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# Logic for Pragmatics

This special issue of *Fundamenta Informaticae* is dedicated to papers related to the two workshops on "Logic for Pragmatics" held at the Dipartimento di Informatica, Università di Verona, Italy, on September 1-5, 2003, and at LACL, Université de Paris 12, Créteil, France, on July 23-24, 2004.

The following note outlines the main ideas of the "Logic for Pragmatics" discussed at those workshops and indicates how the papers in this issue may be relevant for that programme.

The project of a "Logic for Pragmatics" originated in work by Carlo Dalla Pozza and Claudio Garola [5] and later by Dalla Pozza and Gianluigi Bellin [2]: the aim was to capture the logical properties of what are called "illocutionary acts" – asserting, conjecturing, commanding and so on. Consider assertions. In the framework of Dalla Pozza and Garola [5], there is a logic of propositions and a logic of assertions. Propositions can be either true or false, according to classical semantics, assertions are acts that can be justified or unjustified. The distinction between propositions and judgments is due to Frege [1] and has been developed by Martin-Löf [2] Dalla Pozza and Garola's work gives a two-layer theory with a distinctive informal interpretation, according to which propositions have truth conditions, whereas assertions have justifiability conditions.

As a consequence, we can form logical combinations of assertions, and give an interpretation for these combinations along the familiar lines of Heyting's interpretation of intuitionistic connectives: thus an assertion of conditional type $\vartheta_1 \supset \vartheta_2$ is justified by a method that transforms a justification of an assertion of type $\vartheta_1$ into a justification of an assertion of type $\vartheta_2$. The novelty here is that Heyting's semantics is applied to *illocutionary acts*, not to *propositions*. Furthermore, if we endow the underlying propositional logic with an **S4** modality, epistemically interpreted, then the modal translation of intuitionistic logic gives us a translation $(\cdot)^M$ of the pragmatic layer into the propositional layer: $(\vdash \alpha)^M = \Box\alpha$ and $(\vartheta_1 \supset \vartheta_2)^M = \Box(\vartheta_1^M \to \vartheta_2^M)$. And this modal formalism can, of course, be given an interpretation in terms of epistemic Kripke models.

These constructions, of course, cover a great deal of ground, related to XX century mathematical intuitionism. We do not yet have a full semantic, or proof-theoretic, account of them. Nevertheless, this picture gives us a number of suggestions for concrete research, in which the tools of contemporary proof-theory, type theory and category theory may be applied.

[1] For Frege a proposition is the thought which is the content of a judgment and a judgment is the recognition of the truth of its content. Then an *assertion* $\vdash \alpha$ is the expression of a judgment that the proposition $\alpha$ is true; the vertical bar in "$\vdash$" carries the assertoric force (see [6], p.315-6).

[2] For Martin-Löf well-formed complex types are propositions and the terms inhabiting them are witnesses of their truth, intuitionistically understood.

**1** One direction has been to consider the logic of a discourse in which different speech acts, possibly of different illocutionary force, are combined. Natural examples are the propagation of obligations through causal reasoning, as in “Don’t shoot! The gun is loaded”: here the justification of the command follows from the implicit obligation not to kill through implicit causal principles and the explicit assertion (for related work, see [2, 3] and the forthcoming thesis by Ranalter, which presents a deontic logic parametrized with assertions and a categorical semantics for it).

Similarly, Dalla Pozza and Garola’s approach can be extended to the logic of assertions and conjectures [1]. Given elementary illocutionary acts of conjecture of type $\mathcal{H}\alpha$, one defines complex types of conjectural acts. Assuming a form of duality between assertions and conjectures, the conditions for co-intuitionistic conjectural types to be unjustified may be defined in the same way as the justification conditions for intuitionistic assertive types. But conjectures may occur in assertive discourse and conversely, assertions may occur in conjectural discourse: a way to achieve this form of embedding is to assume that some connectives, e.g., negations, can change assertive expressions into conjectural expressions and conversely. This gives a polarization of bi-intuitionistic logic. Different logics arise from different modal interpretations [1]: the one where assertions and conjectures are translated using the *S4* necessity and possibility operators (called $\mathbf{ILP_{AC}}$) is the closest to the intended meaning.

The paper by Biasi and Aschieri in this volume gives two sequent calculus formalisations of this logic, one on them in terms of Herbelin’s $\lambda\overline{\lambda}$ calculus, and proves strong normalisation. [3] The paper also gives a simple ans elegant combinatorial proof of strong normalisation for the simply typed lambda calculus. This proof is due to René David, independently rediscovered by Federico Aschieri and generalized here to the simply typed lambda calculus with pairing and projections.

**2** The work described so far is most closely related to the original programme of formal pragmatics. Most papers in this volume investigate the background and expand the horizon, both conceptual and mathematical.

Graham White has analysed the background notion of *action* which any theory of illocutionary acts will have to use and has made significant progress in this areas: he uses ideas from the AI community to illuminate the philosopher Davidson’s work on actions.

Nicholas Asher has looked at such typing phenomena in linguistics as coercion and co-predication, using concepts from category theory.

The mathematical background is represented by two themes. One theme is the idea of a contexts: the logical systems presented here are strongly typed, and contexts play a large role in their formalisation. We need a unifying treatment of contexts in general, and the paper by Power and Tanaka gives this.

The other theme is the analysis of classical logic. This has strong connections to our programme: the remarkable amount of work since 1990 on the proof theory of classical logic has identified distinctive features of classical proof theory, and we would like to have a conceptual account of them. But there is a big difference between the treatments of disjunctive contexts using the $\mu$-operator and in more “concurrent” syntax, such as proof nets for classical logic. On a more abstract level, there is no treatment of the categorical semantics of classical logic, which may account both for Selinger’s control categories and for the categorical models using polycategories or built from proof-nets.

[3] For a type theoretic study of bi-intuitionistic logic within the $\lambda\mu$-calculus, which is also relevant to the logic of assertions and conjectures, see [4].

This special issue include two original reworking of Krivine's no counterexample interpretation. The original Krivine's interpretation of countable choice is given in a lambda calculus extended with a call-current-continuation operator cc. This is done for efficiency reason, but it also makes the resulting programs more difficult to understand and more difficult to optimize.

The paper by Oliva shows that we can "unfold" Krivine's interpretation in the usual lambda calculus, replacing cc with its usual definition, and modifying all steps of the intepretation accordingly. This makes Krivine's interpretation conceptually simpler, and also allows us to formulate and compare Krivine's interpretation of choice with bar-recursive interpretation of choice.

The paper by Raffalli addresses the second issue, optimization of extracted programs. Raffalli combines Krivine's computational analysis of the axiom of classical choice with some standard idea of dead code elimination, in order to improve the extracted code, and provides some interesting improvements of Krivine's original ideas along this direction.

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