

# A new approach to compensate the geometric distortion in the synthetic aperture ultrasonic imaging system

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**Abstract.** In the field of ultrasonic imaging technology, the problem of geometric distortion is often encountered, especially in the ultrasonic near-field. In this study, a new approach is proposed to compensate for geometric distortion in the synthetic aperture ultrasonic imaging system. This approach is based on the synthetic aperture ultrasonic holographic B-scan (UHB) imaging system, which is a combination of ultrasonic holography based on the backward propagation principle and the conventional B-scan technique. To solve the geometric distortion problem, the operation of the spatial compression and re-sampling in the frequency domain are introduced. The main advantage of the approach is that the real holographic value can be calculated without distortion by using the spatial interpolation function after the spatial frequency compression. After the compensation for geometric distortion is performed, the synthetic aperture technique based on the backward propagation principle is then applied in the process of the two-dimensional numerical imaging reconstruction. Both the simulation and measurement experiment show that the approach is promising. The geometric distortion that is dependent on the wave front angle can be effectively compensated. The spatial resolution is practically uniform throughout the depth range and close to the theoretical limit in the experiments.

Keywords: Synthetic aperture ultrasonic imaging, geometric distortion, spatial compression, re-sampling, numerical imaging reconstruction

## 1. Introduction

Ultrasound imaging is an inexpensive and widely used tool in many applications due to its flexibility and non-invasive character, especially in the diagnosis and discriminate staging of some diseases [1]. A computationally efficient synthetic aperture ultrasonic imaging method, ultrasonic holographic B-images (UHB), was introduced and developed at the University of Oulu in the late 1980's [2]. This imaging method can fully realize digital synthetic aperture imaging without the need

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of a complicated beam focusing through the hardware. UHB imaging is a combination of ultrasonic holography based on the backward propagation principle and the conventional B-scan technique [3-6]. Information about both the phase and the amplitude are recorded in the measurement. The resolution of the image method is guaranteed in the whole imaging area for both the longitudinal and lateral direction.

The image method is appropriate for a wide range of applications. The UHB method has been applied to tissue and diagnostic imaging of the abdomen as well as surgically-exposed brain [7]. In our previous work, we considered UHB imaging for inhomogeneous layers [8] and for a circular array via linear array method [9]. Recently, a UHB 3D system has also been investigated [10-13]. The different distances from each array element on a linear transducer to the same target point results in the difference in the time of the receipt of the echo signal at the same target point so as to cause a geometric distortion of two-dimensional echo data. Different delays are introduced for the echo signals received by different array elements and superposed, and distortion is corrected by receiving dynamic focusing [14-16]. In the synthetic aperture ultrasonic holographic imaging system, the near-field geometric distortion should be compensated, and the so-called rearranging operation in the spatial frequency domain has been proposed [17, 18]. The drawback of this operation is that due to the process of weighted summation, it is difficult to obtain the real holographic value without distortion.

In this paper, a new approach is proposed to compensate the geometric distortion in the synthetic aperture ultrasonic imaging system. The concept of the spatial compression *Rate* is introduced for spatial compression. The geometric distortion compensation can be achieved in the re-sampling process in the spatial frequency domain after the spatial compression, and then, the real holographic value can be calculated without distortion by using the spatial interpolation function. Theoretically, this new approach of spatial compression and re-sampling in the spatial frequency domain can be used to compensate any kind of geometric distortions that are dependent on the wave-front-angle. This new approach has been tested through both simulation and an experiment with the commercial Ultrasonix RP. After the geometric distortion is compensated using the new approach, the wave-front backward propagation principles and the FFT-algorithm are used to achieve the image reconstruction. The spatial resolution about 1mm which is close to the theoretical value can be achieved. The backward propagation principle in ultrasonic imaging requires three steps [18]. First, the Fourier transforms algorithm is used to process echo signals in order to get the spatial frequency. Then, the back-propagation transfer function is applied in the frequency domain. Finally, the ultrasound image can be constructed with the inverse Fourier transforms algorithm.

This paper is organized as follows. In Section II, the theoretical analysis of the new approach is described. A MATLAB simulation is used to theoretically investigate the approach in Section III, and the phantom measurement results that were obtained using the commercial Ultrasonix RP ultrasound system are also presented in this section. Finally, the paper is concluded in Section IV.

## 2. Theoretical analysis

In order to achieve a high spatial resolution and based on the principle of UHB imaging, every element of the linear transducer needs to conduct a fairly wide-angle ( $\beta$ ) beam in the plane of imaging. Meanwhile, the ultrasound beam is focused in the direction that is perpendicular to the plane of the imaging. The wide-angle ultrasonic beam aids in capturing more hologram data that correspond to the one-point target. In the scan sequence, only one element of the linear transducer is valid in each time of transmitting and receiving. The individual transducer element works in a pulse-echo mode and

scans progressively along the direction of the x-axis in the plane of imaging. After the scanning in the plane of imaging is finished, the two-dimensional hologram data can be used for the synthetic aperture ultrasonic holographic imaging.

The blue squares represent the position of the linear transducer elements, and  $\theta$  is the incident wave-front angle. When taking into account the near-field condition, due to the wide-angle beam, the reflected wave-fronts will cause a hyperbolic echo trajectory in the recorded two-dimensional hologram data. Then, the problem of geometric distortion is encountered. The arc  $\widehat{AB}$  indicates geometric distortion in the 2-D hologram data. The distance between the transducer and the object points in the view of the transducer is called the optical distance ( $Od$ ), and the perpendicular distance between the transducer element and the measured object is called the real distance ( $Rd$ ). In order to resolve the geometric distortion problem, adaptive spatial compression is proposed for different curvature data. The purpose is to convert curve  $\widehat{AB}$  into linear  $AB$  via spatial compression. Based on the relationship of  $Od$  and  $Rd$ , as illustrated in Figure 1, the spatial compression *Rate* can be defined as:

$$Rate = Rd / Od = \cos \theta, \tag{1}$$

Eq. (1) presents the spatial compression *Rate* as a function of the wave-front angle  $\theta$ .

Before backward propagation is applied in the numerical imaging reconstruction process, the geometric distortion needs to be compensated. According to Eq. (1), we can calculate  $Rd$  corresponded to  $Od$  multiplied by the spatial compression *Rate*. Obviously, the depth correction becomes very time consuming in the time domain since it must be applied to every depth level and to each measurement point. Fortunately, the geometric correction can be performed efficiently in the spatial frequency domain.

We know that the direction of the incoming waves determines the corresponding spatial angular spectrum  $f_x$  [2]. Moreover, an important relationship between the wave-front angle and the spatial angular spectrum can be expressed as:

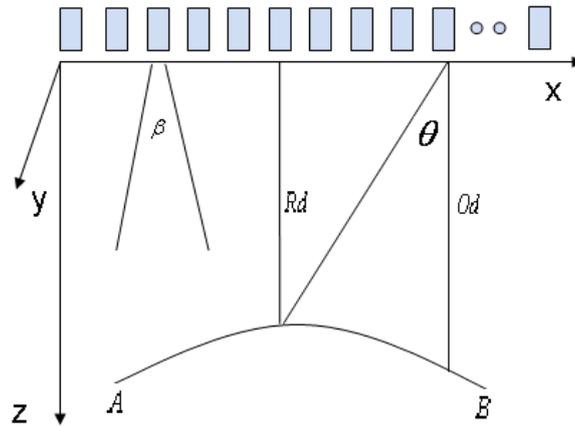


Fig. 1. Geometric distortion in the imaging x-z-plane. The individual transducer element conducts a fairly wide-angle  $\beta$  beam in the plane of imaging.

$$f_x = 2 \sin(\theta) / \lambda, \tag{2}$$

where  $\lambda$  is the wavelength of the ultrasound in medium. The spatial angular spectrum is doubled as the ultrasonic system is operated in the pulse-echo mode. Since both the spatial compression *Rate* and the spatial angular spectrum are both related to the wave-front angle  $\theta$ , through variable substitution, the spatial compression *Rate* in the spatial frequency domain can be expressed as:

$$Rate = \cos \theta = \text{sqrt}[1 - (f_x \lambda / 2)^2], \tag{3}$$

Eq. (3) indicates that the spatial compression operation depends on the wave-front angle and can be performed in the frequency domain in order to compensate for the geometric distortion in the ultrasound near-field.

In geometric compensation, the first step is to apply Fourier transforms to each row of the raw hologram data so as to get the spatial frequencies  $f_x$ . In the frequency domain, the nonzero frequencies  $f_x$  are located at deeper levels than  $Rd$ . In the geometric distortion compensation, each spectral component has a unique compression to  $Rd$ , which is determined by the spatial compression *Rate*. Eq. (4) describes the spatial compression operation:

$$Rd = Od * Rate = Od * \text{sqrt}[1 - (f_x \lambda / 2)^2]. \tag{4}$$

According to Eq. (4), the spatial component depends on the wave-front angle and should be compressed with *Rate*  $\sqrt{1 - (f_x \lambda / 2)^2}$ .

Figure 2 gives an example illustration with fixed  $f_x$ . In the space-frequency domain, through spatial compression, space S1 is converted into space S2. The new series re-sampling points within the compressed S2 are  $X_{r_n}$ . Since the re-sampling points remain a uniformed sampling sequence that

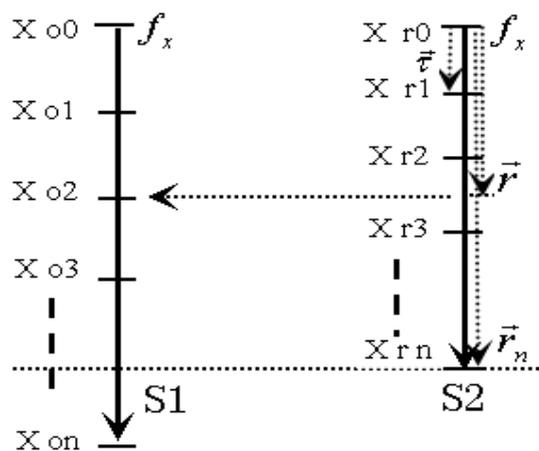


Fig. 2. A spatial frequency component is given as an example so as to illustrate the operation of spatial compression and re-sampling in the frequency domain.

satisfy the sampling theorem in the compressed space S2, the real holographic value corresponding to the original optical depth can be computed by using the spatial interpolation function, which can be expressed as Eq. (5):

$$|\vec{r}| = \sum_1^n |\vec{r}_n| \frac{\sin(\pi(\vec{r} - \vec{r}_n) / \vec{\tau})}{\pi(\vec{r} - \vec{r}_n) / \vec{\tau}}, \quad (5)$$

where  $\vec{r}_n$  is the holographic vector after spatial compression and re-sampling. The desired true spatial holographic vector  $\vec{r}$  can be computed by processing the vector  $\vec{r}_n$  for spatial interpolation. In addition,  $\vec{\tau}$  is the unit gap vector after the spatial compression. The geometric distortion compensation can be achieved with the process of spatial compression and re-sampling in the spatial frequency domain because the desired true holographic value that corresponds to the same optical depth can be calculated without distortion by using the spatial interpolation function.

After using the new approach in the frequency domain to perform the near-field curvature compensation, the synthetic aperture technique based on the ultrasonic backward propagation is then applied in the ultrasound numerical imaging reconstruction process. The one-dimensional (1-D) ultrasound numerical imaging reconstruction process can be described by the following formula:

$$I = F_x^{-1} \left\{ H^{-1} \left\{ C \left[ F_x (X) \right] \right\} \right\}, \quad (6)$$

where  $X$  and  $I$  are the recorded matrix hologram data and the output reconstructed image, respectively.  $F_x$  and  $F_x^{-1}$  respectively denote the linear Fourier transforms and inverse Fourier transforms along the row dimension of matrix  $X$ . Furthermore,  $C$  in Eq. (6) is an abbreviation of “compensation” and represents the operation of geometric distortion compensation in the spatial frequency domain. A detailed description of the operation has been reported above in this paper. If the propagation medium is homogeneous with a constant wave velocity and the wave-front on the array plane satisfies Helmholtz’s equation [17], then the backward propagation transfer function  $H^{-1}$  can be presented as:

$$H^{-1} = \begin{cases} \exp\left\{-jk\Delta z\sqrt{1-(\lambda f_x)^2}\right\}, & (\lambda f_x)^2 < 1 \\ 0, & \text{otherwise} \end{cases}, \quad (7)$$

where  $\lambda$  is the wavelength of the ultrasound in medium, and  $k = 2\pi / \lambda$  is the wave number. Additionally, the parameter  $\Delta z$  presents the distance from the array plane to the object plane. Finally, the one-dimensional ultrasound image of the object can be constructed by using the inverse Fourier transforms algorithm. A two-dimensional image of the object slice can be obtained as a stack of several one-dimensional images.

### 3. Simulations and experiments results

In order to verify the efficiency of the new approach in the compensation of geometric distortion depending on the wave-front angle, a MATLAB simulation was performed. In addition, experiment measurements with phantom were also conducted by employing the commercial Ultrasonix RP ultrasound system, which has an open interface. A sequence with 256 wide beam emissions using the transducer consisting of 128 elements can be achieved by programming the Ultrasonix RP Ultrasound Imaging System, and only one position is transmitting and receiving at a time. The wave-fronts reflected from the target were sampled coherently with a sampling frequency of 20-MHz and a resolution of 12 bits. The normalization factors were the same for all cases. The horizontal axis represented the transducer position (mm), and the vertical axis represented the depth (cm). The main parameters were the same in both the simulation and the experiments; these parameters and their corresponding values can be found in Table 1.

In the simulation, it is assumed that imaging takes place in a homogeneous, no-attenuating medium and that the speed of the ultrasound remains constant. Figure 3(a) shows the simulated images of an object consisting of three point targets. Since it is a wide-angle beam in transmission, each 1-D hologram corresponds to several depths in the hologram data. The geometric distortions can be clearly seen in the near-field of the ultrasound.

To illustrate the new approach's efficient compensation, the backward propagation transfer function is not applied, and the inverse Fourier transforms algorithm gives the output image directly as illustrated in Figure 3(b). After the spatial compress and the re-sampling are performed, the geometric distortion can be effectively compensated. The 1-D hologram corresponds to the same depths instead of several depths in the hologram data.

The inverse Fourier transforms algorithm and the backward propagation principle are applied in the

Table 1  
The simulation and measurement parameters

| Parameters           | Values   |
|----------------------|----------|
| Ultrasonic frequency | 5 MHz    |
| Transducer elements  | 256      |
| Element separation   | 0.25 mm  |
| Sampling frequency   | 20 MHz   |
| Precision (RP)       | 12 bits  |
| Speed of sound       | 1540 m/s |

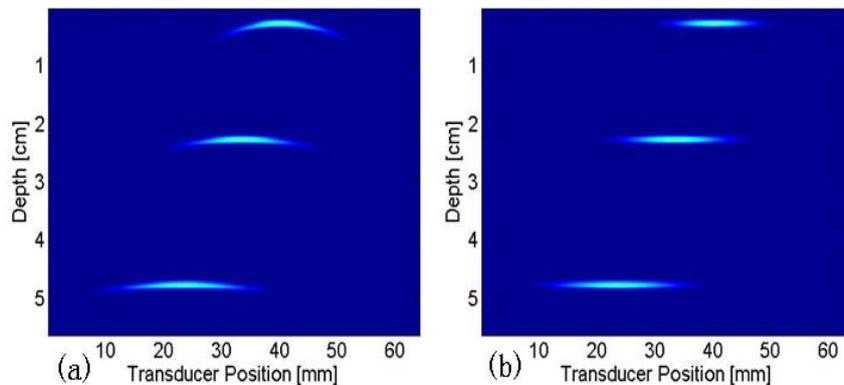


Fig. 3. Geometric distortion in the simulation hologram data without compensation is shown in (a). The compensation result is shown in (b).

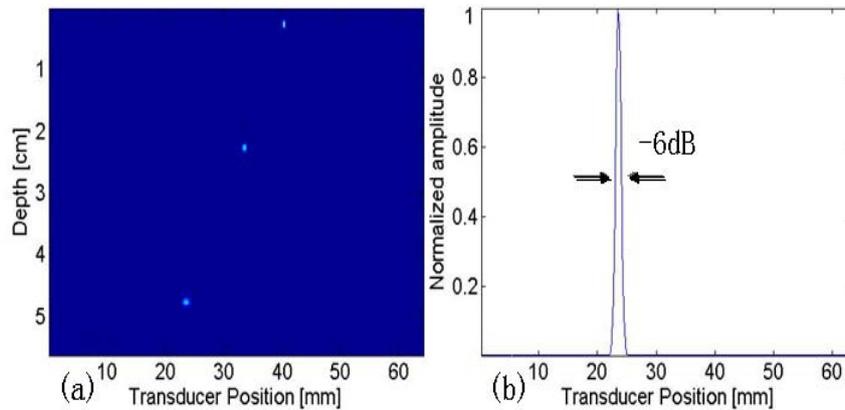


Fig. 4. The synthetic aperture image reconstruction in the simulation and the spatial resolution of the third point target at a depth of 48 mm.



Fig. 5. The overview of the experimental setup. The linear transducer of the commercial Ultrasonix RP ultrasound system was chosen to scan the target phantom (No: 922, model 525).

image reconstruction, and the results can be seen in Figure 4(a). The Point spread function at a depth of 48 mm is also illustrated in Figure 4(b), and it depicts that the -6dB spatial resolution at 48 mm-deep is approximately 1 mm.

The test phantom (No: 922, model 525) was used to evaluate the effectiveness of the new approach of spatial compression and re-sampling in the spatial frequency domain. The speed of sound and mean attenuation in the target medium was 1540 m/sec. All the experiment measurements presented in this paper are from the commercial Ultrasonix RP ultrasound system. The overview of the experimental setup is shown in Figure 5.

Since the transducer conducts a fairly wide-angle beam in the scanning plane, the geometric distortion is encountered in the recorded two-dimensional hologram data. In order to demonstrate the compensation effect more clearly, the one-target point in the red rectangle at a depth of 19 mm is selected, and the corresponding unfocused hologram data are shown in Figure 6(a). The curve is very obvious, which indicates the geometric distortion in the 2-D hologram data. The compensation result is illustrated in Figure 6(b), which demonstrates that the 1-D holography information is selected in the same depth with the new approach.

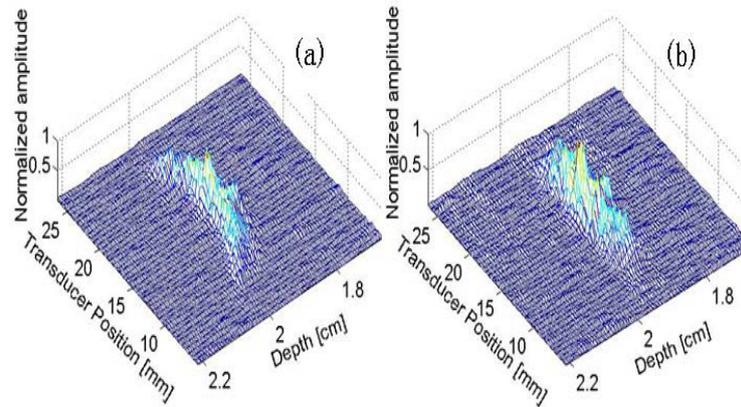


Fig. 6. The recorded one-point hologram data and the geometric distortion compensation result. The one-point in the red rectangle at a depth of 19mm is selected in Figure 7(a). The curve hologram data is converted into linear data, which demonstrates that the geometric distortion compensation is achieved.

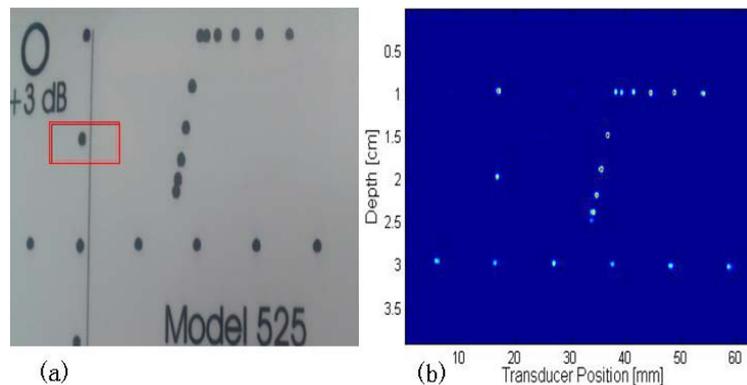


Fig. 7. The part screen capture of phantom and the corresponding synthetic aperture ultrasonic holographic imaging after the geometric distortion compensation.

After the geometric distortion is compensated by using the new approach of spatial compression and re-sampling in the spatial frequency domain, the synthetic aperture technique based on the principle of backward propagation was applied to process all the echo signals for ultrasound imaging. The parameters in Table 1 were used in the image reconstruction. Figure 7(a) is part of the phantom screen, and the minimum target point distance is 1 mm, increasing by 1 mm. The numerical reconstruction image covers a depth range from 5 mm to 35 mm and shows good axial and spatial resolution throughout the image, as is illustrated in Figure 7(b).

All the targets are well resolved, thereby indicating that the spatial two-point resolution is about 1 mm. Three point scatters in Figure 7(b) at transducer position 16 mm and depth 10 mm, 19 mm, 29 mm are selected respectively. The corresponding intensity curves for the lateral direction are shown in Figure 8 for each individual point scatters. It can be seen that as the depth is increased, the -6 dB spatial resolution is still approximately 1 mm.

Both the simulation and the measurement experiment are consistent with theoretical expectations. The geometric distortion dependent on the wave-front angle can be compensated by the new approach of spatial compression and re-sampling in the spatial frequency domain. The spatial resolution is

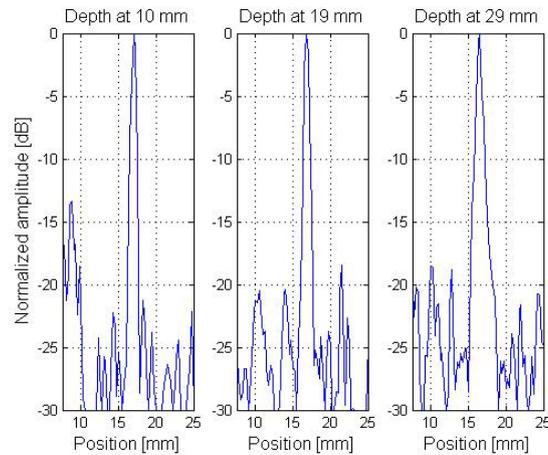


Fig. 8. The -6dB spatial resolution is about 1 mm and is approximately uniform throughout the depth range. Three point scatters in Figure 7(b) at transducer position 16mm and depth 10 mm, 19 mm, 29 mm are selected respectively and their corresponding intensity curves for the lateral direction are also shown.

about 1 mm and is approximately uniform throughout the depth range. This new approach is an essential improvement in the image reconstruction of the synthetic aperture ultrasonic imaging system.

#### 4. Conclusions

A new approach to compensate for geometric distortion in the synthetic aperture ultrasonic imaging system has been presented in this paper. Furthermore, the new concept of spatial compression and re-sampling are introduced in the spatial frequency domain. The real holographic value that corresponds to the original depth can be computed, and the geometric distortion can be compensated by using the new approach. Finally, the two-dimensional numerical imaging reconstruction is conducted by applying the backward propagation principle, and the spatial resolution of the image is improved.

The new approach has been demonstrated through a computer simulation and an experiment employing the commercial Ultrasonix RP system. The spatial resolution is practically uniform throughout the depth range and close to the theoretical limit in the experiments. The approach is expected to compensate the distortion caused by the geometry of the transducer, such as convex and concave as well as circle array.

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