Fundamenta Informaticae 141 (2015) i-xxi
DOI 10.3233/FI-2015-1266
IOS Press

Tomography and Applications

Preface

1. Introduction

The Meeting on Tomography and Applications is an annual meeting organized since 2007, with the main purpose of providing a forum for mathematicians and experimentalists to discuss the state of the art of tomographic research. The Meeting is organized by the Department of Mathematics of Politecnico di Milano. The $8th$ edition of this meeting, which took place in the period May 7-9, 2014 provided a good overview of recent applications of tomography in different areas.

The papers in this Special Issue went through a thorough refereeing process. Some of them discuss, in more detail and depth, topics presented at the meeting, while other present new contributions.

This preface consists of two parts. The first part provides a short overview of each paper accepted for this special issue. The second part provides summaries of papers presented at the meeting - some of these summaries are extended abstracts provided by the authors, and they begin with the “E.A.” label. We believe that such an organization of the preface provides the reader with a good insight into important aspects of current research in tomography.

2. Overview of the papers from this issue

- R. Fiorini, *Computerized Tomography Noise Reduction by CICT*.

  The traditional algebraic approach to the reconstruction problem in CT (Computerized Tomography) suffers from its intrinsical ill-posedness that, in general, prevents a faithful detection of the geometrical properties of the analyzed objects. In addition, all known inspection techniques cannot avoid background noise in the data. In this paper, the author considers the Geometric Science of Information (GSI) approach in order to extend the traditional statistical noise reduction paradigm, that only considers the sum (or average) of the multiple intensities without taking into account the pixel local variance. This approach is motivated by the high impact that the new Computational Information Conservation Theory (CICT) has in the wide area of Systems Theory. So, to obtain a faithful representation of a physical reality, one should use and combine both stochastic methods and combinatorial tools. This paper presents the CITC principles, and shows how they interact in GSI in order to overcame the limits of the classical stochastic representation of noise-affected data (based on the LTR computational approach). The characteristics of CITC to merge classical and quantum information theory, by considering both information from the statistical manifold of model states and from empirical measures of low-level multiplicative noise source generators, can be finally used to define new procedures to achieve dynamical probing field compensation and to improve overall final result by a competitive algorithm.

- N. Hantos, P. Balazs, *Eliminating Switching Components in Binary Matrices by 0-1 Flips and Columns Permutations*.

  The problem studied in this paper is one of the most interesting in Discrete Tomography and concerns the number of solutions that share a given pair of vectors of horizontal and vertical projections. In a simplified standard model, each solution can be represented as a binary matrix, and the ambiguity in its reconstruction (from the horizontal and vertical projections) is related to the presence of a specific 2×2 subpattern called switching component. Searching for switching components inside a binary matrix is one of the most important tasks in discrete image coding and reconstruction, since their absence allows the binary matrix to be stored in a (lossless) compressed form by means of the two vectors of projections. On the other hand, if switching components are present, then there is still a chance to uniquely detect one of the solutions by using some prior information. The authors demonstrate that the knowledge of the positions of suitable chosen elements (i.e. some of those involved in the switching components) suffices to accomplish the task. The aim is to find the minimal number of the elements needed to achieve uniqueness. Since, in general, the problem is NP-hard, the authors develop two polynomial-time heuristics that find the minimal number of switching components by computing a rearrangement of the columns of the matrix according to a certain filling function, instead of exhaustively searching through it. These heuristics are shown to outperform other standard methods on a wide set of uniform random binary matrices as well as on a real life dataset of presence-absence matrices.

- P. Balazs, Z. Ozsvar, T.S. Tasi, L.G. Nyul, *A Measure of hv-Convexity Inspired by Binary Tomography*.

  As discussed already in the previous overview, the faithful reconstruction of an unknown discrete object from projections is, in general, a highly undetermined process which usually needs to be driven by the knowledge of some a priori information about the object itself. The information needed can be very different and range from statistical data, as an example the densities of the object in specific regions of the space, to geometrical data, i.e. hints about its shape perimeter or its topology. Another geometrical information of high relevance is the convexity of the object: it can be regarded as one of the most important shape descriptors both in continuous and discrete domains. Various definitions and measures have been provided: in discrete tomography a special role is played by the horizontal and vertical convexity (briefly hv-convexity) that is strictly related to the discretization process of physical objects throughout their pixel representation, but it acquires relevance in almost all fields where constraint satisfaction problems are present. This study concerns a new smooth vision of the crisp concept of hv-convexity, that is based on the introduction of a measure of hv-convexity for non convex objects to create a basis of novel reconstruction strategies in Discrete Tomography. Thus, following an initial theoretical part, the authors focus on the definition of a fast reconstruction algorithm, that is based on simulated annealing and can deal with noisy projections. The performances of the algorithm are experimentally tested, even on objects that are not fully, but just approximately hv-convex.

- C. Vincze, A. Nagy, *An Algorithm for the Reconstruction of hv-convex Planar Bodies by Finitely Many and Noisy Measurements of their Coordinate X-rays*.

  This paper concerns once again the reconstruction of hv-convex planar bodies from their projections along the coordinate axes. The ill-posedness of this problem when considering a fixed number of projections, can be usually overcome by using an additional knowledge about the body itself. However, no substantial improvements are achieved even in case of classes of planar bodies that are proved to be uniquely determined by the projections. Here, the authors consider a variant of characterizing and reconstructing the class of hv-convex bodies that are uniquely determined by their axes projections. The authors already presented an algorithm for the reconstruction of hv-convex and connected planar bodies (say, hv-convex polyominoes) from their axis projections, and now they extend it to the case when only some measurements of the projections are available. A generalization to noisy data is also provided. These two algorithms converge to a solution by producing a sequence of polyominoes having the given projections almost everywhere, and the uniqueness of the reconstruction process is related to the existence of a single accumulation point. The basic steps towards the reconstruction is to prove that the Hausdorff-convergence of the integration domains implies the convergence of the conic functions provided that we restrict ourselves to the collection of non-empty hv-convex polyominoes contained in a fixed box (reference set) with sides parallel to the coordinate axes. The final reconstruction process turns out to be more efficient in the sense that only finitely many or noisy axes projections can be considered.

- D. Battaglino, A. Frosini, S. Rinaldi, S. Socci, *The Identity Transform of a Permutation and its Applications*.

  Many different variants of the problem of reconstructing an integer matrix from projections have been considered in literature, and one of the most appealing is the one concerning matrices that are superposition of permutation ones, i.e., those binary matrices having exactly one element 1 in each row and column. Although this problem has been proved to be non polynomially solvable even in the case of horizontal, vertical and diagonal projections, if one considers its cylinder variant (i.e., the three projections show a periodical behavior) this problem becomes tractable in polynomial time, and its related consistency version reduces to a check for satisfiability of three trivial properties. Such an apparently surprising result is the *tomographical translation* of a classical theorem by Hall on finite abelian groups: given a multiset C of integers belonging to the cyclic group $\mathbb{Z}_n$, there exists a permutation φ of $[0,..,n-1]$ such that $C = \{a : a + \varphi(a) \in \mathbb{Z}_n\}$ if and only if the elements of C sum to $0 \mod n$. In this paper the authors define an algorithm that provides a new constructive proof of this theorem and that runs in $O(n^2)$, aligning the complexity to the standard one already available in literature. The correctness of this algorithm follows from a series of numerical properties that characterize the set C_n of the sequences of elements of $\mathbb{Z}_n$ having zero sum (modulo n). Furthermore, C_n is proved to be closed under a set of unary operations, and this allows the authors to arrange its elements into equivalence classes and proceed towards its algebraic characterization.

- A. Nagy, *Experimental Study on Multivalued Phantoms Using Different Filters in the DART Algorithm*.

  In Discrete Tomography, several reconstruction algorithms are available, according to some prior geometrical information about the object, to the desired reconstruction accuracy, to the presence of noise in the projections, etc. An approach that relies on Filtered Backprojection is commonly used, but algebraic reconstruction methods, that consider different solution's methods of the linear system associated to the reconstruction problem, are becoming more and more popular in practice. In any case, it is really useful to benefit from the available prior knowledge of the unknown object during the reconstruction process to increase its accuracy, to decrease the required number of projections or simply to speed it up. When the number of different constituents of the object is known, i.e., the number of gray levels in the reconstructed image, a new reconstruction algorithm called $DART$ (Discrete Algebraic Reconstruction Technique) is available. $DART$ iteratively switches between *continuous update steps*, where the reconstructed image has real valued color shading, and *discretization steps*, that impose the constraint about the number of grey levels. In this paper, the author considers variants of DART that incorporate filters during its execution, and he carries on experiments that show how the modified $DART$ algorithm behaves on multivalued images. In particular, an extensive section shows how to tune several parameters of the filters such as the noise level, the number of different possible intensities and their distribution, in order to minimize the number of missed pixels.

- F. Brun, A. Accardo, L. Mancini, D. Dreossi, G. Tromba, G. Kourousias, R. Pugliese, S. Pacilè, F. Zanconati, *Enhanced and Flexible Software Tools for X-ray Tomography Experiments at the Italian Synchrotron Radiation Facility Elettra*.

  A synchrotron light source is a powerful tool to investigate internal structures at micro and nanoscopic scale. The authors present an overview of the workflow that leads to a scientific publications in computerized tomography, pointing out the part of the process that may involve an effective X-ray experiment hold on at a synchrotron radiation facility. At this step, an initial organization of the raw data is often required, and it has to be performed inside the facility itself. On the other hand, there are steps that have to be carried on at users home institution before the data collection such as sample preparation. Finally, there are specific phases in the workflow, i.e., the visualization of reconstructed data, their reduction and analysis, that are usually carried on at users institution, but that could also be arranged directly inside the synchrotron facility, with proper software. The request of these further tasks to the synchrotron is highly increasing, with the purpose of obtaining an on-the-fly analysis of the process to examine the acquired data before the next acquisition is complete, in order to allow a global optimization of the experiment. Sometimes the users may need to optimize one or more steps of the reconstruction process using tools available at their laboratory, and, only after that, they request remote assistance in the final steps of the experiment. The authors emphasize how the answer to all these requirements produces a continuous development of advanced computer technologies as well as refined and extremely efficient digital image processing for Computerized Tomography. They present the SYRMEP Tomo Project suite that provides a fast solution to all the samples' reconstruction routines, combining with the image processing and image analysis software used in a tomography-oriented scientific workflow.

- R. Shkarin, E. Ametova, S. Chilingaryan, T. Dritschler, A. Kopmann, A. Mirone, A. Shkarin, S. Tsapko, M. Vogelgesang, *GPU-optimized Direct Fourier Method for On-line Tomography*.

  The main motivation of this work concerns the need for the on-line visualization of the samples under analysis at a synchrotron facility. As underlined in the previous paper, it is of maximal relevance in the workflow of an experiment to provide the possibility of the visualization of the reconstructed data at each step to set parameter values or to pre-detect anomalies in the process. The authors defined and implemented a parallel GPU-optimized Direct Fourier method (DFM) for on-line tomography that suites with the ultra high-speed $4D$ beamline currently present at the ANKA synchrotron, Karlsruhe Institute of Technology. Usually, in similar frameworks, reconstruction strategies rely either on Filtered Backprojection speeded up with parallel implementations, or on pure DFMs, that reconstruct in the Fourier domain. However, the latter methods suffer from a heavy interpolation process to switch into a viewable Cartesian grid. Furthermore, DFMs are also harder to implement, and significantly more sensitive to the quality of the source data. The parallel implementation of the DFM presented in this paper partially overcame these problems, and so it can be considered as a forefront of research in the field; its performances are compared with classical DFMs both with respect to the achieved quality and with respect to the capability to run on parallel hardware. Finally, the paper includes a comparison between the parallel implementations of DFI and the FBP method. All the obtained benchmarks prove that images with quality comparable to the standard FBP reconstruction can be obtained 2 to 3 times faster up to the chosen platforms. For online monitoring, the authors also boost the performance with small quality degradation by reducing the oversampling rate.

- A. Shkarin, E. Ametova, S. Chilingaryan, T. Dritschler, A. Kopmann, R. Shkarin, S. Tsapko, M. Vogelgesang, *An Open Source Gpu Accelerated Framework for Flexible Algebraic Reconstruction in X-ray CT*.

  A synchrotron can inspect samples both at a high spatial resolution and at a high temporal resolution: this means that fast processes can be detected and temporally reconstructed. It may happen that temporal resolution is limited by the dose the specimen can withstand, so to reduce dose accumulation while maintaining high temporal resolution, one can reduce either exposure time or the number of acquired projections. Unfortunately, both approaches decrease the reconstruction quality. In this scenario, iterative reconstruction methods are usually preferred to algebraic ones, since they can benefit from a priori knowledge about the analyzed samples. A huge variety of algorithms can be found in literature according to the data acquisition process, the detectors geometry, and the type of information that have to be included. Unfortunately all of them rely on a very restricted set of reconstruction methods and, more importantly, on regularization techniques.

  In this paper, the authors present the *UfoIR* framework, i.e., a modular and easily extensible architecture for implementing iterative methods according to different projection models in X-ray imaging at a synchrotron light source. They also provide an interface to arrange methods and models flexibly in a reconstruction chain. The architecture relies on the parallel processing framework UFO, that allows the image processing algorithms to be easily arranged and managed in a graph structure. Finally, the performances of the presented algorithms are benchmarked on different hardware platforms.

3. Outline of the talks presented during the Meeting on Tomography and Applications, Milano, May 20-21, 2013

Power Diagrams for Tomographic Grain Reconstruction

Andreas Alpers, Technische Universität München

In recent years, considerable research efforts have been directed towards the development of methods for reconstructing crystalline structures (so-called grains) from X-ray diffraction tomography data. It has recently been observed that approximate reconstructions can already be obtained by utilizing parts of the data that provide knowledge of the grains' center of mass and their respective volumes. In this talk the speaker discussed how approximate reconstructions can be provided by means of power diagrams (these are special cell decompositions generalizing Voronoi diagrams) that are obtained by solving linear programs. With the linear programming approach, previously applied in the context of farmland consolidation, results have been presented that are superior to the ones reported in [1].

The talk was based on joint work of the speaker with A. Brieden, P. Gritzmann, and H.F. Poulsen.

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Reconstruction of hv-Convex Binary Images: A Survey and Open Questions

Peter Balazs, University of Szeged

Exactly 30 years ago, Attila Kuba published the first paper about the reconstruction of hv-convex binary images from the horizontal and vertical projections. Since then, various aspects of this problem have been studied: computational complexity of the reconstruction, reconstruction of special (e.g., 4-connected or 8-connected) hv-convex images, efficient reconstruction heuristics, number of feasible solutions, uniqueness, generation of hv-convex images according to several parameters, and so on.

The talk provided a survey of the above results, and showed the new perspectives of research in this field. Concerning the latter, the speaker presented results on reconstructing hv-convex images from just one projection, a study of the complexity of the reconstruction in the case of two projections, and a method to measure the hv-convexity of an arbitrary binary image which can serve as a basis for novel-type reconstruction heuristics.

The research was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP-4.2.4.A/2-11/1-2012-0001 'National Excellence Program'.

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Bounded sets in Discrete Tomography
Sara Brunetti, Università di Siena

In the classical mathematical formulation, problems studied in Discrete Tomography concern the recovery of finite sets of points in Z^n from their X-rays in a given set S of directions. Uniqueness of the reconstruction ensures that the computed set of points is the original one.

Both (reconstruction and uniqueness) problems are NP-hard when more than two X-ray directions are considered. In her talk the author focused on bounded set i.e, finite set of points contained in any finite multidimensional grid A [1, 2].

Results on the minimal number of directions ensuring uniqueness have been settled in the planar case as well as in higher dimensions. Moreover, a discussion has been presented concerning some related problems and their computational aspects.

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A Digital Volume Correlation Code for Motion Estimation Based on X-ray Micro-tomography Under in Situ Loading
Roberto Fedele, Politecnico di Milano

During recent years Digital Volume Correlation (DVC) is being extensively applied in material mechanics, to reconstruct time-dependent displacement fields in the inner volume of heterogeneous samples subjected to in situ loading, see e.g. [1]. To this purpose, conventional X-ray microtomography (either by laboratory sources or synchrotron radiation) can be interfaced with ad-hoc micro-apparata [2], allowing for scanning relatively small samples (a few millimeter thick) under in situ loading (usually till to several tens of kilograms). In his talk, the speaker outlined a novel computer implementation for DVC, based on a finite element discretization of the unknown three dimensional kinematic field, see e.g. [3]. Governing equations have been derived from a rigorous variational framework, in which semi-coercivity of the bilinear form can be consistently addressed. Critical issues were discussed concerning image processing, such as intensity field interpolation, numerical differentiation and integration, multi-scale approaches combined with an image pyramid and consequent information transfer along ascending and descending cycles, strategies for memory storage and parallel computing. Experimental results on foam samples under compression have been illustrated, as in [4].

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In-vivo Mapping of the Blood Velocity in Zebrafish with Optical Vector Field Tomography

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E.A.

Introduction The vascular system develops very early during vertebrate embryogenesis, since all other forming organs depend greatly on its ability to distribute oxygen and nutrients. Since long time ago, the development of the vascular system is being studied by means of ex vivo imaging, with proper staining, fluorescent markers and histological sections. Recently, the advent of Confocal Microangiography provided true advances in the field. Microangiography is the elective method for vascular system imaging in transparent embryos, because it allows one to visualize the morphology of the vessel network in 3D, with confocal resolution. However, it is an invasive technique and it is not able to give quantitative information on blood velocity and flow. Nowadays, non-invasive techniques are required in order to provide in vivo measurements over the complete development of the very same specimen. In the zebrafish, blood vessels of the trunk region are particularly important for understanding the mechanisms that drive vasculogenesis and angiogenesis during embryo development. Moreover, it has been proved that the blood flow magnitude and the shear forces applied to vessel walls have a great impact on the vascular system development. In order to provide quantitative information to be used in cardiovascular research, we developed an Optical Vector Field Tomography (OVFT) algorithm for Optical Projection Tomography (OPT) measurements. The technique is able to visualize the vascular system of the zebrafish trunk in three dimensions and to quantitatively analyze and represent in 3D the blood velocity distribution. The contrast is given by the endogenous motion of blood cells.

OPT Setup and OVFT Functioning OPT is the optical analog of Computed Tomography. The light emitted by a white LED propagates trough a diffuser and is projected on the specimen with a telecentric lens. The light transmitted through the sample is imaged by a 5X telecentric objective on a CMOS camera, which acquires 100 frames per second. Telecentric optics are used in order to keep the same magnification throughout the specimen. The sample is then rotated with a stepper motor around its longitudinal axis, with steps of 0.9 over $360°$, leading to 400 acquisitions at different angles. Before calculating blood velocity, it is necessary to detect vessel positions. This can be performed by acquiring

N=40 successive frames of the sample at each angular position. Red Blood Cells (RBCs) attenuate light differently than the surrounding tissue and their distribution pattern inside vessels change continuously because of the movement. As a result, by applying a motion analysis algorithm on the intensities I_N of the set of N frames, it is possible to visualize the position of perfused blood vessels, while rejecting still structures. Given a set of projections corresponding to a specific angular position around the sample, the flow contrast (FC) is calculated, in each pixel, as follows:

$$FC_k = \frac{\sqrt{\sum_{i=1}^{N/2} \left(I_{(2i,k)} - I_{(2i-1,k)}\right)^2}}{\sum_{i=1}^{N} I_{(i,k)}}, \tag{1}$$

where N is the number of acquired frames and $I_{(i,k)}$ is the intensity measured at the k-th pixel of the i-th frame. This method, called Flow-OPT, provides 3D reconstructions of the vascular network by calculating flow contrast maps for all the angles around the sample and by combining these maps using a filtered backprojection algorithm, based on Radon transform, to obtain virtual sections of the circulatory system of the specimen. Furthermore, a Particle Imaging Velocimetry (PIV) algorithm has been applied over the vessels that have been identified by means of Eq.1. PIV analysis consists in a space-time cross-correlation (Ck), between each k-th frame and the successive one and allowed us to measure the speed of RBCs and the direction of the blood flow:

$$C_k = \iint I^*_{\mathrm{ROI},k+1}(\xi, \eta) \cdot I^*_{\mathrm{ROI},k+1}(x + \xi, y + \eta) d\xi d\eta, \tag{2}$$

where I^* denotes the complex conjugate of the intensity I, ξ and η are spatial lag variables, which correspond to pixel shifts of the intensity in x and y directions, respectively.
The PIV method is then repeated on four different angular positions around the sample $(0°, 90°, 180°, 270°)$. The vectors belonging to the four views are geometrically recombined on the 3D virtual volume of the vessels, performing Optical Vector Field Tomography over a large portion of the zebrafish embryo.

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Computerized Tomography Noise Reduction and Minimization by Optimized Exponential Cyclic Sequences (OECS)

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E.A.

In general, a theoretical Computerized Tomography (CT) problem can be formulated as a system of linear equations. The discrete inverse problem of reconstructing finite subsets of the n-dimensional integer lattice Z^n that are only accessible via their line sums (discrete X-rays), in a finite set of lattice directions, results in an even more ill-posed problem, from noisy data. Because of background noise in the data, the reconstruction process is more difficult since the system of equations becomes easily inconsistent. Unfortunately, with every different kind of CT, as with many contemporay advanced instrumentation systems, one is always faced with an additional experimental data noise reduction problem. In the past five decades, trend in Systems Theory, in specialized research area, has shifted from classic single domain information channel transfer function approach (Shannon's noisy channel) into the more structured ODR Functional Sub-domain Transfer Function Approach (Observation, Description and Representation), according to CICT Infocentric Worldview model (theoretically, virtually noise-free data). As a matter of fact, traditional rational number system Q properties allow to generate an irreducible co-domain for every computational operative domain used. Then, computational information usually lost by using classic computational approach only, based on the traditional noise-affected data model stochastic representation (with high-level perturbation computational model under either additive or multiplicative perturbation hypothesis), can be captured and fully recovered to arbitrary precision by a corresponding complementary co-domain, step-by-step. Then, co-domain information can be used to correct any computed result, achieving computational information conservation according to CICT approach. CICT can supply us with co-domain Optimized Exponential Cyclic numeric Sequences (OECS) perfectly tuned to their low-level multiplicative noise source generators, related to experimental high-level overall perturbation. Numeric examples are presented.

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X-ray Polycapillary Optics in 3D Tomography of Low X-ray Contrast Samples

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E.A.

Conventional (transmission) imaging techniques are based on the contrast in the absorption coefficient among different elements. However, light elements feature very close values of the absorption coefficient leading to poor contrast in the case of imaging of soft tissues, tumors vs. healthy tissues, soft materials, plastics, biological samples, etc. In transmission imaging scattered photons are considered a drawback that decreases the image contrast, while the intensity distribution of the scattered X-rays is sensitive to the molecular structure of the target and might, therefore, be exploited as a powerful diagnostic tool, since it provides a signature of the specific cellular structure or of the material structure. When a material is irradiated with X-rays, it emits characteristic ”secondary” (or fluorescent) X-rays, whose energy is characteristic signature of the elements present in sample and whose intensity is related to concentration of the given elements in the sample. The exploitation of the scattered and/or of the fluorescent radiation allows performing non-destructive imaging of the molecular and/or elemental structure of the material. The use of suitably tailored X-ray polycapillary optics [1] coupled to high-performance energy-dispersive X-ray detectors (single elements or area detectors) allows extending the potential of advanced X-ray imaging methodologies (X-ray scatter imaging, X-ray fluorescence imaging) to 3D tomographic sample reconstruction. In the presentation at the TAIR2014 conference we showed the results of the application of X-ray fluorescence imaging (sensitive to the sample elemental composition) and X-ray diffraction imaging (sensitive to the structure of matter at molecular level) to the volumetric reconstruction of samples which suffer poor contrast in conventional X-ray absorption, like biological tissues and light materials (plastics, ...). The use of a polycapillary X-ray collimator allows the angular selection of the scattered or fluorescence radiation thus opening to advanced imaging modalities. An additional key feature in order to achieve high-quality images is the use of a very high-resolution energy dispersive area detector. In the present study we used both single element and area detectors based on the side-ward depletion principle (Silicon Drift Detectors [2] and Controlled-Drift Detectors [3]). The availability of energy discrimination allows the employment of monochromatic as well as polychromatic sources (e.g. conventional x-ray generator) hence easing the translation of advanced imaging techniques from the synchrotron environment to the laboratory. This also opens the way to the fascinating possibility of (simultaneous) acquisition of multi-modality images. The polycapillary collimator further helps to reduce the 3D voxel size and hence to create 3D images well-resolved in the depth of the sample, without the need of sample rotation. Some relevant examples have been presented in the field of tomographic reconstructions of artificial tumor masses loaded with Gold Nanoparticles [4] and of light materials (e.g. plastics, etc.) [5]. The use of polycapillary optics in combination with white X-rays beams pose, however, serious challenge to the reconstruction algorithms due to attenuation correction issues both for the primary and the scattered/fluorescence beam that depends on energy and sample composition, a-priori unknown, due to the energy dependence of the polycapillary lens-response that affects both the primary beam and the scattered/fluorescence radiation and due to the energy-dependent detector response.

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Feature Reconstruction in Tomography
Alfred Louis, Saarland University

In applications like X-ray tomography or vector tomography the same problem has to be solved repeatedly for different right-hand sides. Then it is advantageous to construct an approximate inverse operator which allows for fast and accurate algorithms. Already Likht, in general form and in parallel for an application in geophysics Backus-Gilbert used the idea to calculate linear functionals on the solution. The method of approximate inverse can be viewed as a general approach to construct numerically efficient reconstruction methods using invariance properties of the underlying operator. If the reconstructed object is an image or a movie, then their information content is enhanced by calculating features, like edges or optical flow. In order to combine reconstruction and evaluation in one algorithm a version has been presented of the approximate inverse to directly calculate the features from the measured data very efficiently. The new method is applied on real 3D - data stemming from non-destructive testing (dimensioning) and from a synchrotron to detect fluid fronts.

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Advanced X-ray Microtomography Techniques: Bio-medical and Materials Science Applications
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E.A.

Imaging techniques play an important role in several research fields from medicine to the analysis of biomaterials, from material science to geosciences and cultural heritage studies. As an example, the well known optical and scanning electron microscopy techniques are widely adopted tools for the investigation of the texture and morphology of a larger range of materials. X-ray based techniques are also of particular interest and X-ray microradiography has proved to be useful for the monitoring of the heavy-metal accumulation in vegetal tissues and for medical diagnostics. In recent years, X-ray computed microtomography (-CT) techniques based on both microfocus and synchrotron radiation sources attracted a lot of interest. These techniques produce three-dimensional (3D) images of the internal structure of objects with a spatial resolution at the micron- and submicron- scale. In fact, investigations performed directly in the 3D domain overcome the limitations of stereological methods usually applied to microscopy-based analyses. Moreover, -CT techniques enable to get 3D images of the internal core of a sample in a non-destructive way, more suitable for further complementary analyses and for precious or unique samples (fossils, archeological finds, in-vivo imaging, etc...) [1]. An intriguing challenge lies in the extraction of quantitative measures and indices directly from these kinds of images. Porosity and specific surface area as well as anisotropy, connectivity and tortuosity are interesting descriptors of a 3D model. However, accurate image processing and analysis methods for an effective assessment of these parameters are still an open issue in several applications. To this purpose, the Pore3D software library [2] has been developed at the Elettra - Sincrotrone Trieste laboratory (Italy). Although any kind of 2D image and 3D dataset is a valid input, Pore3D has been mainly conceived for the handling of X-ray -CT images. In fact, the particular case of high-resolution -CT images requires ad hoc software tools able to manage large 3D dataset. This software library assures a complete control of the algorithm implementation and permits different strategies of analysis as a function of the specific scientific application [3, 4]. Several applications of X-ray -CT methodologies for the extraction of quantitative information from -CT images of different kind of materials have been presented and discussed.

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Practical Questions in Discrete Tomography
Antal Nagy, Szeged University

Discrete tomography (DT) is a special kind of tomography that can be applied when the object to be reconstructed consists of only a few known homogeneous materials (e.g., metal and wood). This information can be incorporated into the reconstruction process, giving one the opportunity of reconstructing simple objects from a number of projection values that is much smaller than the number of projections employed for more complex objects.

Generally the discrete tomography problems are under-determined which means that one can have many solutions or, due to the noise distortion, there is no solution at all. Facing these problems during the reconstruction process, one can ask the question: how the projections determine the final result?

In this talk the speaker summarized the latest results concerning such a question, including the direction dependency and local uncertainty as well.

Additionally, a technique was discussed that improves a specific reconstruction algorithm, namely Discrete Algebraic Reconstruction Technique (DART) with a combined filtering technique. The method and the result of the applied framework have also been presented.

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Unique Reconstruction of Convex Bodies by Projection Data via Refinements of the Brunn-Minkowski Inequality

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E.A.

One of the most powerful results in Convex Geometry is the so-called Brunn-Minkowski theorem. It plays a very important role in the theoretical framework of geometric inequalities as well as in applied contexts such as crystallography or some optimization problems relative to algorithms for reconstructing sets by means of some geometric data.

Brunn-Minkowski's inequality (for further details about this result and other related topics we refer to [1, 4]) establishes that the n-th root of the volume of two n-dimensional convex bodies K, E is a concave function, and assuming a common hyperplane projection of K and E it was proved that the volume itself is concave, i.e., $\mathrm{vol}(tK + (1-t)E)) \geq t\mathrm{vol}(K) + (1-t)\mathrm{vol}(E)$.

On the one hand, we have shown that under the sole assumption that K and E have an equal volume projection onto a hyperplane (or a common maximal volume section through parallel hyperplanes to a given one), if the above linear inequality holds with equality for just one value of t in $(0, 1)$, then K may be specifically recovered via $K = L+E$, with L being a segment, i.e., L is a convex body with dimension not greater than 1. Also, this extra assumption is needed in order to obtain such a characterization.

On the other hand, this approach for recovering sausages via linearity on Brunn-Minkowski's inequality (under a common volume projection/maximal volume section onto/through hyperplanes) was possible partially due to its suitable (stronger) linear version. Hence, for possible reconstructions of bodies through refinements of Brunn-Minkowski's inequality, first it would be convenient to know what happens with the suitable (expected) inequality. In this sense, we have been able to show that the expected result of concavity for the k-th root of the volume, when a common projection onto an $(n-k)$-plane (or a common maximal volume $(n-k)$-section) is assumed, is not true by explicitly giving (a family of) convex bodies providing a counterexample for this statement. Nevertheless, other Brunn-Minkowski type inequalities can be derived under this hypothesis; more precisely, one has $\mathrm{vol}(tK + (1-t)E)) \geq t^k \mathrm{vol}(K) + (1-t)^k \mathrm{vol}(E)$. These results, as well as other related topics, can be found in [2, 3].

Furthermore, we would like to point out that the proofs of the above results are constructive, in the sense that, in each case, such bodies can be completely reconstructed by means of the appropriate geometric data.

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Wettability Analyses of Multiphase Interfaces by 3D X-ray Micro Computed Tomography

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Authors reported the application of X-ray micro computed tomography (microCT) for visualizing and analyzing the interaction between fluids and surfaces, a topic that is both challenging and fundamental for comprehension of countless physical problems at microscale. Such understanding is the prerequisite for the design and manufacturing of new nature-inspired (biomimetic) surfaces. X-ray microCT is used for wettability analysis with the objective to overcome limitations of conventional optical acquisitions, in particular for complex surfaces. To test microCT capabilities in this sense, a 3D scan of a drop gently deposited on a smooth surface is acquired, with the aim to reconstruct the drop surface and to perform contact angle measurements on true cross-sections of the drop-surface couple. For comparison, contact angle measurements based on conventional images are also performed. The results demonstrate that the proposed technique is very promising for surface characterization and for getting more accurate and detailed information about wettability characteristics. Additional examples are acquired showing the wetting behavior of complex surfaces. The first one, a water drop on leaves, shows super-hydrophobic properties and is of major interest for the modeling and designing of biomimetics surfaces. A further case is a sessile water drop on an artificial surface: a gas diffusion layer (GDL) developed for fuel cell application that needs to achieve a very high hydrophobicity. Here, a method is also proposed to evaluate the volume and surface of the part of the drop which is enclosed within the apparent external contact line, in order to use them as indicators of the wetting behavior on anisotropic surfaces, where no simplified model can actually be applied due to the topological complexity. Consequently, microCT can also be applied to investigate complex wetting scenarios, and to validate the theoretical models already available for such cases.

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GPU-optimized Direct Fourier Method for On-line Tomography

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To allow on-line monitoring of imaging experiments at synchrotrons, it is essential to have a very fast tomographic software. Direct Fourier methods (DFM) are asymptotically faster than Filtered Backprojection. Multiple DFM have been evaluated, utilizing various interpolation techniques assessing the reconstruction quality and parallelization capacity. The Direct Fourier Inversion (DFI) method using sinc-based interpolation was selected and parallelized for execution on GPUs and other parallel architectures. The speaker also reported on several optimization steps performed to boost performance. He presented the optimization scheme and analyzed quality and performance for several synthetic and experimental data sets.

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UfoART: An Open Source Gpu-accelerated Framework for Flexible Algebraic Reconstruction in X-ray CT

Andrey Shkarin, Karlsruhe Institute of Technology

Modern research often require a dose reduction to make longer experiments. Unfortunately, this leads to a high level of noise, or lack of data. The algebraic reconstruction technique (ART) allows incorporation of a priori knowledge into a reconstruction model and, hence, in some cases can provide a better reconstruction quality than analytical methods. The algebraic methods can be presented as a composition of interacting blocks that perform different tasks, such as: projection selection, minimization, projecting and regularization. Multiple algorithms exists for each of these tasks. The algorithms may differ not only in the performance and quality of reconstruction, but also in the suitability for implementation on the GPU.

The speaker presented a currently developing GPU-accelerated framework which will allow to freely combine various methods and a library of algorithms for ART reconstruction. His talk reviewed existing open-source implementations aimed on ART and presented framework revealing its architecture, currently implemented algorithms, and achieved performance.

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Opportunities and Challenges for Discrete Algebraic Reconstruction Methods

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Iterative tomographic reconstruction has significantly gained interest during the past decade, mainly because they have become computationally more feasible. Compared to analytic reconstruction methods such as filtered back projection, its main advantage is in the possibility to incorporate prior knowledge that can steer the reconstruction process towards high quality images.

One type of prior knowledge concerns assumptions about the grey levels representing the object to be scanned. For example, in quality control of 3D printed objects, the type of material of which the object is composed of is known, but not its exact spatial distribution. Exploiting this prior knowledge into a tomographic reconstruction process often leads to significantly better images.

Discrete tomography focuses on the problem of reconstruction of images that are represented by a (very) small number of grey levels, starting from a small number of acquired projections. Many methods exist to enforce such prior knowledge.

In this lecture the speaker shortly explained the principles of some of these algorithms. Next, he focused on specific applications (in medical CT, electron microscopy, video processing, the world of art) to demonstrate the versatility of this technique.

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Determination of Convex Bodies by Values of Parallel X-ray of Order i or Parallel i-chord Functions.

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Geometric Tomography as we know it today began in 1963 with the following Hammer’s question:

Suppose there is a convex hole in an otherwise homogeneous solid and that X-ray pictures taken are so sharp that the ”darkness” at each point determines the length of a chord along an X-ray line. (No diffusion, please.) How many pictures must be taken to permit exact reconstruction of the body if: a. The X-rays issue from finite point source? b. The X-rays are assumed parallel?

The point X-ray was generalized later by defining point X-ray of order i for any real number i, but the parallel version was never defined. Situation with the i-chord functions is quite similar.

At the beginning of the talk the speaker outlined the definitions of parallel X-ray of order i and of parallel i-chord functions. Later on, basic properties and results concerning determination of convex bodies by parallel X-ray of order i have been presented and discussed.

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Discrete Representation of Local Orientation in Grains Using Diffraction Contrast Tomography.
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The reconstruction of the 3D grain microstructure in the bulk of polycrystalline materials is a problem that presents many challenges, both from an experimental and data analysis point of view. In non-destructive techniques based on monochromatic beam X-Ray diffraction, the local orientation of the crystal lattice, whose knowledge is the aim of the reconstruction, is also responsible for the deformation of the images recorded by the detector. Different computational approaches have been proposed over the years, to deal with the different available experimental techniques. One of the common shortcomings has been the restriction to single voxel optimization. It has been shown that stochastic optimization techniques can be used to approach the solution, matching the diffracted intensities and data acquisition schemes (namely, line beam illumination or 3D volumes from stacking of 2D layers), requiring however long acquisition times. Ideally, reconstruction of the 3D orientation field would be achieved from extended beam diffraction data and the optimization should be performed over the volume of a grain or ultimately the sample volume. X-Ray Diffraction Contrast Tomography is an extended beam tomographic imaging technique which demands shorter acquisition time compared to other techniques. If the reconstruction of the local crystal orientation is extended to the full 3D volume of the grain, the problem will be intrinsically six-dimensional, while if we restrict it only a 2D slice, the reconstruction problem will become five-dimensional, because for each point in the real-space we always assume a three-dimensional orientation-space. In both cases, it will always be possible to describe the reconstruction as the solution to a non-linear system $A(x)x = b$, where x^* is the true solution to the system. Unfortunately, the availability of components in b will be less than required by the number of unknowns in the vector x, and even if the problem could be translated into a classic l2-norm minimization problem over the residual $r(x) = b - A(x)x$, the intrinsic non-linearity of the problem will not guarantee a unique minimum. The developments in mathematical optimization, during the last 15 years, have shown that tools like l1-minimization, used along with sparsifying transformations in the solution domain, can recover specific solutions to heavily under-determined systems.

The speaker proposed a practical representation of the orientation space, which avoids higher degrees of under-determinacy and linearizes the reconstruction problem. He also described the application of the recent developments mathematical optimization to DCT data. This approach overcomes the single voxel optimization restriction. Finally, he presented promising results obtained on simulated data.

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Continuity Properties and their Applications in X-ray Tomography.

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E.A.

Let M be a compact metric space and suppose that $\mathbf{x} \in M$ is given by its continuous image $\mathbf{y} = \Phi(\mathbf{x})$. The continuity of the mapping Φ allows us to approximate $\mathbf{x}$ in the following way: $\Phi(\mathbf{x}_n) \to \mathbf{y}$ implies that any convergent subsequence of $\mathbf{x}_n$ tends to an element in $\Phi^{-1}(\mathbf{y})$. If $\mathbf{x}$ is uniquely determined by the measuring procedure Φ among the elements of M, then $\mathbf{x}_n \to \mathbf{x}$. The question of being uniquely determined is natural and important.

Consider the following optimization problem: minimize $\mathrm{dist}(\mathbf{y}, \Phi(\mathbf{z}))$ subject to $\mathbf{z} \in \mathcal{H}_n \subset M$, where $n \in \mathbb{N}$ and for any open neighbourhood $\mathcal{V}$ of $\mathbf{y}$ we have that $\mathcal{H}_n \cap \Phi^{-1}(\mathcal{V}) \neq \emptyset$ if n is large enough. For the sequence $\mathbf{z}_n$ of the optimal solutions $\Phi(\mathbf{z}_n) \to \mathbf{y}$. Although the solution of the optimization problem is not unique, in general if $\mathbf{x}$ is uniquely determined then the set of the optimal solutions collapses to a single point as $n \to \infty$. *Unfortunately, there is no technique known for determining how many local minimum an NLP has, other than a plain enumeration* [2].

Let K be a compact connected hv - convex planar set given by its coordinate X-rays and consider the function f_K measuring the average taxicab distance from K by integration. We have the sum of univariate integrands weighted by the one-dimensional measures of the slices parallel to the coordinate directions (coordinate X-rays):

$$f_K(x_1, x_2) = \int_{-\infty}^{\infty} |x_1 - t| X_1 K(t)\, dt + \int_{-\infty}^{\infty} |x_2 - s| X_2 K(s)\, ds.$$

It is an accumulation of the coordinate X-rays' information because $f_K = f_L$ iff the coordinate X-rays coincide almost everywhere [3]. The key result is the continuity of the mapping $\Phi\colon L \to f_L$. If a sequence of compact connected hv-convex planar sets L_n tends to K with respect to the Hausdorff metric then $f_{L_n} \to f_K$ with respect to the L_1-norm or the supremum norm over a common axis parallel bounding box of the sets [4]. To reconstruct K we solve the following optimization problem [5]: minimize $\|f_L - f_K\|$ subject to $L \in \mathcal{H}_n$, where $\mathcal{H}_n$ consists of polyomino-like hv-convex sets coming form the divison of the axis parallel bounding box into $n \times n$ congruent subrectangles. For the sequence L_n of the optimal solutions $f_{L_n} \to f_K$ which means that if n is large enough then L_n is close to a set having the same coordinate X-rays as K almost everywhere. The problem of being uniquely determined by the coordinate X-rays among convex planar bodies is due R. J. Gardner [1]. In terms of continuity properties we have a possible reformulation: the inverse mapping Φ^{-1} is lower semi-continuous at f_K iff K is uniquely determined by the coordinate X-rays [4].

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Acknowledgements. We kindly thank all the authors of the papers published in this special issue as well as all the referees, who devoted their precious time to produce thorough reviews providing valuable comments and suggestions for improving the submitted manuscripts. We are also indebted to the speakers of the 8th Meeting on Tomography and Applications for their interesting lectures, and to the agencies supporting the Meeting: Department of Mathematics- Politecnico di Milano, Università Cattolica di Piacenza, and in particular, the European COST Project, since the meeting took place in the framework of the COST Action MP1207 (`http://www.cost.eu/domains_actions/mpns/Actions/MP1207`). Finally, we are grateful to Professor Damian Niwiński for giving us an opportunity to publish this special issue in Fundamenta Informaticae.

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