NOTES

ULTIMATES IN KQKR AND KRKN

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1. INTRODUCTION

For endgames bound to reach their conclusion only after conversion, it has long been customary to compute maximin distances to conversion (*c-distances*). Indeed, to a practical chess-player, the notion of an *ultimate* maximin does not seem relevant, the reasoning being that once the conversion has been enforced, the end is inevitable (and, in any case, for those adhering to the 50-move rule, the 50-move counter is reset after conversion anyhow).

For database constructors the ultimate distance (*u-distance*), i.e., the distance in moves to a mate with a conversion possibly or even necessarily intervening, *is* relevant. It is understood that both distances are computed in a *maximin* sense, that is: with *optimal* play from both sides (but see below).

It has been remarked earlier that the *u*-distance for a given endgame has been found to be *less* than the sum of the *c*-distance of the unconverted endgame *plus* the *u*-distance of the converted endgame (Van Bergen, 1985; Herschberg and Van den Herik, 1985). The reason so far published is that the set of converted endgames is constrained by the fact of their having arisen out of a conversion and so is less numerous and may well have a shorter *u*-distance than the general case of the converted endgame.

Both in order to derive some new quantitative results and in order to verify the theory outlined above, we have analyzed two endgames, viz. KQKR and KRKN, in exhaustive detail. The research was triggered in part by *Complexity Starts at Five* (Dekker *et al.*, 1987).

2. ULTIMATE IN KQKR

Let us start with KQKR. As Ströhlein (1970) and Thompson (as reported by Fenner (1979)) have shown, the *c*-distance is 31. It is known that the *u*-distance of KQK is 10. We find that the *u*-distance of KQKR is 35. Since 35 < 31+10, we here have an instance of the *u*-distance being less by six moves than the sum of the *c*-distance (of KQKR) and the *u*-distance (of KQK).

Moreover, in the KQKR endgame, it turns out that the set of maximin positions with a *c*-distance of 31 contains two essentially distinct positions (being 16 for the full board counting all symmetries). The same two positions also occur in the set of maximin positions with a *u*-distance of 35. However, the latter set is richer, containing 80 positions for the full board. Scrutiny reveals that in this endgame, maximin has an essential choice between *c*-optimal play and *u*-optimal play. In the *c*-optimal sense (applying to both sides), the variations consist of 31 moves to conversion, followed by 0-9 moves to mate (in KQK). It is worthwhile to note that the overall distance-to-mate thus may be below, at or even above 35 moves.

In the *u*-optimal sense (applying to both sides), the variations consist of 35 moves to mate. It is worthwhile to note that in these variations conversion *cannot* occur before move 26.

These apparently conflicting results are best explained by Black, playing c-optimally, deferring conversion as long as possible and thereby failing to postpone mate as long as possible. Experimentally, we have found that u-optimal play for KQKR yields a richer set of maximin positions. In fact, there need not exist a variation which is both c-optimal and u-optimal.

3. SOME EXAMPLES

Example 1

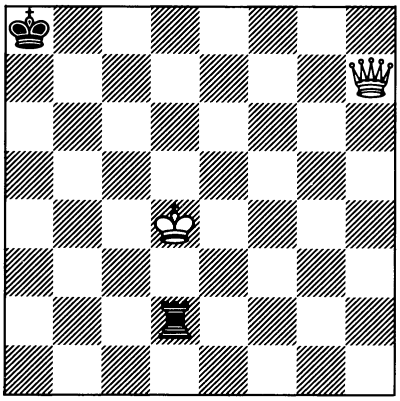

DIAGRAM 1

In Diagram 1, a position is shown where the u- and c-distances coincide, both standing at 7. This is a fairly rare position, since the coincidence of u- and c-distances only occurs in about 1.3 % of all cases, becoming rarer as the c-distances increase (cf. Table 1).

White: Kd4 Qh7;
Black: Ka8 Rd2;
White to move.

Example 2

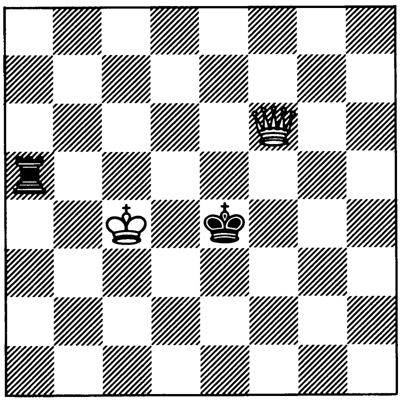

DIAGRAM 2

In Diagram 2, a position is shown with an extreme discrepancy, of nine moves, between the u- and c-distance (u-distance = 21, c-distance = 12). In other words, it is only rarely true that nine moves are needed to mate after conversion. Again, as Table 1 shows, the occurrence is of greater rarity as the c-distances increase. In this respect too the case is extreme.

White: Kc4 Qf6;
Black: Ke4 Ra5;
White to move.

Example 3

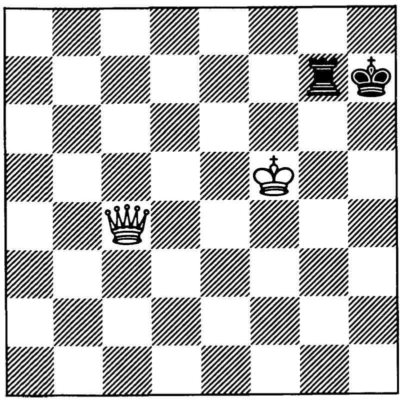

DIAGRAM 3

In Diagram 3, a fairly normal position has been selected in order to point out the difference in mating speed between *c*-based and *u*-based strategies. This position is characterized by the tuple (13, 9) for *u*- and *c*-distances respectively.

White: Kf5 Qc4;
Black: Kh7 Rg7;
White to move.

From Diagram 3, let White play optimally in the *c*-sense; his move will be:
1. Qd5
The corresponding tuple is now (13, 8). The new *c*-distance of 8 is as expected, the *u*-distance of 13 indicates that this move does not speed up the ultimate.

The alternative is to play optimally in the *u*-sense:
1. Qd4
The corresponding tuple now is (12, 9). This indicates a step towards the ultimate, but the conversion is as far ahead as ever.

We return to *c*-optimal play starting from the Diagram 3-position. The first three plies show:
1. Qd5 (13, 8) **Ra7** (13, 8)
2. Kf6 (14, 7).
We now find, somewhat to our surprise, that, for a decreasing *c*-distance, as intended, there is an actual *rise* in the *u*-distance.

Having shown some paradoxical effects we next exhibit, in parallel columns, some outcomes. In both cases White plays optimally in the *c*-sense, in the second case Black counters by playing optimally in the *u*-sense.

c-optimal play

2.	. . .	**Rg7** (9, 7)
3.	**Qh1** (8, 6)	**Kg8** (8, 6)
4.	**Qh5** (7, 5)	**Rg1** (7, 5)
5.	**Qe8** (6, 4)	**Kh7** (6, 4)
6.	**Qe4** (5, 3)	**Kg8** (5, 3)
7.	**Qa8** (4, 2)	**Kh7** (4, 2)
8.	**Qa7** (3, 1)	**Kg8** (3, 1)
9.	**Qxg1** (2)	

u-optimal play by Black

2.	. . .	**Ra6** (14, 6)
3.	**Kf7** (13, 5)	**Ra7** (13, 5)
4.	**Kf8** (12, 4)	**Kg6** (12, 4)
5.	**Qd6** (11, 3)	**Kf5** (11, 2)
6.	**Qc5** (10, 1)	**Ke4** (10, 1)
7.	**Qxa7** (9)	

All we can say is that it will take *at most* 9 moves to capture the Rook and *at least* 13 moves to mate. Strict *u*-optimal play by White implies 1. Qd4 since in the position of Diagram 3 there is no single move which is optimal in both senses. This generates two additional variations for a total of four (White playing *u*-optimal and Black playing *u*-optimal (denoted by (*u*,*u*)), the three other possibilities being (*u*, *c*), (*c*, *u*) and (*c*, *c*), of which only those starting with *c* (for White) have been presented above. In the (*c*, *u*)-strategy, the Rook was captured two moves earlier than in the (*c*, *c*)-strategy, but in the (*c*, *u*)-strategy mate is delayed, *in toto*, by 5 moves.

4. TABULATION

Table 1 shows for KQKR the number of full-board WTM positions having a stated distance to conversion c for a given difference between the ultimate distance u and c. Note that $0 \leq u - c \leq 9$, whence it follows that the KQK maximin of 10 does not occur as a successor of KQKR.

	$u - c$									
c	0	1	2	3	4	5	6	7	8	9
1	90280	25752	47096	147552	265796	480072	722736	935604	693304	155808
2	18808	8856	11336	24272	44824	101616	261876	408724	447632	134000
3	6040	8136	11000	9608	15128	30800	60984	94648	85312	22984
4	1544	6912	5896	5276	8764	19992	38952	61600	53072	12832
5	920	2720	3352	4528	8600	18760	35560	54536	44056	8912
6	232	888	2448	3784	6272	13728	27536	39712	26840	4384
7	**16**	272	2540	2184	5688	10424	19320	27360	14032	1376
8		80	3992	2704	4920	8896	15952	17376	7992	728
9		64	4040	4048	**5832**	8952	13792	10072	3600	368
10		8	3680	5144	9096	11848	12888	5576	1504	112
11			2432	7568	14128	17352	11272	2904	480	160
12			1280	8464	19184	23824	13280	2456	240	**24**
13			560	6568	24216	32728	12704	1312	72	
14			400	5600	26324	43376	12344	1264	8	
15			176	5608	31180	54524	10088	792		
16			184	6360	42400	62884	11048	648		
17			136	6624	53668	74680	11216	312		
18			40	6032	58844	94196	9296	232		
19				4768	70208	112020	6808	64		
20				4248	84240	135544	4216			
21				3016	97564	160740	2352			
22				2176	95452	174976	1616			
23				1064	89100	185020	456			
24				312	78612	164452	264			
25				120	66896	120444	88			
26					47896	72920	24			
27					28220	38188	16			
28					13632	14448				
29					4448	3060				
30					632	64				
31					16					

Table 1: c-distances and u-distances by frequency of occurrence in the KQKR endgame; the column headings show the $u - c$ differences.

5. ULTIMATE IN KRKN

The foregoing results for KQKR had the pleasing property that its 2 ($\times$ 8 = 16) c-maximin positions were a subset of the u-maximin positions. As a warning that this is not generally the case, we append a brief note on the KRKN endgame. With proper multiplicities not yet accounted for, there is only **one** position which is a u-maximin with a u-distance of 40. However, the two known positions which have a c-maximin of 27 do not belong to the set of u-maximin positions.

The three relevant positions are as follows:

(a) Kc4 Rg3 Kd8 Na5 (40, 26)
(b) Ka3 Rh6 Kc1 Nb5 (39, 27)
(c) Ka4 Ra8 Ka2 Nd7 (37, 27)

Position (a) may well represent, to the best of our knowledge, the longest known *u*-maximin sequence in any four-men endgame. The author here appends a warning for all future endgame experimentalists: in order to approximate the truth as closely as possible, it is essential that authors state their results in terms of all combinations of *u*- and *c*- strategies for both sides.

6. REFERENCES

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