i^W), IE(T_+^W) \geqslant 0$, we can state that $0 \leqslant CC_{\wedge}(T_i^W, T_+^W) \leqslant 1$. Similarly, one has $0 \leqslant CC_{\wedge}(T_i^W, T_-^W) \leqslant 1$. The correctness of the properties in part 1 is confirmed. Concerning the necessity in part 4, the presupposition $CC_{\wedge}(T_i^W, T_+^W) = 1$ implies that $CF(T_i^W, T_+^W) = \max\{IE(T_i^W), IE(T_+^W)\} = \max\{IE(T_i^W), n\} = n$; on account of this, $T_i^W = T_+^W$. For the sufficiency in part 4, the prerequisite $T_i^W = T_+^W$ gives rise to $CC_{\wedge}(T_+^W, T_+^W) = CF(T_+^W, T_+^W)/\max\{IE(T_+^W), IE(T_+^W)\} = IE(T_+^W)/IE(T_+^W) = 1$. Therefore, it is acquired that $CC_{\wedge}(T_i^W, T_+^W) = 1$ if and only if $T_i^W = T_+^W$. It is known, just the same, that $CC_{\wedge}(T_i^W, T_-^W) = 1$ if and only if $T_i^W = T_-^W$. As a result, the properties in part 4 are verified. The theorem is proved. □

3.3. *Propounded Multiple-Criteria Choice Method in T-SF Settings*

This subsection attempts to propound an effective and simple-to-implement approach for tackling an uncertain multiple-criteria evaluation issue predicated on the evolved T-SF data-driven correlation measures.

Consider a multiple-criteria choice task embodying the T-SF characteristic T_i and the normalized (standardized) weight w_j of an available choice option $a_i \in A$ and a performance criterion $c_j \in C$, respectively. Place an anchoring parameter $\xi \in [0, 1]$. For each T-SF characteristic, the parameter ξ elucidates the weight of the anchored comparisons relative to universal T-SF sets, while $(1 - \xi)$ depicts the weight of the anchored comparisons relative to null T-SF sets. In what follows, this study contributes two constructive T-SF comprehensive correlation indices as the measurements of deciding the relative prioritization for available choice options.

Definition 13. Denote $\underline{CC}_{\surd}^{+} = \min_{i'=1}^{m} CC_{\surd}(T_{i'}^W, T_+^W)$, $\overline{CC}_{\surd}^{+} = \max_{i'=1}^{m} CC_{\surd}(T_{i'}^W, T_+^W)$, $\underline{CC}_{\surd}^{-} = \min_{i'=1}^{m} CC_{\surd}(T_{i'}^W, T_-^W)$, and $\overline{CC}_{\surd}^{-} = \max_{i'=1}^{m} CC_{\surd}(T_{i'}^W, T_-^W)$ for the "square root function" type. Denote $\underline{CC}_{\wedge}^{+} = \min_{i'=1}^{m} CC_{\wedge}(T_{i'}^W, T_+^W)$, $\overline{CC}_{\wedge}^{+} = \max_{i'=1}^{m} CC_{\wedge}(T_{i'}^W, T_+^W)$, $\underline{CC}_{\wedge}^{-} = \min_{i'=1}^{m} CC_{\wedge}(T_{i'}^W, T_-^W)$, and $\overline{CC}_{\wedge}^{-} = \max_{i'=1}^{m} CC_{\wedge}(T_{i'}^W, T_-^W)$ for the "maximum function" type. By the agency of $CC_{\surd}$ and $CC_{\wedge}$ on a_i, the T-SF comprehensive correlation indices $CI_{\surd}(a_i)$ and $CI_{\wedge}(a_i)$, respectively, are delineated along these lines:

$$CI_{\surd}(a_i) = \xi \cdot \frac{CC_{\surd}(T_i^W, T_+^W) - \underline{CC}_{\surd}^{+}}{\overline{CC}_{\surd}^{+} - \underline{CC}_{\surd}^{+}} + (1 - \xi) \cdot \frac{\overline{CC}_{\surd}^{-} - CC_{\surd}(T_i^W, T_-^W)}{\overline{CC}_{\surd}^{-} - \underline{CC}_{\surd}^{-}}, \quad (11)$$

$$CI_{\wedge}(a_i) = \xi \cdot \frac{CC_{\wedge}(T_i^W, T_+^W) - \underline{CC}_{\wedge}^{+}}{\overline{CC}_{\wedge}^{+} - \underline{CC}_{\wedge}^{+}} + (1 - \xi) \cdot \frac{\overline{CC}_{\wedge}^{-} - CC_{\wedge}(T_i^W, T_-^W)}{\overline{CC}_{\wedge}^{-} - \underline{CC}_{\wedge}^{-}}. \quad (12)$$

Theorem 7. *The T-SF comprehensive correlation indices* $CI_{\surd}(a_i)$ *and* $CI_{\wedge}(a_i)$ *fulfill the following favoutirable features*:

1. $0 \leqslant CI_{\surd}(a_i) \leqslant 1$ *and* $0 \leqslant CI_{\wedge}(a_i) \leqslant 1$;
2. $CI_{\surd}(a_i) = 1$ *and* $CI_{\wedge}(a_i) = 1$ *for all* $\xi \in [0, 1]$ *if* $T_i = T_+$;
3. $CI_{\surd}(a_i) = 0$ *and* $CI_{\wedge}(a_i) = 0$ *for all* $\xi \in [0, 1]$ *if* $T_i = T_-$.

Proof. Utilizing the foregoing delineation, it is realized that $CC_{\surd}(T_i^W, T_+^W) \leqslant \overline{CC}_{\surd}^+$ $(= \max_{i'=1}^m CC_{\surd}(T_{i'}^W, T_+^W))$, which follows that $CC_{\surd}(T_i^W, T_+^W) - \underline{CC}_{\surd}^+ \leqslant \overline{CC}_{\surd}^+ - \underline{CC}_{\surd}^+$, thereby gaining $(CC_{\surd}(T_i^W, T_+^W) - \underline{CC}_{\surd}^+)/(\overline{CC}_{\surd}^+ - \underline{CC}_{\surd}^+) \leqslant 1$. It is apparent to observe that $CC_{\surd}(T_i^W, T_+^W) - \underline{CC}_{\surd}^+ \geqslant 0$ and $\overline{CC}_{\surd}^+ - \underline{CC}_{\surd}^+ \geqslant 0$ for the reason that $CC_{\surd}(T_i^W, T_+^W) \geqslant \min_{i'=1}^m CC_{\surd}(T_{i'}^W, T_+^W)$ and $\max_{i'=1}^m CC_{\surd}(T_{i'}^W, T_+^W) \geqslant \min_{i'=1}^m CC_{\surd}(T_{i'}^W, T_+^W)$. Accordingly, one has $0 \leqslant (CC_{\surd}(T_i^W, T_+^W) - \underline{CC}_{\surd}^+)/(\overline{CC}_{\surd}^+ - \underline{CC}_{\surd}^+) \leqslant 1$. In a similar fashion, $0 \leqslant (\overline{CC}_{\surd}^- - CC_{\surd}(T_i^W, T_-^W))/(\overline{CC}_{\surd}^- - \underline{CC}_{\surd}^-) \leqslant 1$. Taking $0 \leqslant \xi \leqslant 1$ into consideration, it is deduced that $0 \leqslant CI_{\surd}(a_i) \leqslant 1$. By the same token, one has $0 \leqslant CI_{\wedge}(a_i) \leqslant 1$, which demonstrates the truth of the properties in part 1. Next, it is realized that $T_+^W = T_+$ and $T_-^W = T_-$ based on Theorem 2. The prerequisite $T_i = T_+ (= T_+^W)$ brings about $CC_{\surd}(T_i^W, T_+^W) = 1$ and $\overline{CC}_{\surd}^+ = 1$, which indicates that $(1 - \underline{CC}_{\surd}^+)/(1 - \underline{CC}_{\surd}^+) = 1$. Moreover, the condition $T_i = T_+ (= T_+^W)$ leads to $CC_{\surd}(T_i^W, T_-^W) = 0$ and $\underline{CC}_{\surd}^- = 0$, which indicates that $(\overline{CC}_{\surd}^- - 0)/(\overline{CC}_{\surd}^- - 0) = 1$. From this basis, it is obtained that $CI_{\surd}(a_i) = \xi \cdot 1 + (1 - \xi) \cdot 1 = 1$ for all $\xi \in [0, 1]$. The correctness of $CI_{\wedge}(a_i) = 1$ is analogously corroborated, which produces proof of part 2. The properties in part 3 are verified similarly. The theorem is proved. □

Definition 14. Given two choice options a_i and $a_{i'}$ involving T-SF characteristics T_i and $T_{i'}$, respectively, the prioritization procedure of a_i and $a_{i'}$ can be elucidated using the subsequent relations "$\succ_{\surd}$" (indicating "better than"), "$\sim_{\surd}$" (indicating "indefinite or indifferent"), and "$\prec_{\surd}$" (indicating "worse than") (or "$\succ_{\wedge}$", "$\sim_{\wedge}$", and "$\prec_{\wedge}$"), like this:

1. Based on the "square root function" type:
   a) If $CI_{\surd}(a_i) > CI_{\surd}(a_{i'})$, then it is convinced that $a_i \succ_{\surd} a_{i'}$;
   b) If $CI_{\surd}(a_i) = CI_{\surd}(a_{i'})$, then $a_i \sim_{\surd} a_{i'}$;
   c) If $CI_{\surd}(a_i) < CI_{\surd}(a_{i'})$, then it is convinced that $a_i \prec_{\surd} a_{i'}$.
2. Based on the "maximum function" type:
   a) If $CI_{\wedge}(a_i) > CI_{\wedge}(a_{i'})$, then it is convinced that $a_i \succ_{\wedge} a_{i'}$;
   b) If $CI_{\wedge}(a_i) = CI_{\wedge}(a_{i'})$, then $a_i \sim_{\wedge} a_{i'}$;
   c) If $CI_{\wedge}(a_i) < CI_{\wedge}(a_{i'})$, then it is convinced that $a_i \prec_{\wedge} a_{i'}$.

The framework of the propounded multiple-criteria choice method on grounds of T-SF data-driven correlation measures is depicted in Fig. 2. As exhibited in this framework, the evolved methodology comprises four phases, i.e. the organization of a multiple-criteria choice issue in Phase I, the computation of weighted performance information with T-SF sets in Phase II, the generation of T-SF data-driven correlation measures in Phase III, and decision making for treating multiple-criteria choice analysis in Phase IV.

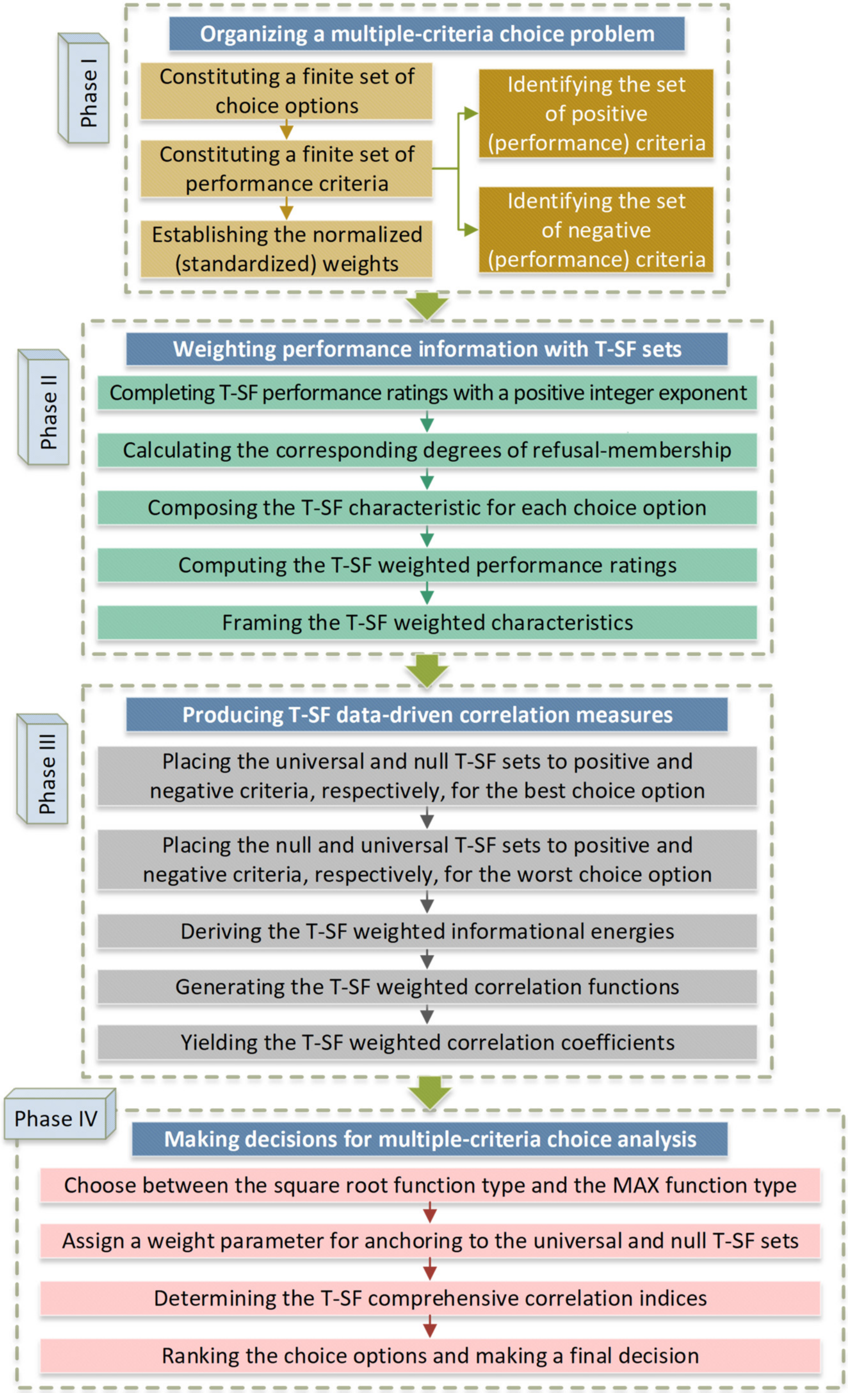


Fig. 2. The framework of the propounded methodology.

To implement the propounded methodology, this study provides a new algorithm to perform the procedural steps pragmatically in order to facilitate the decision-maker's multiple-criteria analysis. The following algorithm is expressed using a sequence of simple operations (consisting of Steps 1 and 2 in Phase I, Steps 3–5 in Phase II, Steps 6–8 in Phase III, and Steps 9 and 10 in Phase IV) for conducting the initiated multiple-criteria choice method with T-SF data-driven correlation measures:

Step 1. Place a limited set of choice options $A = \{a_1, a_2, \dots, a_m\}$ and a limited set of performance criteria $C = \{c_1, c_2, \dots, c_n\}$. Separate C into two parts: one is the collection of positive criteria C_{Po}; the other is the collection of negative criteria C_{Ne}.

Step 2. Generate the normalized (standardized) weight w_j with the conditioning of weight normalization for each performance criterion c_j.

Step 3. Specify a suitable positive-integer exponent q and form a T-SF performance rating t_{ij} signified as the T-SF number $(\mu_{ij}, \eta_{ij}, \nu_{ij})$ with the refusal-membership γ_{ij}.

Step 4. Assemble the T-SF characteristic T_i in Eq. (1) by gathering all T-SF performance rating t_{ij} regarding a choice option a_i across all criteria in C.

Step 5. Employ Eq. (3) to derive the T-SF weighted performance rating t_{ij}^w (with refusal-membership γ_{ij}^w) for the sake of framing the T-SF weighted characteristic T_i^W in Eq. (2).

Step 6. Utilize the universal and null T-SF sets to signify the T-SF characteristics T_+ and T_- for the best choice option a_+ and the worst choice option a_-, respectively. Moreover, the T-SF weighted characteristics $T_+^W = T_+$ and $T_-^W = T_-$.

Step 7. Derive the T-SF weighted informational energy $IE(T_i^W)$ using Eq. (4) and the T-SF weighted correlation functions $CF(T_i^W, T_+^W)$ and $CF(T_i^W, T_-^W)$ using Eqs. (5) and (6), respectively.

Step 8. Proceed to either **Step 8-1** or **Step 8-2**.

Step 8-1. Use the "square root function" type to produce the T-SF weighted correlation coefficients $CC_{\surd}(T_i^W, T_+^W)$ and $CC_{\surd}(T_i^W, T_-^W)$ using Eqs. (7) and (8), respectively.

Step 8-2. Exploit the "maximum function" type to produce the T-SF weighted correlation coefficients $CC_{\wedge}(T_i^W, T_+^W)$ and $CC_{\wedge}(T_i^W, T_-^W)$ using Eqs. (9) and (10), respectively.

Step 9. Assign an anchoring parameter ξ to determine the T-SF comprehensive correlation index $CI_{\surd}(a_i)$ (or $CI_{\wedge}(a_i)$) using Eq. (11) (or Eq. (12)).

Step 10. Rank the m choice options in A supported by $CI_{\surd}(a_i)$ (or $CI_{\wedge}(a_i)$) in descending order to identify the prioritization relations "$\succ_{\surd}$", "$\sim_{\surd}$", and "$\prec_{\surd}$" (or "$\succ_{\wedge}$", "$\sim_{\wedge}$", and "$\prec_{\wedge}$"). Make a final decision for completing the multiple-criteria choice task.

4. Practical Application and Comparative Research

This section intends to exemplify the functionality and suitability of the propounded methodology for applications in a location selection issue for a construction company in

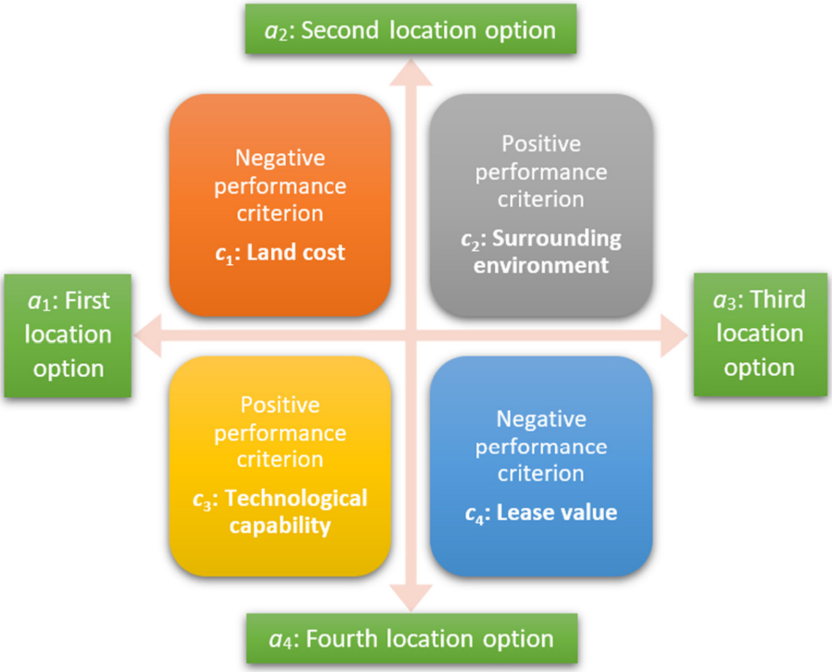


Fig. 3. Profile of the location selection issue of a construction company for building new apartments.

complex uncertain circumstances. Moreover, this section puts into effect two comparative studies to scrutinize the helpfulness and merits of the current technique.

4.1. *Realistic Application and Discussions*

The multiple-criteria choice case investigated by Chen *et al.* (2021) focused on the issue of a construction company finding an appropriate location to put up a new apartment. In order to find the most suitable location, the construction company evaluates four location options ($a_1 - a_4$) for constructing new apartments predicated on the four performance criteria. The performance criteria consist of land cost (c_1), surrounding environment (c_2), technological capability (c_3), and lease value (c_4). Fig. 3 provides a profile of the location selection issue under study.

In Step 1, the two limited sets of choice options and performance criteria were designated as $A = \{a_1, a_2, a_3, a_4\}$ and $C = \{c_1, c_2, c_3, c_4\}$, respectively. Herein, the set C was separated into two parts: one is the collection of positive criteria $C_{Po} = \{c_2, c_3\}$; the other is the collection of negative criteria $C_{Ne} = \{c_1, c_4\}$. In Step 2, in conformity with the expert's professional opinions, the normalized (standardized) weights were given by $(w_1, w_2, w_3, w_4) = (0.2, 0.1, 0.3, 0.4)$. In Step 3, the expert evaluated the location options one by one based on the four performance criteria, and the relevant evaluation data were expressed in terms of T-SF information, as revealed in Table 2. The data fields contain the T-SF performance rating $t_{ij} =$

Table 2
Data of the T-SF performance rating t_{ij} (with the refusal-membership γ_{ij}) in the location selection problem.

c_j	$t_{1j} = (\mu_{1j}, \eta_{1j}, \nu_{1j})$	γ_{1j}	$t_{2j} = (\mu_{2j}, \eta_{2j}, \nu_{2j})$	γ_{2j}	$t_{3j} = (\mu_{3j}, \eta_{3j}, \nu_{3j})$	γ_{3j}	$t_{4j} = (\mu_{4j}, \eta_{4j}, \nu_{4j})$	γ_{4j}
c_1	(0.43, 0.20, 0.61)	0.88	(0.14, 0.32, 0.74)	0.82	(0.75, 0.12, 0.41)	0.80	(0.35, 0.44, 0.83)	0.67
c_2	(0.54, 0.35, 0.63)	0.82	(0.26, 0.17, 0.26)	0.99	(0.59, 0.29, 0.13)	0.92	(0.91, 0.12, 0.49)	0.50
c_3	(0.81, 0.62, 0.11)	0.61	(0.77, 0.23, 0.55)	0.71	(0.56, 0.22, 0.36)	0.92	(0.63, 0.11, 0.27)	0.90
c_4	(0.18, 0.33, 0.66)	0.88	(0.61, 0.34, 0.57)	0.82	(0.11, 0.14, 0.45)	0.97	(0.31, 0.36, 0.84)	0.69

Table 3
Outcomes relevant to the T-SF weighted performance rating t_{ij}^w and the refusal-membership γ_{ij}^w ($q = 3$).

a_i	c_j	$t_{ij}^w = (\mu_{ij}^w, \eta_{ij}^w, \nu_{ij}^w)$	γ_{ij}^w	$(\mu_{ij}^w)^q$	$(\eta_{ij}^w)^q$	$(\nu_{ij}^w)^q$	$(\gamma_{ij}^w)^q$	Squared sum*
a_1	c_1	(0.4003, 0.1867, 0.5750)	0.9042	0.0641	0.0065	0.1901	0.7393	0.5868
	c_2	(0.4046, 0.2683, 0.5031)	0.9233	0.0662	0.0193	0.1274	0.7871	0.6405
	c_3	(0.8422, 0.6136, 0.1060)	0.5544	0.5974	0.2310	0.0012	0.1704	0.4393
	c_4	(0.2104, 0.3841, 0.7406)	0.8082	0.0093	0.0567	0.4062	0.5278	0.4469
a_2	c_1	(0.1300, 0.2974, 0.7002)	0.8564	0.0022	0.0263	0.3433	0.6282	0.5132
	c_2	(0.1919, 0.1258, 0.1928)	0.9946	0.0071	0.0020	0.0072	0.9838	0.9679
	c_3	(0.8036, 0.2345, 0.5538)	0.6682	0.5189	0.0129	0.1699	0.2983	0.3873
	c_4	(0.6963, 0.3758, 0.6098)	0.7259	0.3376	0.0531	0.2267	0.3826	0.3146
a_3	c_1	(0.7080, 0.1156, 0.3965)	0.8345	0.3549	0.0015	0.0623	0.5812	0.4677
	c_2	(0.4446, 0.2244, 0.1009)	0.9654	0.0879	0.0113	0.0010	0.8998	0.8175
	c_3	(0.5914, 0.2307, 0.3766)	0.8994	0.2068	0.0123	0.0534	0.7275	0.5750
	c_4	(0.1286, 0.1637, 0.5210)	0.9480	0.0021	0.0044	0.1414	0.8521	0.7461
a_4	c_1	(0.3254, 0.4109, 0.8012)	0.7255	0.0344	0.0694	0.5143	0.3818	0.4163
	c_2	(0.7542, 0.1171, 0.5083)	0.7595	0.4289	0.0016	0.1313	0.4381	0.3932
	c_3	(0.6634, 0.1147, 0.2812)	0.8812	0.2920	0.0015	0.0222	0.6843	0.5540
	c_4	(0.3615, 0.4165, 0.8922)	0.5544	0.0472	0.0722	0.7101	0.1704	0.5408

Squared sum*: $((\mu_{ij}^w)^q)^2 + ((\eta_{ij}^w)^q)^2 + ((\nu_{ij}^w)^q)^2 + ((\gamma_{ij}^w)^q)^2$ ($q = 3$).

$(\mu_{ij}, \eta_{ij}, \nu_{ij})$ and its associated refusal-membership γ_{ij}, where the positive-integer exponent $q = 3$ and $\gamma_{ij} = \sqrt[3]{1 - (\mu_{ij})^3 - (\eta_{ij})^3 - (\nu_{ij})^3}$. Taking $t_{13} = (0.81, 0.62, 0.11)$ as an illustration, $\gamma_{13} = \sqrt[3]{1 - 0.81^3 - 0.62^3 - 0.11^3} = 0.61$. In Step 4, the T-SF characteristics were generated by $T_i = \{\langle c_j, t_{ij}\rangle \mid c_j \in C\} = \{\langle c_j, (\mu_{ij}, \eta_{ij}, \nu_{ij})\rangle \mid c_j \in \{c_1, c_2, c_3, c_4\}\}$ for each location option a_i. For example, $T_1 = \{\langle c_1, (0.43, 0.20, 0.61)\rangle, \langle c_2, (0.54, 0.35, 0.63)\rangle, \langle c_3, (0.81, 0.62, 0.11)\rangle, \langle c_4, (0.18, 0.33, 0.66)\rangle\}$.

In Step 5, the T-SF weighted performance rating t_{ij}^w was computed using Eq. (3). To give an instance, $t_{13}^w = (n \cdot w_3) \odot t_{13} = (4 \times 0.3) \odot t_{13} = ([1 - (1 - 0.81^3)^{1.2}]^{1/3}, [(1 - 0.81^3)^{1.2} - (1 - 0.81^3 - 0.62^3)^{1.2}]^{1/3}, [(1 - 0.81^3 - 0.62^3)^{1.2} - (1 - 0.81^3 - 0.62^3 - 0.11^3)^{1.2}]^{1/3}) = (0.8422, 0.6136, 0.1060)$, where the refusal-membership $\gamma_{13}^w = \sqrt[3]{1 - 0.8422^3 - 0.6136^3 - 0.1060^3} = 0.5544$. The computed outcomes of t_{ij}^w and γ_{ij}^w are revealed in the third and fourth columns of Table 3. Moreover, the T-SF weighted characteristics were determined by the use of $T_i^W = \{\langle c_j, t_{ij}^w\rangle \mid c_j \in C\} = \{\langle c_j, (\mu_{ij}^w, \eta_{ij}^w, \nu_{ij}^w)\rangle \mid c_j \in \{c_1, c_2, c_3, c_4\}\}$ for each a_i. As an illustration, $T_1^W = \{\langle c_1, (0.4003, 0.1867, 0.5750)\rangle, \langle c_2, (0.4046, 0.2683, 0.5031)\rangle, \langle c_3, (0.8422, 0.6136, 0.1060)\rangle, \langle c_4, (0.2104, 0.3841, 0.7406)\rangle\}$.

Table 4
Outcomes relevant to the T-SF data-driven correlation measures.

a_i	$IE(T_i^W)$	$CF(T_i^W, T_+^W)$	$CF(T_i^W, T_-^W)$	$CC_{\surd}(T_i^W, T_+^W)$	$CC_{\surd}(T_i^W, T_-^W)$	$CC_{\wedge}(T_i^W, T_+^W)$	$CC_{\wedge}(T_i^W, T_-^W)$
a_1	2.1135	1.2599	0.2020	0.4333	0.0695	0.3150	0.0505
a_2	2.1830	1.0960	0.5169	0.3709	0.1749	0.2740	0.1292
a_3	2.6062	0.4984	0.4115	0.1544	0.1274	0.1246	0.1029
a_4	1.9043	1.9454	0.2352	0.7049	0.0852	0.4864	0.0588

In Step 6, based on the universal and null T-SF sets, the T-SF characteristic $T_+ = \{\langle c_1, (0, 0, 1)\rangle, \langle c_2, (1, 0, 0)\rangle, \langle c_3, (1, 0, 0)\rangle, \langle c_4, (0, 0, 1)\rangle\}$ because of $C_{Po} = \{c_2, c_3\}$ and $C_{Ne} = \{c_1, c_4\}$. Moreover, $T_- = \{\langle c_1, (1, 0, 0)\rangle, \langle c_2, (0, 0, 1)\rangle, \langle c_3, (0, 0, 1)\rangle, \langle c_4, (1, 0, 0)\rangle\}$. According to Theorem 2, it was acquainted with $T_+^W = T_+$ and $T_-^W = T_-$. In Step 7, the T-SF weighted informational energies were yielded using Eq. (4). Specifically, $IE(T_1^W) = \sum_{j=1}^{4}[((\mu_{1j}^w)^q)^2 + ((\eta_{1j}^w)^q)^2 + ((\nu_{1j}^w)^q)^2 + ((\gamma_{1j}^w)^q)^2]$. The respective computation results of $(\mu_{ij}^w)^q$, $(\eta_{ij}^w)^q$, $(\nu_{ij}^w)^q$, and $(\gamma_{ij}^w)^q$ are shown in the fifth to eighth columns of Table 3. Moreover, their corresponding squared sum, i.e. $((\mu_{ij}^w)^q)^2 + ((\eta_{ij}^w)^q)^2 + ((\nu_{ij}^w)^q)^2 + ((\gamma_{ij}^w)^q)^2$, can be directly derived, and the results are demonstrated in the last column of Table 3. In conformity with these outcomes, it was derived that $IE(T_1^W) = 0.5868 + 0.6405 + 0.4393 + 0.4469 = 2.1135$. In the same fashion, $IE(T_2^W) = 2.1830$, $IE(T_3^W) = 2.6062$, and $IE(T_4^W) = 1.9043$, as shown in the second column of Table 4. Next, the T-SF weighted correlation functions $CF(T_i^W, T_+^W)$ and $CF(T_i^W, T_-^W)$ were acquired using Eqs. (5) and (6), respectively. To give an example, $CF(T_1^W, T_+^W) = \sum_{c_j \in C_{Po}}(\mu_{1j}^w)^q + \sum_{c_j \in C_{Ne}}(\nu_{1j}^w)^q = ((\mu_{12}^w)^q + (\mu_{13}^w)^q) + ((\nu_{11}^w)^q + (\nu_{14}^w)^q) = (0.0662 + 0.5974) + (0.1901 + 0.4062) = 1.2599$. The outcomes of $CF(T_i^W, T_+^W)$ and $CF(T_i^W, T_-^W)$ are displayed in the third and fourth columns of Table 4.

In Step 8, if the "square root function" type was employed, this study would comply with Step 8-1 to determine the T-SF weighted correlation coefficients $CC_{\surd}(T_i^W, T_+^W)$ and $CC_{\surd}(T_i^W, T_-^W)$ using Eqs. (7) and (8), respectively. It was recognized that $IE(T_+^W) = IE(T_-^W) = n = 4$ following Theorem 3. To give an instance, $CC_{\surd}(T_1^W, T_+^W) = CF(T_1^W, T_+^W)/\sqrt{IE(T_1^W) \cdot IE(T_+^W)} = 1.2599/\sqrt{2.1135 \times 4} = 0.4333$, and $CC_{\surd}(T_1^W, T_-^W) = CF(T_1^W, T_-^W)/\sqrt{IE(T_1^W) \cdot IE(T_-^W)} = 0.2020/\sqrt{2.1135 \times 4} = 0.0695$. The obtained outcomes of $CC_{\surd}(T_i^W, T_+^W)$ and $CC_{\surd}(T_i^W, T_-^W)$ are indicated in the fifth and sixth columns, respectively, of Table 4. On the flip side, if the "maximum function" type was utilized, this study would comply with Step 8-2 to generate the T-SF weighted correlation coefficients $CC_{\wedge}(T_i^W, T_+^W)$ and $CC_{\wedge}(T_i^W, T_-^W)$. The yielded outcomes are manifested in the last two columns of Table 4. For example, $CC_{\wedge}(T_i^W, T_+^W) = CF(T_1^W, T_+^W)/\max\{IE(T_1^W), IE(T_+^W)\} = 1.2599/\max\{2.1135, 4\} = 0.3150$, and $CC_{\wedge}(T_i^W, T_-^W) = CF(T_1^W, T_-^W)/\max\{IE(T_1^W), IE(T_-^W)\} = 0.2020/\max\{2.1135, 4\} = 0.0505$.

In Step 9, in the light of Definition 13, the following minimal and maximal correlation coefficients were produced as: $\underline{CC}_{\surd}^{+} = \min_{i'=1}^{4} CC_{\surd}(T_{i'}^W, T_+^W) = \min\{0.4333, 0.3709, 0.1544, 0.7049\} = 0.1544$, $\overline{CC}_{\surd}^{+} = \max_{i'=1}^{4} CC_{\surd}(T_{i'}^W, T_+^W) = 0.7049$, $\underline{CC}_{\surd}^{-} = \min_{i'=1}^{4} CC_{\surd}(T_{i'}^W, T_-^W) = \min\{0.0695, 0.1749, 0.1274, 0.0852\} = 0.0695$, and

$\overline{CC}^{-}_{\surd} = \max_{i'=1}^{4} CC_{\surd}(T_{i'}^{W}, T_{-}^{W}) = 0.1749$ for the "square root function" type. In a similar fashion, it was yielded that $\underline{CC}^{+}_{\wedge} = \min\{0.3150, 0.2740, 0.1246, 0.4864\} = 0.1246$, $\overline{CC}^{+}_{\wedge} = 0.4864$, $\underline{CC}^{-}_{\wedge} = 0.0505$, and $\overline{CC}^{-}_{\wedge} = 0.1292$. Letting the anchoring parameter $\xi = 0.5$, the T-SF comprehensive correlation indices were calculated using Eq. (11) for the "square root function" type. That is, $CI_{\surd}(a_1) = 0.6 \times (CC_{\surd}(T_1^W, T_+^W) - \underline{CC}^{+}_{\surd})/(\overline{CC}^{+}_{\surd} - \underline{CC}^{+}_{\surd}) + 0.4 \times (\overline{CC}^{-}_{\surd} - CC_{\surd}(T_1^W, T_-^W))/\ (\overline{CC}^{-}_{\surd} - \underline{CC}^{-}_{\surd}) = 0.6 \times (0.4333 - 0.1544)/(0.7049 - 0.1544) + 0.4 \times (0.1749 - 0.0695)/(0.1749 - 0.0695) = 0.7040$, $CI_{\surd}(a_2) = 0.2360$, $CI_{\surd}(a_3) = 0.1801$, and $CI_{\surd}(a_4) = 0.9402$. Next, for the "maximum function" type, the T-SF comprehensive correlation index $CI_{\wedge}(a_i)$ was generated using Eq. (12). Specifically, $CI_{\wedge}(a_1) = 0.6 \times (0.3150 - 0.1246)/(0.4864 - 0.1246) + 0.4 \times (0.1292 - 0.0505)/(0.1292 - 0.0505) = 0.7157$, $CI_{\wedge}(a_2) = 0.2478$, $CI_{\wedge}(a_3) = 0.1339$, $and CI_{\wedge}(a_4) = 0.9578$.

Finally, in Step 10, the four location options were ranked in descending order of the $CI_{\surd}(a_i)$ values for the "square root function" type, which rendered the prioritization ranking $a_4 \succ_{\surd} a_1 \succ_{\surd} a_2 \succ_{\surd} a_3$. Moreover, the prioritization ranking $a_4 \succ_{\wedge} a_1 \succ_{\wedge} a_2 \succ_{\wedge} a_3$ was yielded in descending order of $CI_{\wedge}(a_i)$ for the "maximum function" type. Regardless of the usage of the square root function and the maximum function, the solution outcomes generated by the current correlation-focused approach are concordant with the final ranking rendered by the technique using T-SF group-generalized hybrid geometric (GGHG) operators in Chen *et al.* (2021).

The conclusions of the application of the propounded methodology to the pragmatic problem for location selection are consistent with the consequences of the existing literature. The new approach centered on T-SF correlation-focused measurements in this study is not only rigorous in concept but also simple and easy to implement. Findings in practical applications are also consistent with existing literature and expectations.

4.2. *Comparative Analysis with Other Relevant Approaches*

This subsection intends to conduct a comparative analysis to analyse the solution outcomes with those yielded by other T-SF multiple-criteria assessment approaches. As described in the state-of-the-art literature review in Table 1, many studies have explored the modularity of evaluation and decision-making methods involving T-SF information by T-SF averaging aggregation operations. Given the large body of related work that has concentrated on models of aggregated or averaged operations, this comparative analysis will provide a comprehensive discussion of the applied results rendered by some newly-developed aggregating or averaging operations regarding the location selection issue of the construction company to build new apartments. Such comparisons and analyses focus on the process of investigating the solution outcomes with each other and distinguishing their similarities and differences.

The T-SF averaging aggregation operations used for this comparative research cover the T-SF weighted averaging (WA) and T-SF weighted geometric (WG) operators advanced by Ullah *et al.* (2020a), the T-SF Frank weighted averaging (FWA) and T-SF Frank weighted geometric (FWG) operators initiated by Mahnaz *et al.* (2022), and the

T-SF Aczel-Alsina weighted averaging (AAWA) and T-SF Aczel-Alsina weighted geometric (AAWG) operators advocated by Hussain *et al.* (2022b). From the arithmetic mean perspective, the technique using T-SF WA operators is a generally recognized T-SF aggregation algorithm. Moreover, the techniques using T-SF FWA or T-SF AAWA operators are rising T-SF aggregation algorithms with great potential. From the geometric mean viewpoint, the technique established on the T-SF WG operator provides a well-known T-SF aggregation algorithm. Furthermore, the techniques using T-SF FWG or T-SF AAWG operators are recently up-and-coming T-SF aggregation algorithms. Next, the mathematical expressions of the aforementioned arithmetic mean operators (i.e. T-SF WA, T-SF FWA, and T-SF AAWA) and the geometric mean operators (i.e. T-SF WG, T-SF FWG, and T-SF AAWG) will be described later.

To perform averaging aggregation operations under T-SF uncertainty, the direction of the negative criteria in the collection C_{Ne} should be reversed to be consistent with the direction of the positive criteria in the collection C_{Po}. Let $t'_{ij} = (\mu'_{ij}, \eta'_{ij}, \nu'_{ij})$ signify the normalized T-SF performance rating associated with t_{ij}. Using the means of the complement set operation, the T-SF characteristic T_i can be transformed into the normalized T-SF characteristic T'_i using the following formula:

$$\begin{aligned} T'_i &= \{\langle c_j, t'_{ij}\rangle \mid c_j \in C\} \\ &= \{\langle c_j, (\mu_{ij}, \eta_{ij}, \nu_{ij})\rangle \mid c_j \in C_{Po}, \langle c_j, (\nu_{ij}, \eta_{ij}, \mu_{ij})\rangle \mid c_j \in C_{Ne}\}. \end{aligned} \tag{13}$$

This comparative study endeavours to aggregate the normalized T-SF performance rating t'_{ij} across all $c_j \in C$ concerning each a_i into a T-SF comprehensive evaluation value by employing the aggregation operators propounded by Ullah *et al.* (2020a), Mahnaz *et al.* (2022), and Hussain *et al.* (2022b). Let $\phi > 1$ and $\Phi \geqslant 1$ denote the parameters contained in Mahnaz *et al.*'s and Hussain *et al.*'s formulations, respectively. The T-SF comprehensive evaluation value of $t'_{i1}, t'_{i2}, \dots, t'_{in}$ using the T-SF WA, T-SF FWA, and T-SF AAWA operators are determined sequentially along these lines:

$$WA(t'_{i1}, t'_{i2}, \dots, t'_{in}) = \left(\sqrt[q]{1 - \prod_{j=1}^{n}\left(1 - (\mu'_{ij})^q\right)^{w_j}}, \prod_{j=1}^{n}(\eta'_{ij})^{w_j}, \prod_{j=1}^{n}(\nu'_{ij})^{w_j}\right), \tag{14}$$

$$\begin{aligned} &FWA(t'_{i1}, t'_{i2}, \dots, t'_{in}) \\ &= \left(\sqrt[q]{1 - \log_\phi\left(1 + \prod_{j=1}^{n}\left(\phi^{1-(\mu'_{ij})^q} - 1\right)^{w_j}\right)}, \right. \\ &\quad \sqrt[q]{\log_\phi\left(1 + \prod_{j=1}^{n}\left(\phi^{(\eta'_{ij})^q} - 1\right)^{w_j}\right)}, \\ &\quad \left. \sqrt[q]{\log_\phi\left(1 + \prod_{j=1}^{n}\left(\phi^{(\nu'_{ij})^q} - 1\right)^{w_j}\right)}\right), \end{aligned} \tag{15}$$

$$AAWA(t'_{i1}, t'_{i2}, \dots, t'_{in}) = \Bigg(\sqrt[q]{1 - \exp\Bigg(-\Bigg\{\sum_{j=1}^{n} w_j\big[-\ln\big(1 - (\mu'_{ij})^q\big)^{\Phi}\big]\Bigg\}^{1/\Phi}\Bigg)}, \sqrt[q]{\exp\Bigg(-\Bigg\{\sum_{j=1}^{n} w_j\big[-\ln\big((\eta'_{ij})^q\big)^{\Phi}\big]\Bigg\}^{1/\Phi}\Bigg)}, \sqrt[q]{\exp\Bigg(-\Bigg\{\sum_{j=1}^{n} w_j\big[-\ln\big((\nu'_{ij})^q\big)^{\Phi}\big]\Bigg\}^{1/\Phi}\Bigg)}\Bigg). \tag{16}$$

From the geometric mean perspective, the T-SF comprehensive evaluation value of $t'_{i1}, t'_{i2}, \dots, t'_{in}$ using the T-SF WG, T-SF FWG, and T-SF AAWG operators are calculated sequentially in the following manner, where $\phi > 1$ and $\Phi \geqslant 1$:

$$WG(t'_{i1}, t'_{i2}, \dots, t'_{in}) = \Bigg(\prod_{j=1}^{n} (\mu'_{ij})^{w_j}, \prod_{j=1}^{n} (\eta'_{ij})^{w_j}, \sqrt[q]{1 - \prod_{j=1}^{n} \big(1 - (\nu'_{ij})^q\big)^{w_j}}\Bigg), \tag{17}$$

$$FWG(t'_{i1}, t'_{i2}, \dots, t'_{in}) = \Bigg(\sqrt[q]{\log_{\phi}\Bigg(1 + \prod_{j=1}^{n} \big(\phi^{(\mu'_{ij})^q} - 1\big)^{w_j}\Bigg)}, \sqrt[q]{1 - \log_{\phi}\Bigg(1 + \prod_{j=1}^{n} \big(\phi^{1-(\eta'_{ij})^q} - 1\big)^{w_j}\Bigg)}, \sqrt[q]{1 - \log_{\phi}\Bigg(1 + \prod_{j=1}^{n} \big(\phi^{1-(\nu'_{ij})^q} - 1\big)^{w_j}\Bigg)}\Bigg), \tag{18}$$

$$AAWG(t'_{i1}, t'_{i2}, \dots, t'_{in}) = \Bigg(\sqrt[q]{\exp\Bigg(-\Bigg\{\sum_{j=1}^{n} w_j\big[-\ln\big((\mu'_{ij})^q\big)^{\Phi}\big]\Bigg\}^{1/\Phi}\Bigg)}, \sqrt[q]{1 - \exp\Bigg(-\Bigg\{\sum_{j=1}^{n} w_j\big[-\ln\big(1 - (\eta'_{ij})^q\big)^{\Phi}\big]\Bigg\}^{1/\Phi}\Bigg)}, \sqrt[q]{1 - \exp\Bigg(-\Bigg\{\sum_{j=1}^{n} w_j\big[-\ln\big(1 - (\nu'_{ij})^q\big)^{\Phi}\big]\Bigg\}^{1/\Phi}\Bigg)}\Bigg). \tag{19}$$

This study exploited a well-grounded score function advanced by Zeng *et al.* (2019) to help compare the obtained T-SF comprehensive evaluation values. Let $t'_i = (\mu'_i, \eta'_i, \nu'_i)$ signify the T-SF comprehensive evaluation value produced by the T-SF WA, FWA, AAWA, WG, FWG, or AAWG operators, where its degree of refusal-membership $\gamma'_i = \sqrt[q]{1 - (\mu'_i)^q - (\eta'_i)^q - (\nu'_i)^q}$. Following Zeng *et al.*'s formulation, the aggregated score

Table 5
Outcomes of the T-SF comprehensive evaluation value t_i' yielded by the comparative approaches.

Method	$t_1' = (\mu_1', \eta_1', \nu_1')$	$t_2' = (\mu_2', \eta_2', \nu_2')$	$t_3' = (\mu_3', \eta_3', \nu_3')$	$t_4' = (\mu_4', \eta_4', \nu_4')$
The aggregation technique using Ullah *et al.*'s (2020a) operators				
T-SF WA	(0.7051, 0.3629, 0.2095)	(0.6765, 0.2787, 0.4045)	(0.4999, 0.1672, 0.2343)	(0.8093, 0.2353, 0.3190)
T-SF WG	(0.6771, 0.3629, 0.3607)	(0.6076, 0.2787, 0.5287)	(0.4846, 0.1672, 0.4894)	(0.7749, 0.2353, 0.3379)
The aggregation technique using Mahnaz *et al.*'s (2022) operators				
T-SF FWA	(0.7030, 0.3647, 0.2103)	(0.6740, 0.2789, 0.4081)	(0.4993, 0.1673, 0.2367)	(0.8071, 0.2359, 0.3192)
T-SF FWG	(0.6790, 0.4574, 0.3583)	(0.6124, 0.2981, 0.5272)	(0.4851, 0.1921, 0.4825)	(0.7780, 0.3321, 0.3375)
The aggregation technique using Hussain *et al.*'s (2022b) operators				
T-SF AAWA	(0.7443, 0.3161, 0.1735)	(0.7185, 0.2668, 0.2913)	(0.5247, 0.1596, 0.1716)	(0.8375, 0.1869, 0.3117)
T-SF AAWG	(0.6510, 0.5505, 0.5025)	(0.5024, 0.3168, 0.5703)	(0.4733, 0.2307, 0.6497)	(0.7254, 0.3811, 0.3896)

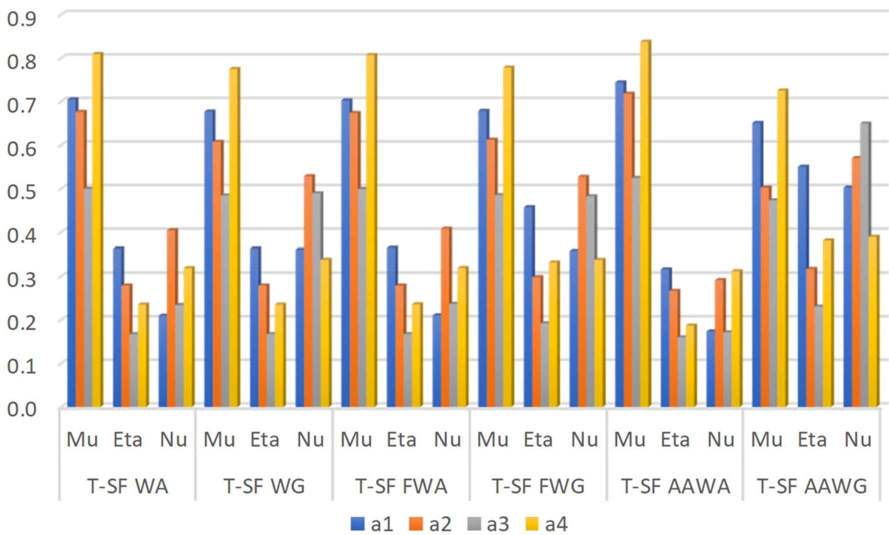


Fig. 4. Juxtaposition of three components of positive-, neutral-, and negative-membership in t_i'.

value of t_i' is elucidated like this:

$$AS(t_i') = (\mu_i')^q - (\eta_i')^q - (\nu_i')^q + (\gamma_i')^q\left(\frac{\exp((\mu_i')^q - (\eta_i')^q - (\nu_i')^q)}{\exp((\mu_i')^q - (\eta_i')^q - (\nu_i')^q) + 1} - \frac{1}{2}\right). \tag{20}$$

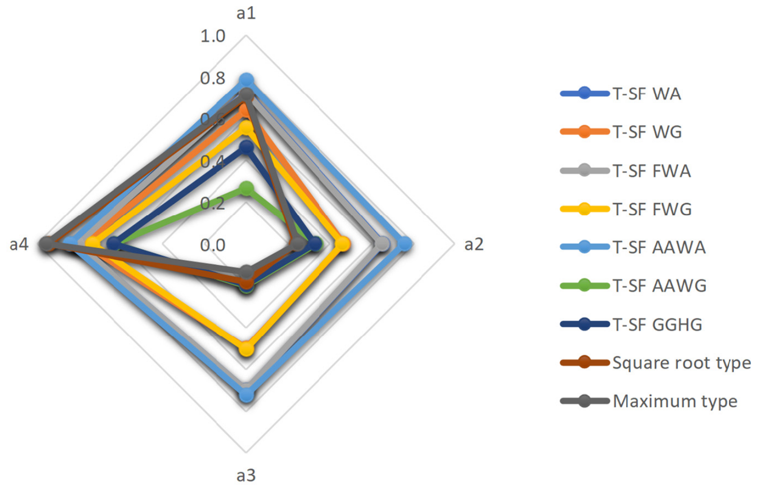
In the light of the location selection issue of a construction company for building new apartments, this research exploited Eqs. (14)–(19) to produce the T-SF comprehensive evaluation value t_i', and the determination outcomes are displayed in Table 5. Herein, referring to the specifications by Mahnaz *et al.* (2022) and Hussain *et al.* (2022b), the two parameters ϕ and Φ were designated as $\phi = 2$ in Eqs. (15) and (18) and $\Phi = 5$ in Eqs. (16) and (19). To get a general idea of the obtained T-SF comprehensive evaluation values, the juxtaposition results of the three components of positive-, neutral-, and negative-membership (i.e. μ_i', η_i', and ν_i', respectively) contained in t_i' are sketched in Fig. 4.

Table 6
The aggregated score value and the T-SF comprehensive correlation index with their rank orders.

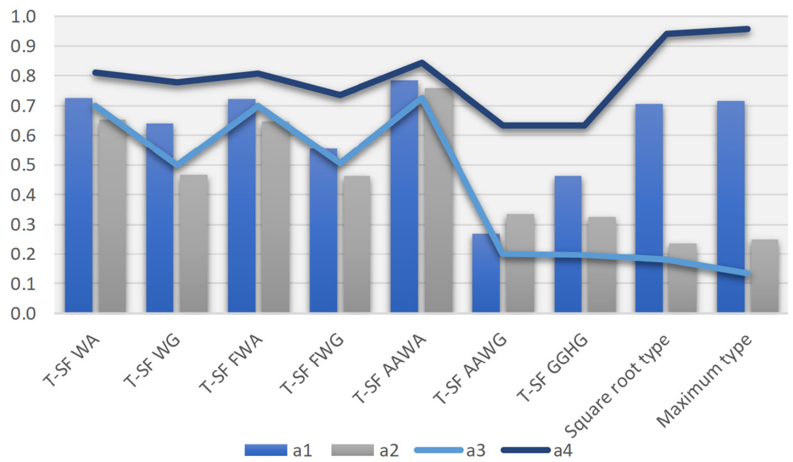
Source of methods	Comparative approach	a_1	a_2	a_3	a_4
Ullah *et al.* (2020b)	T-SF WA operator	0.7252 (2)	0.6520 (4)	0.6991 (3)	0.8091 (1)
	T-SF WG operator	0.6397 (2)	0.4664 (4)	0.5001 (3)	0.7768 (1)
Mahnaz *et al.* (2022)	T-SF FWA operator	0.7224 (2)	0.6472 (4)	0.6982 (3)	0.8074 (1)
	T-SF FWG operator	0.5550 (2)	0.4624 (4)	0.5048 (3)	0.7355 (1)
Hussain *et al.* (2022b)	T-SF AAWA operator	0.7855 (2)	0.7575 (3)	0.7245 (4)	0.8423 (1)
	T-SF AAWG operator	0.2675 (3)	0.3334 (2)	0.1994 (4)	0.6316 (1)
Chen *et al.* (2021)	T-SF GGHG operator	0.4620 (2)	0.3257 (3)	0.1951 (4)	0.6322 (1)
Current paper	Square root function type	0.7040 (2)	0.2360 (3)	0.1801 (4)	0.9402 (1)
	Maximum function type	0.7157 (2)	0.2478 (3)	0.1339 (4)	0.9578 (1)

Next, this study used Eq. (20) to generate the aggregated score value $AS(t_i')$ and then identify the corresponding prioritization ranking order, as revealed in Table 6. Over and above that, to conduct a baseline analysis, the technique using the T-SF GGHG operator evolved by Chen *et al.* (2021) will be exploited to be a beginning point used for comparisons. The aggregated score values generated by the T-SF GGHG operator are exhibited in Table 6. As described in the previous subsection, when employing the propounded methodology in this study, the T-SF comprehensive correlation indices ($CI_{\surd}(a_i)$ and $CI_{\wedge}(a_i)$ based on the square root and maximum functions, respectively) are also displayed in Table 6. Moreover, the numbers in parentheses are the orders of precedence for each choice option. The techniques using the T-SF WA, WG, FWA, and FWG operators generated the identical prioritization ranking $a_4 \succ a_1 \succ a_3 \succ a_2$. The techniques using the T-SF AAWA and GGHG operators and the current multiple-criteria choice method using the square root and maximum functions generated the same prioritization ranking $a_4 \succ a_1 \succ a_2 \succ a_3$. The use of the technique using the T-SF AAWG operator yielded a particularly different ordering result $a_4 \succ a_2 \succ a_1 \succ a_3$. Of all the comparative approaches, only the solution results yielded by the T-SF AAWA operator and the current methodology ranked the same as the benchmark method using the T-SF GGHG operator. The Spearman correlation between the benchmark ranking (i.e. $a_4 \succ a_1 \succ a_2 \succ a_3$) and the solution outcome based on the T-SF WA, WG, FWA, and FWG operators is equal to 0.8. The Spearman correlation between the benchmark ranking and the solution outcome based on the T-SF AAWG operator is also equal to 0.8. It is noted that the Spearman correlation between the two prioritization rankings $a_4 \succ a_1 \succ a_3 \succ a_2$ and $a_4 \succ a_2 \succ a_1 \succ a_3$ reduces to 0.4.

The aggregated score values and T-SF comprehensive correlation indices yielded by the T-SF averaging aggregation operations and the evolved multiple-criteria choice method, respectively, are contrasted in Fig. 5. In particular, Fig. 5(a) reveals the comparisons among the four choice options under distinct comparative approaches. Furthermore, consider that the choice option a_4 performed the best among all comparative approaches, while the choice option a_3 performed the worst among most comparative approaches (i.e. the T-SF AAWA, AAWG, GGHG operators, and the current method based on the square root and maximum functions). The relative performances associated with the best and comparatively worst choice options (i.e. a_4 and a_3, respectively) are contrasted in Fig. 5(b) to highlight their juxtaposition.



(a) Contrast outcomes among choice options under distinct comparative approaches.



(b) Relative performance associated with the best and relatively worst choice options.

Fig. 5. Comparison results of the aggregated score values/T-SF comprehensive correlation indices.

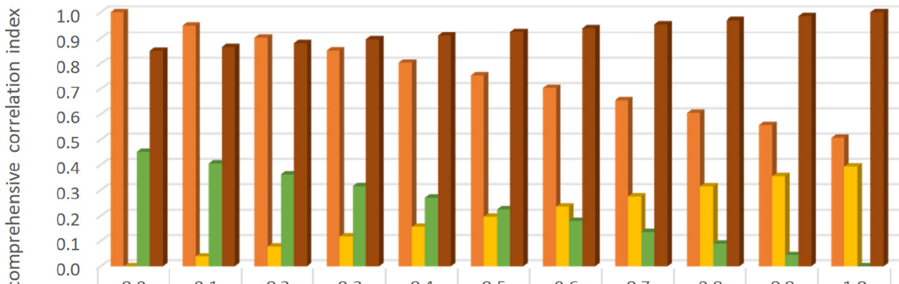
Going a step further, this study attempts to examine the solution outcomes produced by the comparative approaches with a benchmark ranking by Chen *et al.* (2021). The prioritization ranking (i.e. $a_4 \succ a_1 \succ a_3 \succ a_2$) obtained by the techniques using the T-SF WA, WG, FWA, and FWG operators differs from the benchmark ranking (i.e. $a_4 \succ a_1 \succ a_2 \succ a_3$) based on the T-SF GGHG operator in the outranking relationship between a_2 and a_3. The difference between the prioritization ranking (i.e. $a_4 \succ a_2 \succ a_1 \succ a_3$) rendered by the technique using the T-SF AAWG operator and the benchmark ranking based on the T-SF GGHG operator lies in the outranking relationship between a_1 and a_2. Different from the techniques using the aggregation operators initiated by Ullah *et al.* (2020a), Mahnaz *et al.* (2022), and Hussain *et al.* (2022b), the prioritization rankings yielded by the two approaches based on square root and maximum functions in this study are consistent with the benchmark ranking determined from the T-SF GGHG operator. Therefore, the comparative investigation of the application outcomes supports the superiority of the proposed multiple-criteria choice method grounded in T-SF data-driven correlation measures.

4.3. *More Comparative Discussion Based on Parametric Analysis*

This subsection has the objective of conducting a comprehensive comparative analysis from a problem-oriented point of view. In the first comparative study, different settings of the anchoring parameter are explored and the yielded outcomes of T-SF comprehensive correlation indices under each scenario are discussed holistically. In the second comparative study, the best and worst choice options that are constituted by the universal and null T-SF sets are replaced by the positive and negative ideal schemes, respectively, to be a benchmark for exploring the effects on the T-SF correlation-focused measurements.

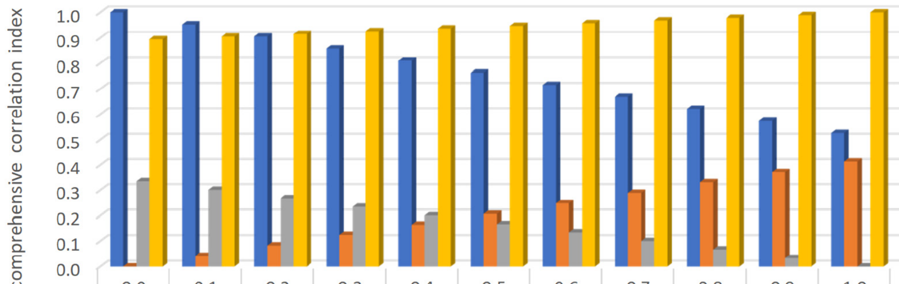
The first comparative study gives thought to distinct assigned values of the anchoring parameter ξ and investigates the yielded consequences of T-SF comprehensive correlation indices under various parameter settings. By conducting such a comparative study, the effect of the distinct controlling or deciding of the parameter ξ on the T-SF comprehensive correlation indices $CI_{\surd}(a_i)$ (based on the square root function) and $CI_{\wedge}(a_i)$ (based on the maximum function) can be obtained; moreover, the stability and controllability of the prioritization ranking results can be investigated. In the comparative analysis, the values of the anchoring parameter ξ were set to $0.0, 0.1, \ldots, 1.0$. The juxtaposition and comparisons of $CI_{\surd}(a_i)$ and $CI_{\wedge}(a_i)$ for distinct values of ξ are portrayed in Fig. 6(a) and Fig. 6(b), respectively.

As depicted in Fig. 6(a), the three prioritization rankings $a_1 \succ_{\surd} a_4 \succ_{\surd} a_3 \succ_{\surd} a_2$, $a_4 \succ_{\surd} a_1 \succ_{\surd} a_3 \succ_{\surd} a_2$, and $a_4 \succ_{\surd} a_1 \succ_{\surd} a_2 \succ_{\surd} a_3$ were generated when $\xi = 0.0, 0.1, 0.2$, $\xi = 0.3, 0.4, 0.5$, and $\xi = 0.6, 0.7, \ldots, 1.0$, respectively. As revealed in Fig. 6(b), the rankings $a_1 \succ_{\wedge} a_4 \succ_{\wedge} a_3 \succ_{\wedge} a_2$, $a_4 \succ_{\wedge} a_1 \succ_{\wedge} a_3 \succ_{\wedge} a_2$, and $a_4 \succ_{\wedge} a_1 \succ_{\wedge} a_2 \succ_{\wedge} a_3$ were produced when $\xi = 0.0, 0.1$, $\xi = 0.2, 0.3, 0.4$, and $\xi = 0.5, 0.6, \ldots, 1.0$, respectively. In this respect, the prioritization ranking outcomes using the square root function were not much different from those using the maximum function. The main discrimination was that the ranking outcomes in the case of $\xi = 0.2$ and $\xi = 0.5$ are inconsistent. On the other hand, it is worth mentioning that the obtained $CI_{\surd}(a_i)$ and $CI_{\wedge}(a_i)$ values gave rise to identical rankings (i.e. the prioritization rankings $a_4 \succ_{\surd} a_1 \succ_{\surd} a_2 \succ_{\surd} a_3$ and $a_4 \succ_{\wedge} a_1 \succ_{\wedge} a_2 \succ_{\wedge} a_3$ when $\xi = 0.6, 0.7, \ldots, 1.0$ and $\xi = 0.5, 0.6, \ldots, 1.0$, respectively) in comparison to the ranking outcome rendered by Chen *et al.* (2021). Thus, the efficacy and reasonableness of the proposed methodology can be corroborated because of consistent ranking results in most cases. Furthermore, somewhat different rankings $a_4 \succ_{\surd} a_1 \succ_{\surd} a_3 \succ_{\surd} a_2$ (based on the square root function) and $a_4 \succ_{\wedge} a_1 \succ_{\wedge} a_3 \succ_{\wedge} a_2$ (based on the maximum function) were acquired when $\xi = 0.3, 0.4, 0.5$ and $\xi = 0.2, 0.3, 0.4$, respectively. Nonetheless, different outcomes were yielded in face of the small values of ξ, namely $a_1 \succ_{\surd} a_4 \succ_{\surd} a_3 \succ_{\surd} a_2$ and



	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a1	1.0000	0.9507	0.9013	0.8520	0.8027	0.7533	0.7040	0.6547	0.6054	0.5560	0.5067
a2	0.0000	0.0393	0.0787	0.1180	0.1573	0.1967	0.2360	0.2753	0.3147	0.3540	0.3933
a3	0.4502	0.4052	0.3602	0.3152	0.2701	0.2251	0.1801	0.1351	0.0900	0.0450	0.0000
a4	0.8505	0.8655	0.8804	0.8954	0.9103	0.9253	0.9402	0.9552	0.9701	0.9851	1.0000

(a) Juxtaposition of $CI_{\surd}(a_i)$ for various values of ξ.



	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a1	1.0000	0.9526	0.9052	0.8579	0.8105	0.7631	0.7157	0.6684	0.6210	0.5736	0.5262
a2	0.0000	0.0413	0.0826	0.1239	0.1652	0.2065	0.2478	0.2891	0.3304	0.3717	0.4130
a3	0.3347	0.3012	0.2677	0.2343	0.2008	0.1673	0.1339	0.1004	0.0669	0.0335	0.0000
a4	0.8944	0.9050	0.9155	0.9261	0.9366	0.9472	0.9578	0.9683	0.9789	0.9894	1.0000

(b) Juxtaposition of $CI_{\wedge}(a_i)$ for various values of ξ.

Fig. 6. Contrasts of the T-SF comprehensive correlation indices in distinct settings of the anchoring parameter.

$a_1 \succ_{\wedge} a_4 \succ_{\wedge} a_3 \succ_{\wedge} a_2$ when $\xi = 0.0, 0.1, 0.2$ and $\xi = 0.0, 0.1$, respectively. Overall, stable and justified consequences can be generated under most settings of ξ. When $\xi = 0.0, 0.1, 0.2$ based on the "square root function" type or $\xi = 0.0, 0.1$ based on the "maximum function" type, distinct prioritization ranking outcomes can be rendered to reflect the change of the ξ values, which gives substance to the pliability of the current methods by adjusting the anchoring parameter ξ. The comparison consequence demonstrates that by controlling the parameter values, stable and flexible prioritization rankings can be produced by using the propounded methodology.

In the second comparative study, the best choice option a_+ and the worst choice option a_- (composed of the universal T-SF set and the null T-SF set) are replaced by the positive and negative ideal schemes, respectively, as an alternate benchmark for calculating the T-SF correlation-focused measurements. To accommodate the change of the reference points, this study would like to yield the corresponding T-SF correlation-focused measurements, so that the subsequent practical data processing and multiple-criteria evaluation procedures can operate smoothly. Through the juxtaposition and comparison of the solution outcomes, the influence of distinct points of reference on the yielded results can be clarified. Moreover, through the side-by-side comparison, the advantages of taking a_+ and a_- as points of reference can be demonstrated and justified.

The positive and negative ideal schemes would be exploited to replace the best and worst choice options, respectively, to explore the influences of different points of reference on the T-SF correlation-focused measurements and resolution consequences. More specifically, instead of the universal and null T-SF sets, the T-SF characteristics of the ideal schemes would be established using the union and intersection operations. Let a_* indicate the positive ideal scheme, where the T-SF characteristic $T_* = \{\langle c_j, t_{*j}\rangle \mid c_j \in C\} = \{\langle c_j, (\mu_{*j}, \eta_{*j}, \nu_{*j})\rangle \mid c_j \in C\}$. Let $a_\#$ signify the negative ideal scheme, where the T-SF characteristic $T_\# = \{\langle c_j, t_{\#j}\rangle \mid c_j \in C\} = \{\langle c_j, (\mu_{\#j}, \eta_{\#j}, \nu_{\#j})\rangle \mid c_j \in C\}$. Utilizing the set operations $\cup$ and $\cap$, T_* and $T_\#$ are delineated in this fashion:

1. $T_* = \Big\{\Big\langle c_j, \Big(\max_{i=1}^m \mu_{ij}, \min_{i=1}^m \eta_{ij}, \min_{i=1}^m \nu_{ij}\Big)\Big\rangle \Big| c_j \in C_{Po}, \Big\langle c_j, \Big(\min_{i=1}^m \mu_{ij}, \min_{i=1}^m \eta_{ij}, \max_{i=1}^m \nu_{ij}\Big)\Big\rangle \Big| c_j \in C_{Ne}\Big\}$;
2. $T_\# = \Big\{\Big\langle c_j, \Big(\min_{i=1}^m \mu_{ij}, \min_{i=1}^m \eta_{ij}, \max_{i=1}^m \nu_{ij}\Big)\Big\rangle \Big| c_j \in C_{Po}, \Big\langle c_j, \Big(\max_{i=1}^m \mu_{ij}, \min_{i=1}^m \eta_{ij}, \min_{i=1}^m \nu_{ij}\Big)\Big\rangle \Big| c_j \in C_{Ne}\Big\}$.

Recall that $C_{Po} = \{c_2, c_3\}$ and $C_{Ne} = \{c_1, c_4\}$ in the location selection problem. Using the aforesaid manner, the T-SF characteristics of a_* and $a_\#$ were identified as follows: $T_* = \{\langle c_1, (0.14, 0.12, 0.83)\rangle, \langle c_2, (0.91, 0.12, 0.13)\rangle, \langle c_3, (0.81, 0.11, 0.11)\rangle, \langle c_4, (0.11, 0.14, 0.84)\rangle\}$ and $T_\# = \{\langle c_1, (0.75, 0.12, 0.41)\rangle, \langle c_2, (0.26, 0.12, 0.63)\rangle, \langle c_3, (0.56, 0.11, 0.55)\rangle, \langle c_4, (0.61, 0.14, 0.45)\rangle\}$. The corresponding T-SF weighted characteristics were given by: $T_*^W = \{\langle c_1, (0.1300, 0.1156, 0.8012)\rangle, \langle c_2, (0.7542, 0.1171, 0.1009)\rangle, \langle c_3, (0.8422, 0.1147, 0.1060)\rangle, \langle c_4, (0.1286, 0.1637, 0.8922)\rangle\}$ and $T_\#^W = \{\langle c_1, (0.7080, 0.1156, 0.3965)\rangle, \langle c_2, (0.1919, 0.1171, 0.5083)\rangle, \langle c_3, (0.5914, 0.1147, 0.5538)\rangle, \langle c_4, (0.6963, 0.1637, 0.5210)\rangle\}$. The T-SF weighted informational energies were derived as: $IE(T_*^W) = 2.1053$ and $IE(T_\#^W) = 2.0837$. The comparisons of the T-SF weighted correlation functions $CF(T_i^W, T_+^W)$, $CF(T_i^W, T_*^W)$, $CF(T_i^W, T_-^W)$, and $CF(T_i^W, T_\#^W)$ are manifested in Fig. 7. Furthermore, the T-SF weighted correlation coefficients $CC_{\surd}(T_i^W, T_+^W)$, $CC_{\surd}(T_i^W, T_*^W)$, $CC_{\surd}(T_i^W, T_-^W)$, and $CC_{\surd}(T_i^W, T_\#^W)$ are contrasted in Fig. 8(a), while the comparisons of $CC_\wedge(T_i^W, T_+^W)$, $CC_\wedge(T_i^W, T_*^W)$, $CC_\wedge(T_i^W, T_-^W)$, and $CC_\wedge(T_i^W, T_\#^W)$ are exhibited in Fig. 8(b).

First, consider the contrast outcomes of the T-SF weighted correlation functions concerning the best choice option a_+ and the positive ideal scheme a_*, as revealed in Fig. 5. The differences among the $CF(T_i^W, T_+^W)$ values of the four choice options ($a_1 - a_4$) were significantly higher than the differences among the $CF(T_i^W, T_*^W)$ values. In particular, the gap between the maximum value (i.e. $CF(T_4^W, T_+^W)$) and the minimum value (i.e. $CF(T_3^W, T_+^W)$) was quite pronounced. However, the gap between the maximum value

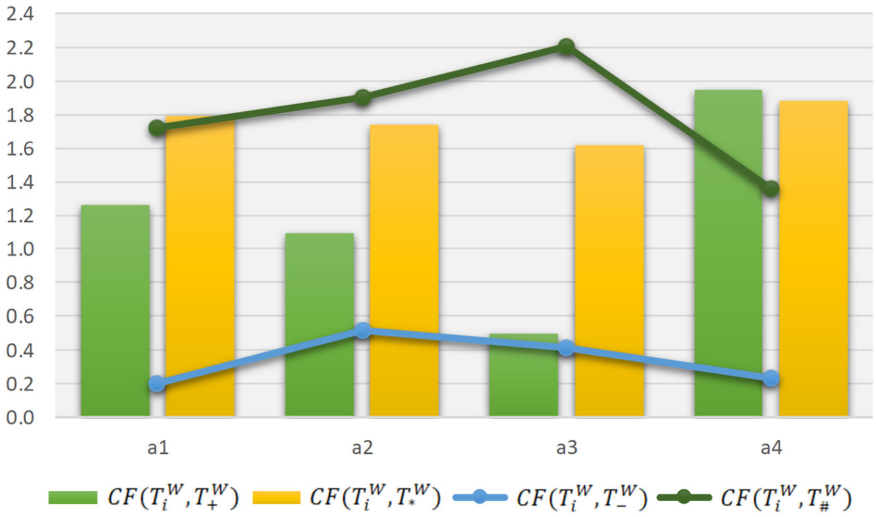
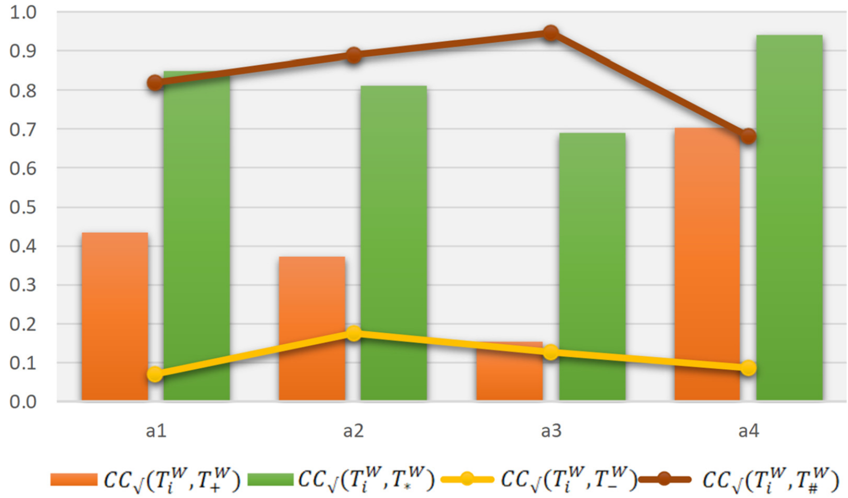


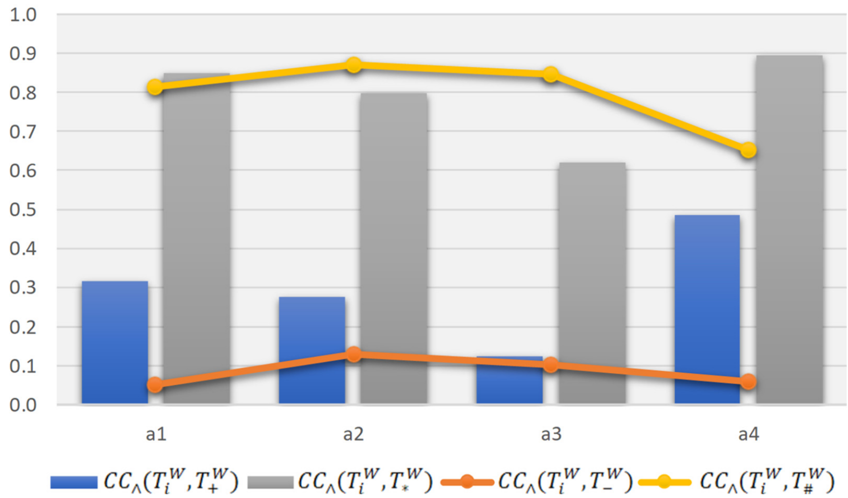
Fig. 7. Contrast outcomes of the T-SF weighted correlation functions concerning distinct points of reference.

(i.e. $CF(T_4^W, T_*^W)$) and the minimum value (i.e. $CF(T_3^W, T_*^W)$) did not show a particularly significant difference. Next, concerning the T-SF weighted correlation functions toward the worst choice option a_- and the negative ideal scheme $a_\#$, the maximum value of $CF(T_i^W, T_-^W)$ and the maximum value of $CF(T_i^W, T_\#^W)$ correspond to different options; the same is true for the minimum value of $CF(T_i^W, T_-^W)$ (or $CF(T_i^W, T_\#^W)$). To be precise, the options a_2 and a_1 enjoy the largest and smallest values, respectively, of $CF(T_i^W, T_-^W)$; a_3 and a_4 enjoy the largest and smallest values, respectively, of $CF(T_i^W, T_\#^W)$.

Next, consider the comparisons of the T-SF weighted correlation coefficients with relevance to two types of points of reference (i.e. one type for the best and worst choice options and the other type for the positive and negative ideal schemes). Let us investigate the contrast outcomes in Fig. 8(a) using the "square root function" type. The $CC_{\surd}(T_i^W, T_*^W)$ values of the four choice options were significantly higher than the $CC_{\surd}(T_i^W, T_+^W)$ values; this phenomenon was also found in the comparisons of the values of $CC_{\surd}(T_i^W, T_\#^W)$ and $CC_{\surd}(T_i^W, T_-^W)$. The higher the value of $CC_{\surd}(T_i^W, T_+^W)$ (or $CC_{\surd}(T_i^W, T_*^W)$), the higher the correlation between the corresponding option a_i and a_+ (or a_*). Accordingly, the decision-maker expects to choose the option that is highly correlated with the best choice option (or the positive ideal scheme). The lower the value of $CC_{\surd}(T_i^W, T_-^W)$ (or $CC_{\surd}(T_i^W, T_\#^W)$), the lower the correlation between a_i and a_- (or $a_\#$). In this regard, the decision-maker expects to choose the option that lowly correlates with the worst choice option (or the negative ideal scheme). In Fig. 8(a), the numerical orders of the T-SF weighted correlation coefficients for mutual relationships with a_+ and a_* were $CC_{\surd}(T_4^W, T_+^W) > CC_{\surd}(T_1^W, T_+^W) > CC_{\surd}(T_2^W, T_+^W) > CC_{\surd}(T_3^W, T_+^W)$ and $CC_{\surd}(T_4^W, T_*^W) > CC_{\surd}(T_1^W, T_*^W) > CC_{\surd}(T_2^W, T_*^W) > CC_{\surd}(T_3^W, T_*^W)$, respectively. Different from the identical ranking orders above, the numerical orders of the T-SF weighted correlation coefficients for mutual relationships with a_- and $a_\#$



(a) Outcomes using the "square root function" type



(b) Outcomes using the "maximum function" type

Fig. 8. Contrast outcomes of the T-SF weighted correlation coefficients concerning distinct points of reference. (a) Outcomes using the "square root function" type. (b) Outcomes using the "maximum function" type.

were $CC_{\surd}(T_1^W, T_-^W) < CC_{\surd}(T_4^W, T_-^W) < CC_{\surd}(T_3^W, T_-^W) < CC_{\surd}(T_2^W, T_-^W)$ and $CC_{\surd}(T_4^W, T_\#^W) < CC_{\surd}(T_1^W, T_\#^W) < CC_{\surd}(T_2^W, T_\#^W) < CC_{\surd}(T_3^W, T_\#^W)$, respectively. In Fig. 8(b), the findings concerning the contrast outcomes using the "maximum function" type were about the same as those using the "square root function" type except for the results of $CC_{\wedge}(T_i^W, T_\#^W)$. Specifically, the numerical orders of $CC_{\wedge}(T_i^W, T_\#^W)$ were given by $CC_{\wedge}(T_4^W, T_\#^W) < CC_{\wedge}(T_1^W, T_\#^W) < CC_{\wedge}(T_3^W, T_\#^W) < CC_{\wedge}(T_2^W, T_\#^W)$. In the matter of the numerical orders of the T-SF weighted correlation coefficients among the four options, the two ranking outcomes employing $CC_{\surd}(T_i^W, T_+^W)$ and $CC_{\surd}(T_i^W, T_*^W)$ were consistent; however, the two ranking outcomes via $CC_{\surd}(T_i^W, T_-^W)$ and $CC_{\surd}(T_i^W, T_\#^W)$ were somewhat different. To unify the inconsistent numerical orders, this study continued to calculate the T-SF comprehensive correlation indices for the final decision.

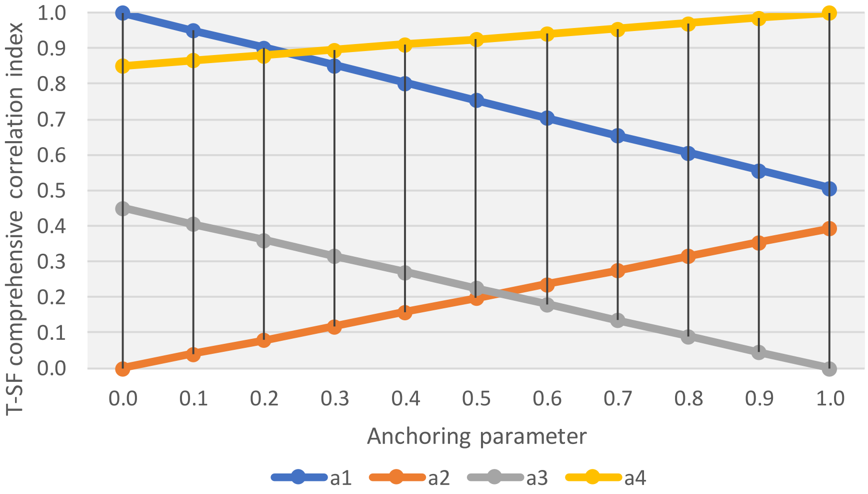
To facilitate discussing the effects of the anchoring parameter ξ regarding the solution consequences, this study set eleven different values for the parameter to calculate the T-SF comprehensive correlation indices and identify the ultimate ranking outcome under various scenarios. This study designated the anchoring parameter ξ ranging from 0 to 1, wherein $\xi = 0.0, 0.1, \ldots, 1.0$. Let $\underline{CC}^*_{\surd} = \min_{i'=1}^4 CC_{\surd}(T_{i'}^W, T_*^W)$, $\overline{CC}^*_{\surd} = \max_{i'=1}^4 CC_{\surd}(T_{i'}^W, T_*^W)$, $\underline{CC}^\#_{\surd} = \min_{i'=1}^4 CC_{\surd}(T_{i'}^W, T_\#^W)$, and $\overline{CC}^\#_{\surd} = \max_{i'=1}^4 CC_{\surd}(T_{i'}^W, T_\#^W)$ for the "square root function" type. Let $\underline{CC}^*_{\wedge} = \min_{i'=1}^4 CC_{\wedge}(T_{i'}^W, T_*^W)$, $\overline{CC}^*_{\wedge} = \max_{i'=1}^4 CC_{\wedge}(T_{i'}^W, T_*^W)$, $\underline{CC}^\#_{\wedge} = \min_{i'=1}^4 CC_{\wedge}(T_{i'}^W, T_\#^W)$, and $\overline{CC}^\#_{\wedge} = \max_{i'=1}^4 CC_{\wedge}(T_{i'}^W, T_\#^W)$ for the "maximum function" type. On the grounds of the ideal schemes a_* and $a_\#$, the T-SF comprehensive correlation indices $CI'_{\surd}(a_i)$ and $CI'_{\wedge}(a_i)$ are elucidated in this fashion:

$$CI'_{\surd}(a_i) = \xi \cdot \frac{CC_{\surd}(T_i^W, T_*^W) - \underline{CC}^*_{\surd}}{\overline{CC}^*_{\surd} - \underline{CC}^*_{\surd}} + (1-\xi) \cdot \frac{\overline{CC}^\#_{\surd} - CC_{\surd}(T_i^W, T_\#^W)}{\overline{CC}^\#_{\surd} - \underline{CC}^\#_{\surd}}, \quad (21)$$

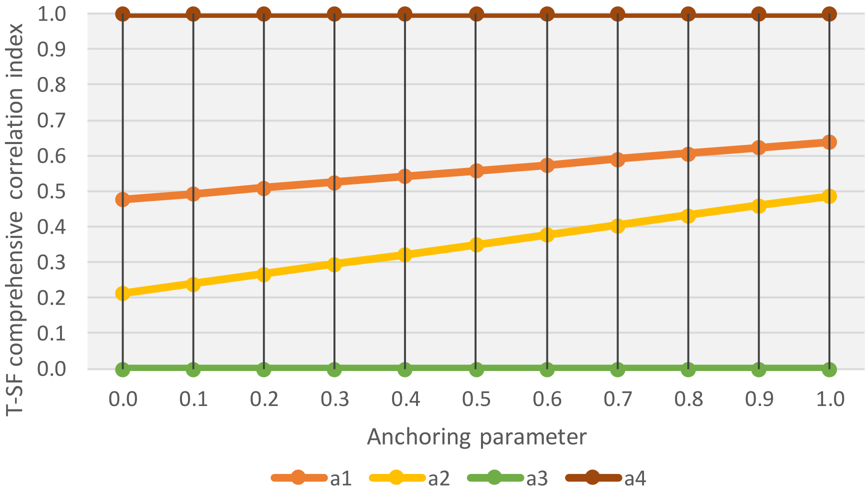
$$CI'_{\wedge}(a_i) = \xi \cdot \frac{CC_{\wedge}(T_i^W, T_*^W) - \underline{CC}^*_{\wedge}}{\overline{CC}^*_{\wedge} - \underline{CC}^*_{\wedge}} + (1-\xi) \cdot \frac{\overline{CC}^\#_{\wedge} - CC_{\wedge}(T_i^W, T_\#^W)}{\overline{CC}^\#_{\wedge} - \underline{CC}^\#_{\wedge}}. \quad (22)$$

Regarding the "square root function" type, the juxtaposition of the T-SF comprehensive correlation indices $CI_{\surd}(a_i)$ and $CI'_{\surd}(a_i)$ for various values of ξ are displayed in Fig. 9. Specifically, Fig. 9(a) reveals the contrast outcomes concerning the points of reference a_+ and a_-, while Fig. 9(b) demonstrates the comparisons in connection with the points of reference a_* and $a_\#$. For the "maximum function" type, the juxtaposition of the T-SF comprehensive correlation indices $CI_{\surd}(a_i)$ and $CI'_{\surd}(a_i)$ for various values of ξ are sketched in Fig. 10. Herein, Fig. 10(a) shows the comparison consequence on the grounds of a_+ and a_-, while Fig. 10(b) exemplifies the contrasts based on a_* and $a_\#$.

On the grounds of the reference points of the ideal schemes a_* and $a_\#$, the contrast outcomes of the T-SF comprehensive correlation indices among the four options presented moderately unreasonable patterns; moreover, these results may be difficult to be accepted by the decision-maker. To be specific, the unusual consequences were produced using the "square root function" type, i.e. $CI'_{\surd}(a_3) = 0$ and $CI'_{\surd}(a_4) = 1$ for all $\xi = 0.0, 0.1, \ldots, 1.0$, as displayed in Fig. 9(b). Additionally, it was received that $CI'_{\wedge}(a_4) = 1$ predicated on the "maximum function" type for all $\xi = 0.0, 0.1, \ldots, 1.0$, as displayed in Fig. 10(b). Regardless of how the value of the anchoring parameter ξ changed, the indices $CI'_{\surd}(a_3)$, $CI'_{\surd}(a_4)$, and $CI'_{\wedge}(a_4)$ were fixed at 0, 1, and 1, respectively. These findings revealed that the multiple-criteria analysis approach that exploited the positive and negative ideal schemes as a benchmark for reference was not sensitive enough to reflect changes in various ξ values. On the contrary, the propounded methodology predicated on the best and worst choice options (i.e. a_+ and a_-) that were established on the universal and null T-SF sets generated reasonable and desirable consequences.



(a) Outcomes with reference to a_{+} and a_{-}

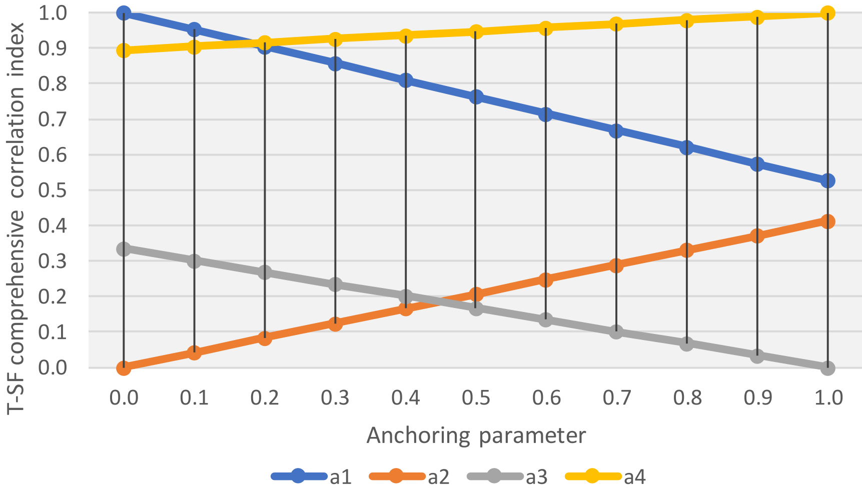


(b) Outcomes with reference to a_{*} and $a_{\#}$

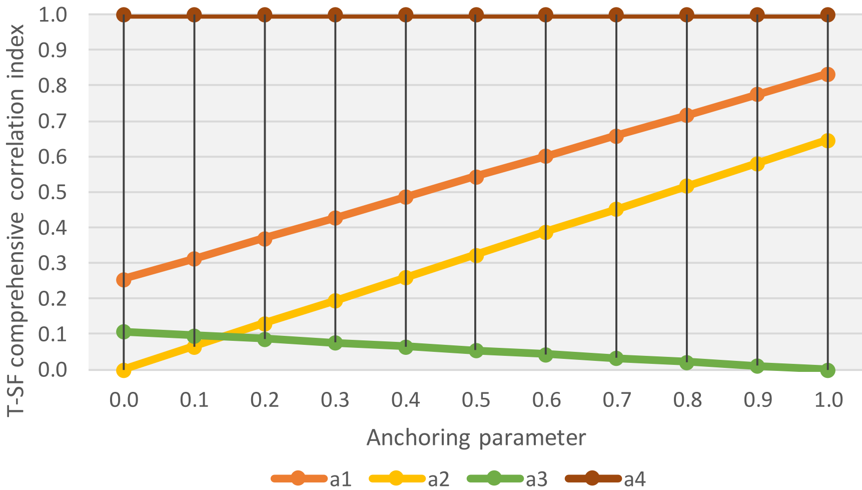
Fig. 9. Contrast outcomes of $CI_{\surd}(a_i)$ and $CI'_{\surd}(a_i)$ for various values of ξ.

5. Conclusions and Future Research Avenues

The framework based on T-spherical fuzziness provides an important tool for overcoming complex uncertainties in multiple-criteria choice issues by manipulating the four membership degrees involving positive, neutral, negative, and refusal components. One of the recent developments of multiple-criteria analysis techniques under T-SF conditions, the notion of correlation coefficients, has an increased uncertainty modelling capacity for decision-making. This paper creates some valuable concepts of T-SF data-driven correla-



(a) Outcomes with reference to a_+ and a_-



(b) Outcomes with reference to a_* and $a_\#$

Fig. 10. Contrast outcomes of $CI_\wedge(a_i)$ and $CI'_\wedge(a_i)$ for various values of ξ.

tion measures predicated on correlation coefficients in T-SF settings. Furthermore, this paper formulates a beneficial multiple-criteria choice method through a correlation-focused approach, which assists with computational intelligence in uncertain decision analysis. Following the anchored comparisons relative to the universal T-SF set and the null T-SF set, this paper constructs the T-SF weighted correlation coefficients using the types of square root and maximum functions. This paper also institutes the T-SF comprehensive correlation indices to determine the relative prioritization of all competing options and decide on the most appropriate scheme.

The evolved methodology is applied to a location selection issue to support a construction company in constructing new apartments. In addition to an applicable illustration, two types of comparative analyses (by changing anchoring parameters and reference T-SF sets) are performed to examine the robustness and merits of the developed techniques. It was found that the obtained T-SF comprehensive correlation indices render relatively stable but adjustable ranking results in different scenarios of anchoring parameters. Moreover, comparative analyses demonstrate that the T-SF comprehensive correlation indices predicated on the universal and null T-SF sets are more reasonable and justifiable than the yielded outcomes on the other reference T-SF sets.

Although the advantages of the multiple-criteria choice methodology are demonstrated through practical applications and comparative studies, the current methods still struggle with research limitations and disadvantages. The core concept of schematizing the evolved methodology is the notion of the T-SF data-driven correlation measures, which are derived from the T-SF correlation coefficients. The T-SF correlation coefficients can be utilized in statistical analysis or machine learning, and they mainly measure the degree of linear correlation between two T-SF sets. In other words, the T-SF correlation coefficients can explore whether there is a linear relationship between two T-SF sets (or multiple T-SF sets). However, if the relationship between two T-SF sets is nonlinear, the T-SF correlation coefficients may not precisely represent the relationship between them. This limitation may reduce the accuracy or sensitivity of the T-SF data-driven correlation measures in distinguishing between superior and inferior T-SF (weighted) characteristics.

Future research avenues can be improved and extended in two aspects. First, the proposed methodology and techniques can be exploited for other relevant high-order fuzzy configurations, such as uncertain sets of interval-valued spherical fuzziness, (complex) T-spherical fuzziness, (complex) q-rung orthopair fuzziness, T-spherical hesitant fuzziness, and neutrosophic fuzziness. Second, colloquially referred to as a normalized measurement of the covariance using correlation coefficients in applied statistics, the initiated T-SF data-driven correlation measures can also be employed for a variety of tasks in data analysis, decision aiding, engineering, intelligence sciences, and other areas.

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