NOTES

KARPOV AND KASPAROV: THE END IS PERFECTION

Lewis Stiller and the Editors

Department of Computer Science
Johns Hopkins University

University of Limburg/
Delft University of Technology

[Below, this Journal reports on an investigation prompted by a burning question: did Karpov and Kasparov play correctly the endgame when they met in October 1991 in Tilburg? The short answer is that they did. The interest of the report to our readers may be centered on the mechanics of consulting and as it happens even reconstructing a database when it suddenly is urgently necessary for consultation. Lewis Stiller must be regarded as the prime author of this contribution which has been given a somewhat unusual form for reasons which will become clear as the story unfolds. - Eds.]

The problem

It happened when Karpov first met Kasparov in the Interpolis tournament in Tilburg 1991 (they were due to meet again in a second round robin). Excitingly, the game ended in a pawnless KBNNKR configuration and it dawned upon some of the cognoscenti that they henceforth were treading on possibly known ground. Had not Stiller (1991a) let it be known that nowadays databases are available for pawnless six-piece endgames? And was the KBNNKR database among them?

As it happened, Karpov quite easily drove Kasparov's King into a corner and thereafter confined the BK to that area. Somewhat surprisingly the outcome still was a draw, achieved by a stalemate forced by Kasparov sacrificing his Rook. To chess commentators all over the globe, this gave rise to a burning question: could Karpov have achieved a win, as would seem not unlikely by virtue of the superiority of his forces?

Specifically IGM Raymond Keene, The Times' chess editor, wished to determine, once for all, whether and if so where the contestants had deviated from the optimal path.

A joint forensic operation

At this point, the search for the truth of the matter became a joint forensic operation with Keene in the lead, Stiller owner of the relevant database as the ultimate arbiter and your Editors as intermediaries, with Victor Allis serving as an indispensable liaison officer.

When asked by Email to analyze the Karpov-Kasparov endgame, Stiller replied that the database would have to be recomputed, because it had not been saved. What *was* still at hand was a statistical summary of results, insufficient to answer questions about the game theoretical results of any particular manifestation of KBNNKR. From what was retained, one could conclude that generally but not invariably, the value of the game was drawn, and that the maximin consisted of 49 moves, – tantalizing but insufficient information.

Lewis Stiller undertook to settle the specific issue by requesting time for his computer to reconstruct the database in question. The CM-2, however, was temporarily shut down due to a water leak in the building in which it is housed; and Stiller did not know when it would again be accessible. Fortunately, Stiller's lab was greatly intrigued by the request and the reasons behind it, so much so that Stiller was given the assurance that his request would be considered for eventual scheduling. Cautiously he stipulated that both the decision to schedule and the scheduling itself might take some time.

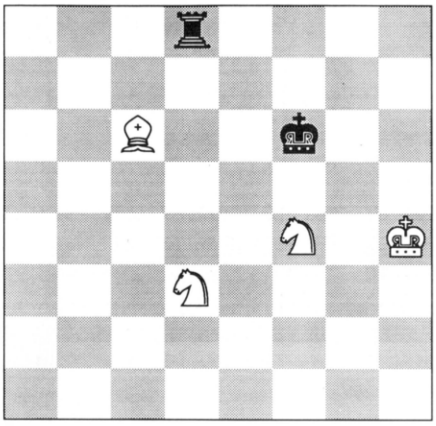
Conclusions

When time was obtained – at a shorter notice than anyone dared hope – on the original computer, it took some two hours on the CM-2 (7 MHz SIMD bit-serial 8 gigabyte RAM 16-cube with SUN front end) to prove that, given K-K's initial position of KBNNKR, a draw would be the outcome of perfect play.

Moreover, it was shown that neither contestant ever departed from optimality where we note, of course, that in a game as complex as this there is a multitude of equi-optimal paths.

Karpov-Kasparov

In Diagram 1, the beginning position of the Karpov-Kasparov KBNNKR endgame (Tilburg, 1991) has been depicted. The game proceeded as follows.



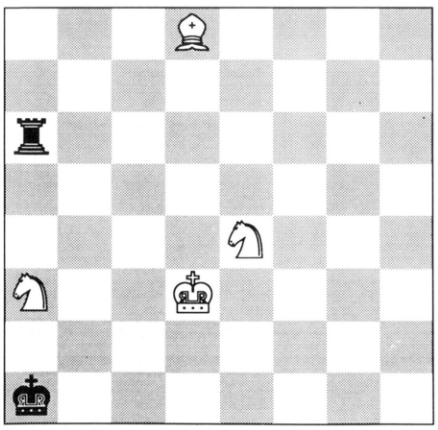
Stiller Diagram #1

Diagram 1: Position after 63. Kxh4.

63. ... Rg8 64. Be4 Rg1 65. Nh5+ Ke6 66. Ng3 Kf6 67. Kg4 Ra1 68. Bd5 Ra5 69. Bf3 Ra1 70. Kf4 Ke6 71. Nc5+ Kd6 72. Nge4+ Ke7 73. Ke5 Rf1 74. Bg4 Rg1 75. Be6 Re1 76. Bc8 Rc1 77. Kd4 Rd1+ 78. Nd3 Kf7 79. Ke3 Ra1 80. Kf4 Ke7 81. Nb4 Rc1 82. Nd5+ Kf7 83. Bd7 Rf1+ 84. Ke5 Ra1 85. Ng5+ Kg6 86. Nf3 Kg7 87. Bg4 Kg6 88. Nf4+ Kg7 89. Nd4 Re1+ 90. Kf5 Rc1 91. Be2 Re1 92. Bh5 Ra1 93. Nfe6+ Kh6 94. Be8 Ra8 95. Bc6 Ra1 96. Kf6 Kh7 97. Ng5+ Kh8 98. Nde6 Ra6 99. Be8 Ra8 100. Bh5 Ra1 101. Bg6 Rf1+ 102. Ke7 Ra1 103. Nf7+ Kg8 104. Nh6+ Kh8 105. Nf5 Ra7+ 106. Kf6 Ra1 107. Ne3 Re1 108. Nd5 Rg1 109. Bf5 Rf1 110. Ndf4 Ra1 111. Ng6+ Kg8 112. Ne7+ Kh8 113. Ng5 Ra6+ 114. Kf7 Rf6+ draw.

A maximin

In Diagram 2 a maximin position is presented. In the optimal line of play, equi-optimal moves are given in parentheses.



Stiller Diagram #2

Diagram 2: A maximin KBNNKR position.

1. Na3-c4 Ra6-e6 2. Ne4-f2 Re6-e8 3. Bd8-g5 Re8-f8 4. Nf2-d1 Rf8-g8 5. Bg5-e7 Rg8-e8 6. Be7-b4 Re8-b8 7. Bb4-a5 Ka1-b1 8. Nd1-c3 Kb1-a1 9. Nc3-e4 Ka1-b1 10. Ba5-c7 Rb8-c8 11. Bc7-b6 Kb1-a2 12. Kd3-c3 Rc8-b8 13. Bb6-d4 Rb8-d8 14. Nc4-d6 Rd8-b8 15. Ne4-c5 Rb8-g8 16. Nd6-c4 Rg8-g4 17. Bd4-e5 Rg4-g5 18. Be5-d6 Rg5-g6 19. Bd6-e7 Rg6-g7 20. Be7-h4 Rg7-h7 (Rg7-g4) 21. Bh4-e1 Rh7-h1 22. Be1-d2 Rh1-d1 23. Bd2-f4 Rd1-f1 24. Bf4-c7 Rf1-c1 25. Kc3-d3 Ka2-b1 26. Bc7-f4 Rc1-g1 (Rc1-h1) 27. Kd3-c3 Rg1-g4 28. Bf4-c7 Rg4-g7 29. Bc7-a5 Kb1-c1 30. Nc4-b2 Kc1-b1 (Rg7-g5) 31. Nc5-d3 Rg7-g5 (Rg7-a7) 32. Ba5-b4 Rg5-b5 33. Bb4-d6 Rb5-b6 (Rb5-d5) 34. Bd6-f4 Rb6-f6 35. Bf4-d2 Rf6-f7 (Rf6-f8 Rf6-d6) 36. Nb2-c4 Rf7-d7 37. Bd2-e3 (Bd2-f4 Bd2-g5 Bd2-h6) Rd7-e7 38. Be3-c5 (Be3-c1) Re7-c7 39. Bc5-a3 Rc7-c8 40. Kc3-b3 (Nd3-b4) Rc8-b8 41. Nd3-b4 Rb8-d8 42. Nc4-b6 Rd8-f8 (Rd8-g8 Rd8-h8 Rd8-d2) 43. Nb6-d5 (Nb6-a4) Rf8-f2 44. Nd5-c3 Kb1-a1 45. Nb4-c6 Rf2-g2 (Rf2-h2 Rf2-c2) 46. Nc6-d4 Rg2-d2 47. Ba3-c1 Rd2-f2 (Rd2-g2 Rd2-h2) 48. Kb3-a3 (Nd4-e2) Rf2-a2 (Rf2-f3 Rf2-f4 Rf2-f5 Rf2-f6 Rf2-f7 Rf2-f8 Rf2-g2 Rf2-h2 Rf2-f1 Rf2-e2 Rf2-d2 Rf2-c2 Rf2-b2) 49. Nc3xa2

Some Statistics

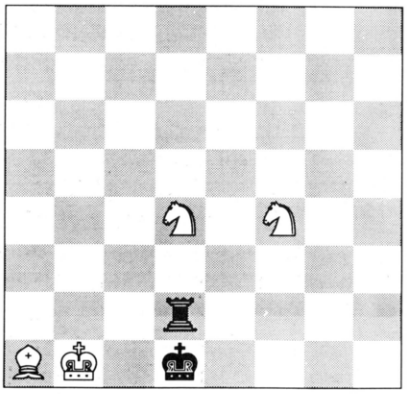
In Table 1 the distribution of WTM win-in-*n* positions is exhibited.

Move	Count	Move	Count	Move	Count	Move	Count	Move	Count
1.	3089594176*	11.	443084	21.	163800	31.	30030	41.	1950
2.	188530100	12.	346710	22.	142374	32.	25908	42.	1382
3.	11952424	13.	296734	23.	124490	33.	21272	43.	698
4.	2995578	14.	273968	24.	113828	34.	16238	44.	348
5.	1670082	15.	288174	25.	102082	35.	11626	45.	214
6.	897998	16.	299784	26.	89342	36.	6930	46.	54
7.	769590	17.	265396	27.	75626	37.	4396	47.	82
8.	714796	18.	238110	28.	56724	38.	2688	48.	30
9.	666430	19.	221456	29.	46160	39.	2592	49.	24
10.	584452	20.	197258	30.	37596	40.	2336	50.	0

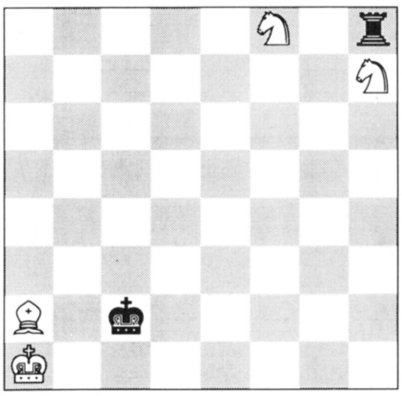
Table 1: The distribution of WTM win-in-*n* positions.
* This figure includes Black in check.

Zugzwangs

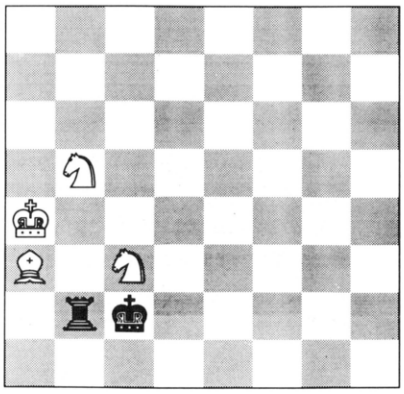
Noam Elkies, a Harvard University mathematician and talented endgame composer, had been a collaborator of Stiller's throughout the quest to solve 6-piece endgames. Dr. Elkies designed the max-to-mate algorithm, suggested numerous testing procedures, determined the most promising 6-piece endgames to solve, and helped Stiller understand and implement aspects of the program modules that rely on abstract algebra (Stiller, 1991b), among other contributions. He also designed the module that computes mutual zugzwangs, positions from which White can win if and only if Black is to move. This module found 1270 such mutual zugzwangs, three of which are listed in Diagrams 3a-c. (Note that 3b is particularly elegant.)



Mutual zugzwang type 1



Mutual zugzwang type 2



Mutual zugzwang type 3

Diagrams 3a-c: Three mutual zugzwang positions of different type.

References

Stiller, L. (1991a). Some Results from a Massively Parallel Retrograde Analysis. *ICCA Journal*, Vol. 14, No. 3, pp. 129-134.

Stiller, L. (1991b). Group Graphs and Computational Symmetry on Massively Parallel Architecture. *Journal of Supercomputing*, Vol. 5, No. 2, pp. 99-117.

Acknowledgements

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HOW TO WIN WITH A KNIGHT AHEAD

H.J.J. Nefkens

Rotterdam, the Netherlands

In a regular chess game, being a Knight ahead is normally sufficient to win a game. In the endgame, by contrast, a draw may be the result. However, there are some exceptions. The KNNK endgame is considered a draw, but giving the weak side an extra Knight increases the winning chances. The maximum number of moves to win is 7 (see Diagram 1) and is given on Ken Thompson's (1990) CD-ROM.

A solution reads: 1. Na6+ Kb7 2. Nc5+ Kb8 3. Ne7 Ng3 4. Nc6+ Ka8 5. Kc7 Nf1 6. Nd7 Ne3 7. Nb6 mate.

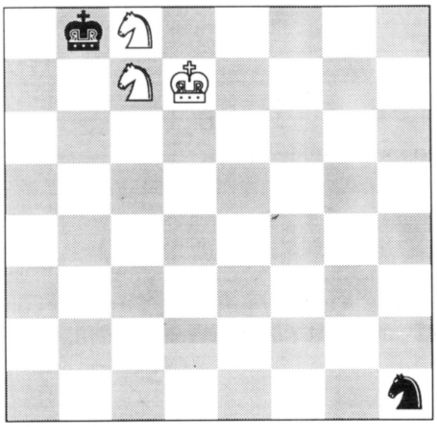

Diagram 1: White to move (mate in 7).

Most of the 5-piece endgames with a Knight ahead are on the CD-ROM. These endgames are normally a draw, but some interesting wins are documented in the chess literature. For instance, in Euwe (1950, p. 30) a win in 7 is given for the following KQNKQ endgame (see Diagram 2).

The solution reads: 1. Qc7+ Ka8 2. Qa5+ Kb7 3. Nc5+ Kb8 4. Qb6+ Kc8 5. Qb7+ Kd8 6. Kd2! Qe7 7. Qb8 mate.

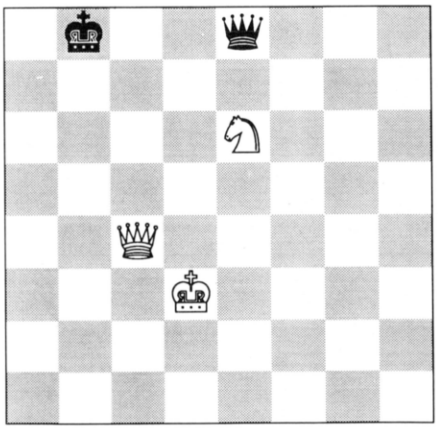

Diagram 2: White to move (a win in 7).