Argument & Computation 11 (2020) 103–149
DOI 10.3233/AAC-190479
IOS Press

Similarity notions in bipolar abstract argumentation

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Abstract. The notion of similarity has been studied in many areas of Computer Science; in a general sense, this concept is defined to provide a measure of the semantic equivalence between two pieces of knowledge, expressing how “close” their meaning can be regarded. In this work, we study similarity as a tool useful to improve the representation of arguments, the interpretation of the relations between arguments, and the semantic evaluation associated with the arguments in the argumentative process. In this direction, we present a novel mechanism to determine the similarity between two arguments based on descriptors representing particular aspects associated with these arguments. This mechanism involves a comparison process influenced by the context in which the process develops, where this context provides the relevant aspects that need to be analyzed in the application domain. Then, we use this similarity measure as a quantity to compute the result of attacks and supports in the argumentation process. These valuations, applied to a Bipolar Argumentation Frameworks, allowed us to refine the argument relations, providing the tools to establish a family of new argumentation semantics that considers the similarity between arguments as a crucial part for the argumentation process.

Keywords: Bipolar Argumentation, similarity, cohesion and controversy values

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1. Introduction

Human commonsense reasoning involves in many occasions a process of analysis over a set of (potentially contradictory) alternatives and the evaluation of their support. The study of such cognitive process has led to the development of several formalisms that were introduced in an attempt to provide a formal model for this mechanism. In this direction, argumentation has become a topic of significant impact in the field of *Artificial Intelligence* (AI). The basic idea is to identify arguments in favor and against an affirmation and then select which arguments are acceptable among them, with the goal of resolving whether the reasoner can accept the assertion. Thus, Argumentation Theory provides reasoning mechanisms able to handle contradictory information concerning specific issues.

In the course of the research on the argumentation field, several argument-based formalisms have emerged to study the various possible relations among arguments. In [32], Dung proposes *Abstract Argumentation Frameworks* (AF) to model real-world situations by representing attack relations between abstract entities called *arguments*, providing different acceptability semantics for determining which sets of arguments are acceptable. Subsequently, Cayrol and Lagasquie-Schiex in [22] extended Dung's framework [32] taking into account two independent types of interaction between arguments by adding the relation of *support* to the original relation of *attack* in abstract argumentation frameworks. The resulting *Bipolar Argumentation Frameworks* (BAFs) allow to model situations in which an argument can, for instance, reinforce another argument providing more reasons to believe in it; moreover, they adapt Dung's acceptability semantics adding the consideration of the support relationship. Also, there exist some argumentative formalisms that represent the attributes associated to arguments providing more information to determine arguments acceptability [15,19,20,23], while others consider relations between the arguments to calculate the acceptability and safety of a given set of arguments [10].

Leaving aside the advances in argumentation frameworks [15,20,23], several representational aspects of the argumentation process still require further study. For example, a very natural tool for argument-based reasoning is the notion of *similarity* among arguments: during an argumentation process we sometimes tend to group arguments according to their shared characteristics or to the topics to which they refer. It can be argued that any comparison process requires the definition of a context in which such comparison can be meaningful [25,43,73]. The same applies to arguments: two arguments may be similar in a given context, but they may be entirely unrelated (or even incomparable) under different circumstances of analysis. Argumentative reasoning that gives importance to the similarities between pieces of knowledge represents a natural form of everyday human reasoning [76,77,79]. As computational argumentation aims to the definition of useful systems based on common sense, it seems reasonable, and desirable, to formalize the notion of similarity between arguments. As it happens with the similarity analysis in other settings where entities are compared provided they have some essential aspects in common, similarities between arguments also require the consideration of the context where they are issued identifying the aspects that are relevant in the comparison and the assessment of their importance in the similarity analysis. Notwithstanding the usefulness of exploring these problems concerning the argumentation process, similarity relations between arguments have not been deeply explored in the argumentation literature.

Example 1. As a running example that we will develop over this work, consider the following situation where a user wants to decide about which activities to perform considering the weather conditions. The user has the following set of arguments which wants to use to decide whether going out for a walk or staying at home.

A: *On rainy days we should eat chocolate since chocolate lifts the mood and makes us happy.*
B: *If we are happy, we want to go out for a walk.*
C: *In rainy days we are happy and in a good mood, therefore, we go out shopping.*
D: *Rainy days may be depressing. Since today is raining, I prefer to stay to keep the house.*
E: *Sunny days are optimal for outdoor activities, in that way help to release endorphins.*
F: *It is not a good idea to go for a walk to relax, because we can hurt our feet; thus, it is better to watch a movie instead.*
G: *If we go shopping, we take a walk and burn calories; therefore, it is a good plan.*
H: *Keeping the house is stressful since it is a job that requires several days, therefore, it is better to hire somebody else to do it.*

This example illustrates how the knowledge used to decide can be naturally structured as arguments; to reach a decision it is necessary to consider the relationships between these arguments, and how strong, or weak, those connections are. In particular, this scenario shows a support relation between arguments A, B, and C. However, it would be necessary to analyze how strong the ties in this set are, i.e., how cohesive the set is, to have a measure of the strength of the support among the arguments in it; similarly, it is possible to examine how controversial a set of conflicting arguments is. For instance, regarding the conflict relation between arguments C, D, and H, we can analyze the (dis)similarity between these arguments given a context that will depend on the application domain; furthermore, the comparison process must be performed under the expressed user's preferences.

Notice that the similarity between a pair of arguments can be judged differently by each user, and consequently, the argumentation process where the relations between arguments are analyzed should reflect this important aspect. Although the perceived similarity among arguments is not the only tool that can be used to define the strength of the relations of support and attack it provides a natural alternative that can help to weight the relationships based on the (dis)similarity of the different situations in which the arguments are issued.

We will propose a *Similarity-based Bipolar Argumentation Framework*, which provides mechanisms for considering the context of the comparison between arguments, based on a set of descriptors that are common to the arguments which are being analyzed. Thus, we can represent and determine similarities between arguments introducing means to enrich the representation of the relationships between them and to be able to distinguish among arguments that are weakly related from those whose relationship is stronger. In this direction, we use an arguments' similarity degree, computed from the descriptors that arguments have in common, combined with the weight those descriptors have in the process comparison. Thus, we determine a *cohesion value* between supporting arguments and a *controversy value* between conflicting arguments. Based on this analysis, we can refine the acceptability process provided by a BAF to obtain a new family of argumentation semantics. It is important to remark that the descriptors attached to arguments are additional knowledge representation devices added by this proposal. The basic assignment of the particular values to the descriptors is a task that can be performed in different ways. On the one hand, this can be performed by hand by a knowledge engineer; on the other hand, several tools have been introduced in the computational argumentation field by the research of argument mining techniques for which we will provide references below since the topic is outside the reach of this work.

The presentation will be structured as follows. We begin with an introduction to the BAFs is presented in Section 2. In Section 3 a brief, general description of the similarity concept will be addressed. The development of the tools for similarity comparison between arguments in the argumentation domains is presented in Section 4. Then, Section 5 contains the proposed argumentation formalism, the

Similarity-based Bipolar Argumentation Framework, where an interpretation of supporting and attacking arguments is given based on the notion of similarity, also studying how this interpretation affects the acceptability process. In Section 6, we offer a case study by using an example in the food and nutrition domain. Finally, in Section 7 and 8, we present related works and offer conclusions, respectively.

2. Bipolar Argumentation Framework

When using arguments to reason, different types of relationships between them can be considered. One possible view is that arguments exhibit a "bipolar" behavior since reasons in favor of an affirmation can be considered as being positive while reasons against it can be viewed as negative. This approach was taken in the presentation of *Bipolar Argumentation Frameworks* (BAFs) that was proposed by Cayrol and Lagasquie-Schiex in [22], extending Dung's notion of acceptability by distinguishing two independent forms of interaction between arguments: *support* and *attack*. We will assume that the reader is familiar with the formalism of Abstract Argumentation Frameworks [32].

Definition 1 (Bipolar Argumentation Framework (BAF)). A Bipolar Argumentation Framework is a 3-tuple $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$, where $\mathtt{Arg}$ is a set of arguments, $\mathtt{R}_a$ and $\mathtt{R}_s$ are two disjoint binary relations defined on $\mathtt{Arg}$ called *attack* and *support*, respectively.

To provide a graphical representation of *BAF*s, Cayrol and Lagasquie-Schiex also extended argumentation graphs presented by Dung in [32] by adding the representation of the support relation between arguments. This argumentation model introduced a starting point to analyze human reasoning adding the consideration of the bipolar aspect that naturally occurs in any debate to our modeling toolbox. Furthermore, they introduced the notions of *supported* and *secondary* attack that combine a sequence of *supports* with a *direct attack* considering the interaction between supporting and attacking arguments. These relations are defined as follows:

Definition 2 (Attacks in BAF). Let $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a *BAF*, and $\mathtt{A}, \mathtt{B} \in \mathtt{Arg}$ two arguments. Then:

- There is a direct attack from $\mathtt{A}$ to $\mathtt{B}$ if $\mathtt{A}\ \mathtt{R}_a\ \mathtt{B}$.
- There is a supported attack from $\mathtt{A}$ to $\mathtt{B}$ if there exists a sequence $\mathtt{A}_1\ \mathtt{R}_1 \ldots \mathtt{R}_{n-1}\ \mathtt{A}_n$, with $n \geqslant 3$, where $\mathtt{A}_1 = \mathtt{A}$ and $\mathtt{A}_n = \mathtt{B}$, such that $\forall i = 1, \ldots, n-2$, $\mathtt{R}_i = \mathtt{R}_s$ and $\mathtt{R}_{n-1} = \mathtt{R}_a$.
- There is a secondary attack from $\mathtt{A}$ to $\mathtt{B}$ if there exists a sequence $\mathtt{A}_1\ \mathtt{R}_1 \ldots \mathtt{R}_{n-1}\ \mathtt{A}_n$, with $n \geqslant 3$, where $\mathtt{A}_1 = \mathtt{A}$ and $\mathtt{A}_n = \mathtt{B}$, such that $\mathtt{R}_1 = \mathtt{R}_a$ and $\forall i = 2, \ldots, n-1$, $\mathtt{R}_i = \mathtt{R}_s$.

Cayrol and Lagasquie-Schiex argued in [22] that a set of arguments must keep in some sense a minimum of coherence to be able to model one side of any reasonable dispute adequately. They propose that the coherence of an acceptable set of arguments can be kept *internally* by requiring the set not to contain an argument that attacks another in the same set, and *externally* by requiring the set not to include both a supporter and an attacker of the same argument. Internal coherence can be obtained by extending the definition of *conflict free set* proposed in [32] and external coherence can be captured by the notion of a *safe* set.

Definition 3 (Conflict-freeness and safety properties in BAF). Let $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a *BAF*, and $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments. We say that $\mathtt{S}$ is *conflict-free* iff $\nexists \mathtt{A}, \mathtt{B} \in \mathtt{S}$ s.t. there is an attack (direct, or supported, or secondary) from $\mathtt{A}$ to $\mathtt{B}$. We say that $\mathtt{S}$ is *safe* iff $\nexists \mathtt{A} \in \mathtt{Arg}$ and $\nexists \mathtt{B}, \mathtt{C} \in \mathtt{S}$ s.t. there is an attack (direct, or supported, or secondary) from $\mathtt{B}$ to $\mathtt{A}$, and either there is a sequence of support from $\mathtt{C}$ to $\mathtt{A}$, or $\mathtt{A} \in \mathtt{S}$.

The notion of conflict-freeness requires taking in consideration the direct, supported, and secondary attacks. Additionally, Cayrol and Lagasquie-Schiex show that the notion of safe set is powerful enough to encompass the concept of conflict-freeness, i.e., if a set is safe, it is also conflict-free. The closure under $\mathtt{R}_s$ was introduced in [22] is a requirement that only concerns the support relation.

Definition 4 (Closure property in BAF)**.** Let $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be an *BAF*. $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments. $\mathtt{S}$ is closed under $\mathtt{R}_s$ iff $\forall \mathtt{A} \in \mathtt{S}$, $\forall \mathtt{B} \in \mathtt{Arg}$ if $\mathtt{A}\, \mathtt{R}_s\, \mathtt{B}$ then $\mathtt{B} \in \mathtt{S}$.

Using these ideas, Cayrol and Lagasquie-Schiex in [22] extended the notions of defense for an argument with respect to a given set, where they take into account the relations of support and conflict.

Definition 5 (Defense from a set of arguments $\mathtt{S}$ to a single argument $\mathtt{A}$ in BAF)**.** Let $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments, and $\mathtt{A} \in \mathtt{Arg}$ be an argument. We say $\mathtt{S}$ collectively defends $\mathtt{A}$ iff $\forall \mathtt{B} \in \mathtt{Arg}$ if $\mathtt{B}$ is an attacker (direct, or supported, or secondary) for $\mathtt{A}$ then $\exists \mathtt{C} \in \mathtt{S}$ such that $\mathtt{C}$ is an attacker (direct, or supported, or secondary) for $\mathtt{B}$. In this case, it can be said that $\mathtt{C}$ defends $\mathtt{A}$ from $\mathtt{B}$.

The authors proposed three different definitions of admissibility which reflect three different levels of generality. The most general, called *d-admissibility*, is based on Dung's admissibility capturing the internal coherence requirement through the conflict-free property. Then, the authors refine d-admissibility in two directions: one that captures the external coherence notion through the safe property, called *s-admissibility*; and the other, that considers the closure property over a conflict-free set, called *c-admissibility*. Formally:

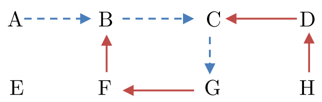
Definition 6 (Admissibility notions in BAF)**.** Let $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a *BAF*. Let $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments. Then, the admissibility of $\mathtt{S}$ is defined as follows:

– $\mathtt{S}$ is d-admissible if $\mathtt{S}$ is conflict-free and defends all its elements.
– $\mathtt{S}$ is s-admissible if $\mathtt{S}$ is safe and defends all its elements.
– $\mathtt{S}$ is c-admissible if $\mathtt{S}$ conflict-free, closed for $\mathtt{R}_s$ and defends all its elements.

Note that, a c-admissible extension is a set of arguments that is conflict-free and closed under support. Naturally, this extension is also a d-admissible extension since it is conflict-free and defends all its elements. Furthermore, the arguments that belong to the extension cannot attack an argument that they support, because this action violates the conflict-freeness property. Thus, this extension satisfies the external coherence property obtaining that a c-admissible extension is also a s-admissible extension. On the other hand, the s-admissible extension satisfies the conflict-freeness property, since a safe set of arguments is a conflict-free set that satisfies the external coherence. Roughly speaking, we can note that a d-admissible extension is refined to consider: external coherence in a s-admissible set, and closure under support condition in a c-admissible set.

Cayrol and Lagasquie-Schiex in [22] proposed different semantics for computing acceptability. These semantics consider the previous admissibility notions, redefining the classical ones proposed in [32] by Dung.

Definition 7 (Stable extension in BAF)**.** Let $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a *BAF*, and $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments. $\mathtt{S}$ is a stable extension of Θ if $\mathtt{S}$ is conflict-free and $\forall \mathtt{A} \notin \mathtt{S}$, there is an attacker (using a direct or supported or secondary attack) of $\mathtt{A}$ in $\mathtt{S}$.



Extensión	{A,E,F,H}	{A,B,C,G,E,H}
d-admissible	X	X
s-admissible		X
c-admissible		X
d-preferred	X	X
s-preferred		X
c-preferred		X
stable	X	X

Fig. 1. Representation of attack and support relations in *BAF*.

Definition 8 (Preferred extensions in BAF)**.** Let $\Theta = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s \rangle$ be a *BAF*, and $\mathtt{S} \subseteq \mathtt{Arg}$ be a set of arguments. $\mathtt{S}$ is a d-preferred (resp. s-preferred, c-preferred) extension if $\mathtt{S}$ is maximal (for set-inclusion) among the d-admissible (resp. s-admissible, c-admissible) subsets of $\mathtt{Arg}$.

Example 2. Our running example can be represented by a *BAF*, characterized by $\Theta = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s \rangle$, where:

$$\mathtt{Arg} = \{\mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{D}, \mathtt{E}, \mathtt{F}, \mathtt{G}, \mathtt{H}\},$$
$$\mathrm{R}_a = \big\{(\mathtt{F}, \mathtt{B}), (\mathtt{G}, \mathtt{F}), (\mathtt{H}, \mathtt{D}), (\mathtt{D}, \mathtt{C})\big\},$$
$$\mathrm{R}_s = \big\{(\mathtt{A}, \mathtt{B}), (\mathtt{B}, \mathtt{C}), (\mathtt{C}, \mathtt{G})\big\}.$$

Figure 1 represents the attack and support relation between arguments, using solid arrows and dashed arrows, respectively. Observe that there is a supported attack from $\mathtt{A}$ to $\mathtt{F}$, a secondary attack from $\mathtt{F}$ to $\mathtt{G}$ and a secondary attack from $\mathtt{D}$ to $\mathtt{G}$. Besides, the set $\mathtt{S}_1 = \{\mathtt{A}, \mathtt{B}, \mathtt{D}, \mathtt{E}\}$ is conflict-free but is not safe due to the fact that $\mathtt{B}$ supports $\mathtt{C}$, that is attacked by $\mathtt{D}$; also $\mathtt{S}_1$ is not closed under R_s; furthermore $\mathtt{S}_1$ is not d-admissible because there is no argument that defends $\mathtt{D}$ from $\mathtt{H}$ attacks.

On another hand, the set $\mathtt{S}_2 = \{\mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{G}, \mathtt{H}, \mathtt{E}\}$ is conflict-free, safe, and closed under R_s; moreover, $\mathtt{S}_2$ is d-admissible, s-admissible, and c-admissible, given the characteristics already analyzed and adding that $\mathtt{H}$ defends $\mathtt{C}$ from $\mathtt{D}$ attacks, and $\mathtt{G}$ defends $\mathtt{B}$ from $\mathtt{F}$ attacks. So, $\mathtt{S}_2$ is a stable, d-preferred, s-preferred, and c-preferred extension of Θ. Furthermore, the set $\mathtt{S}_3 = \{\mathtt{A}, \mathtt{F}, \mathtt{H}, \mathtt{E}\}$ is conflict-free, but it is not safe and not closed under R_s since $\mathtt{F}$ attacks $\mathtt{B}$ which is supported by $\mathtt{A}$. Thus, $\mathtt{S}_3$ is only d-admissible. So, $\mathtt{S}_3$ is a stable and d-preferred extension of Θ.

A bipolar argumentation framework obtains an acceptable set of arguments based on a specific analysis that considers the support and conflict relations between the arguments involved in a dispute. However, this formalism does not provide with tools to adequately analyze how *cohesive* or *controversial* are the arguments in a discussion. In the following sections, we extend the basic *BAF* to refine the argumentation analysis in this sense.

3. Preliminaries on similarity and similarity measures

The concept of similarity has been studied in many areas of Computer Science, in general sense, that is considering the similarity between two pieces of text. Thus, the textual semantic similarity is a measure of the equivalence of meaning between these two texts, which expresses the similarity or resemblance between them [1]. However, similarity can refer to syntactical or semantical aspects, and therefore the concept has been studied by various areas of AI such as natural language processing and argumentation. From the perspective of natural language processing, there exists, for example, several well-known techniques for disambiguation of words [2,28,57] or induction of word meaning [3,27,72]. From the field of argumentation, numerous advances have occurred in the area of argumentation mining such as the development of techniques linked to the processing of natural language [41,56,64]. However, the use of a measurement of similarity between arguments for reasoning that involves uncertain or potentially contradictory information is an area where, to the best of our knowledge, no previous work addresses the issue in depth.

Lin et al. in [54] present an independent definition of similarity, which compares the information included in the description of two objects A and B definitions, based on the information Theory, comparing the amount of information contained in the definition of the two objects. They argue that it is also essential to establish a measure of the difference between the objects to be compared because the more differences there are between the objects, the less similar they are. In that work, the authors propose that the similarity between two objects A and B be calculated from the relationship between the average of the information held in the description of the objects and the average of the information referring to their common aspects. Formally:

$$\mathrm{simil}(A, B) = \frac{\log P(I(\mathrm{common}(A, B)))}{\log P(I(\mathrm{description}(A, B)))}.$$

This measure of similarity is quite general and, therefore, can be applied to different domains. Some examples are the determination of semantic similarity in a taxonomy, or the calculation of the similarity between the words in a text according to the distribution that words follow in it, or its application to feature vectors which are extensively used in machine learning techniques to find those words that they derived from the same root.

On the other hand, numerous studies have put the focus on finding a measure of similarity between objects, according to the domain of application to which these objects belong. One of the best-known measures is the *cosine similarity* [44] from the representation of the entities in the *Bag-Of-Words* (BOW) format [50]. The BOW format consists of finding a set of words that describe an entity, e.g., those that are present in a text, and each word has associated a weight representing the importance of the word in that text. This format allows to calculate the similarity between two entities using the measure of cosine similarity, and it is especially useful for information retrieval [44]. More clearly, in each text, it is possible to identify the words that will be part of the BOW and the weight assigned to each word may be the frequency of its appearance in the text; so, the terms that appear most frequently in the text are those that will have a higher weight.

In [74] the authors suggest a practical use of the BOW format to compare two entities using an ontology and to determine the semantic similarity between these entities. In this specific case, the process is carried out to find the semantic similarity between domains, using two sets of words obtained from the application of web mining techniques in search and recommendation systems. From the ontology, it is feasible to find concepts related to the entities under consideration by applying a spreading process.

In a similar direction, Rusu et al. [71] presents a summary of different possible metrics to determine the similarity between concepts in an ontology, highlighting the theoretical models proposed by cognitive psychology. Among them are mentioned: *measures based on definitions* like *Lesk algorithm* [53];

measures based on structures, for example distance Rada [69], the *Leacock and Chodorow Similarity* [51] and the *Wu and Palmer Similarity* [80]; and *measures based on the information content*, such as *Resnik Measure* [70] or *Jiang Distance* [48]. These measurements have their origin in a geometric model [26], or in the coincidence of features or characteristics models [75]. The first represents the concepts and relationships between them, stored in a computer memory that simulates human memory, using three hierarchical levels of storage: the concept, its category, and its properties. The second one mentions the comparison between objects of any nature to find similarities and dissimilarities between them, beyond the distance between points, by comparing characteristic features [75]. The proposal of Rusu et al. in [71] is also based on the distance between concepts defined on an ontology, but concepts and relationships are weighted; then, the similarity between the concepts according to the weight of the shortest path between them. Finally, Amgoud et al. in [11] explore several similarity measures between logical arguments and define a very general function denoted as a similarity measure. Then, they define a set of basic principles that a similarity measure should satisfy, such as syntax independence, maximality, symmetry, substitution, monotony, and dominance. While in [12], the authors work with logical entities representing the structures of the arguments and propose a mechanism to calculate similarity measures using concise refinements of arguments based on the arguments that only contain useful information in their premises to infer the conclusion. Furthermore, in [8], the authors use the similarity between arguments as the means to analyze an individual argument and its attackers and the concept to introduce a new semantics. Also, they present properties that this semantics should satisfy. This brief description summarizes some of the results that can be found in the literature on the subject, and which are the basis for the approach taken in our proposal.

4. Introducing similarity measures for arguments

A crucial component in the determination of the similarity between arguments is the definition of the *conditions* under which such comparison is performed [77]. In [54], D. Lin states that the main problem with existent similarity measures is that they assume a particular domain model, i.e., the conditions over which the similarity is calculated are considered preexistent. Although the similarity is related to the properties shared between the two entities being compared, the comparison of two arguments largely depends on an agent's perception which can be influenced by the mental status of the agent, e.g., her beliefs, goals, or by external, possible unknown external variables. All these factors are part of a context that affects the assessment of the similarities between two arguments. The intuition that we have just presented is essential to define a *similarity measure* between arguments.

In this section, we present a method that allows us to determine the similarity between arguments, according to a given *context*. In general, this method consists of the following three stages: (i) *the specification of argument's descriptors*; (ii) *the setting of the context based on argument's descriptors*; and (iii) *the computation of the similarity degree among the arguments being compared*. In what follows, we present and develop these three stages.

4.1. Specification of argument's descriptors

As suggested above, a *descriptor* is a *word*, a *tag*, or a *label* that describes an aspect to which the argument is in some way connected [21]. In this sense, the set of descriptors associated with each argument involved in a debate should be representative of the domain of such discussion. Finding the particular set of descriptors associated with an argument can be done by an analyst (knowledge engineer) or can involve the use of argumentation mining techniques, which is currently an important topic in the area of computational argumentation; however, this task is complex and exceeds the scope of this paper, we refer the reader to the comprehensive work in [49,55,66] for further details. We assume this information is available and that an argument comes with a set of descriptors and a mapping from these descriptors to values in a corresponding application domain; still, as it is usual in abstract argumentation frameworks, no reference to the underlying structure of the argument will be made, here we are interested in giving significance to what this argument refers to, without delving into its the logical details.

As a prelude to the next definitions, we introduce a few notational conventions. We denote as $\mathcal{D}$ the set of possible descriptors that represent the characteristics of a specific argumentation domain and as $\mathcal{V}$ the set of semantic values associated with descriptors in $\mathcal{D}$. Given $d \in \mathcal{D}$, $\mathcal{V}_d$ will be the set of semantic values associated with a descriptor d in the application domain. The descriptors and their values characterize a conceptualization of this domain, and they can be, for instance, just words in natural language or more complex concepts provided by an ontology [40].

Definition 9 (Enriched argument)**.** Given a set of arguments $\mathtt{Arg}$, an *enriched argument* is a pair $\mathbb{A} = \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle$, where $\mathtt{A}$ is an abstract argument in $\mathtt{Arg}$, $\delta_{\mathtt{A}}$ is a finite non-empty set of pairs $(d, \mathcal{V}_d^{\mathtt{A}})$ such that $d \in \mathcal{D}$ and $\mathcal{V}_d^{\mathtt{A}} \subseteq \mathcal{V}_d$, and for every two pairs $(d, \mathcal{V}_d^{\mathtt{A}})$ and $(d', \mathcal{V}_{d'}^{\mathtt{A}})$ in $\delta_{\mathtt{A}}$, we have that $d \neq d'$. We denote with $\mathbb{A}\mathrm{rg}$ the set of all enriched arguments w.r.t. $\mathtt{Arg}$.

Intuitively, an enriched argument $\mathbb{A} = \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle$ consists of the actual argument $\mathtt{A}$ and an additional structure $\delta_{\mathtt{A}}$, that is a set of pairs representing the descriptors mentioned in $\mathbb{A}$ and the semantic values associated to such descriptors. Given a set of enriched arguments $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$, there will be occasions in which we may want to refer specifically to the set of (classical) arguments involved in it; for that, we will introduce in the following definition how to obtain the set of arguments involved in a set of enriched argument.

Definition 10 (Arguments from a set of enriched arguments)**.** Let $\Theta = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s \rangle$ be a bipolar argumentation framework and $\mathbb{A}\mathrm{rg}$ be the set of enriched arguments produced from $\mathtt{Arg}$. Given $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$, we introduce the function $\Pi_\Theta(\mathbb{S})$, defined as $\Pi_\Theta(\mathbb{S}) = \{\mathtt{A} \mid \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle \in \mathbb{S}\}$, that will obtain the set of arguments $\mathtt{S} \subseteq \mathtt{Arg}$ involved in the set of enriched arguments $\mathbb{S}$.

Next, we analyze the example presented in the introduction in order to identify the descriptors associated to each argument, and the values of these descriptors according to the information provided by the arguments.

Example 3. Continuing with Example 1 presented in the introduction, we can instantiate the universe of descriptors for this specific domain as the set:

$$\begin{aligned}\mathcal{D} = \{&\mathit{weather\_conditions}, \mathit{general\_activity}, \mathit{frame\_of\_mind},\\ &\mathit{food}, \mathit{action\_food}, \mathit{consequence\_activity}\}.\end{aligned}$$

Then, we can provide the set of descriptors and the corresponding structure for the enriched arguments as follows:

$$\begin{aligned}\delta_{\mathtt{A}} = \big\{&\big(\mathit{weather\_conditions}, \{\mathit{rainy\_day}\}\big); \big(\mathit{general\_activity}, \{\mathit{eat}\}\big); \big(\mathit{food}, \{\mathit{chocolate}\}\big);\\ &\big(\mathit{action\_food}, \{\mathit{lifts\_mood}\}\big); \big(\mathit{frame\_of\_mind}, \{\mathit{happy}\}\big)\big\},\end{aligned}$$

$$\delta_{\mathrm{B}} = \{(general\_activity, \{go\_out, walk\}); (frame\_of\_mind, \{happy\})\},$$

$$\delta_{\mathrm{C}} = \{(weather\_conditions, \{rainy\_day\}); (general\_activity, \{go\_out, shopping\}); (frame\_of\_mind, \{happy, good\_mood\})\},$$

$$\delta_{\mathrm{D}} = \{(weather\_conditions, \{rainy\_day\}); (frame\_of\_mind, \{depressing\}); (general\_activity, \{keeping\_house\})\},$$

$$\delta_{\mathrm{E}} = \{(weather\_conditions, \{sunny\_day\}); (general\_activity, \{outdoor\_activities\}); (consequence\_activity, \{release\_endorphins\})\},$$

$$\delta_{\mathrm{F}} = \{(general\_activity, \{walk, watch\_movie, go\_out\}); (consequence\_activity, \{hurt\_feet\})\},$$

$$\delta_{\mathrm{G}} = \{(general\_activity, \{go\_out, shopping, walk, spend\_calories\})\},$$

$$\delta_{\mathrm{H}} = \{(general\_activity, \{keeping\_house\}); (frame\_of\_mind, \{stressful\})\}.$$

All the arguments, except E, F, and G, share the descriptor "*frame_of_mind*", so they might be compared using it. Additionally, the value of "*frame_of_mind*" is the same for arguments A and B but only matches one of the values corresponding to "*frame_of_mind*" for argument C. Also, the value taken by "*frame_of_mind*" is totally different for the arguments D and H. These differences mean that, although the arguments have the same descriptors and therefore they could be compared in that regard, there is no assurance that they could be similar. The reason for this is that the value taken by each descriptor could be different for particular arguments. This analysis can be performed separately considering each descriptor associated with the arguments.

As several descriptors can be associated with an argument, it is necessary to establish contexts in which to conduct the comparisons and the means of prioritizing the descriptors in such comparisons as well. We borrow the formal elements of the mechanism for computing similarities among arguments from work proposed in [21]; however, in that framework, there is no way of judging the importance of the descriptors in the comparison. Next, we extend the mechanism to establish a comparison based on context, in which each descriptor has a weight or degree of importance associated.

4.2. Setting the context

The descriptors of the arguments may refer to different topics, a fact that makes more complicated the comparison process. For that reason, as part of the knowledge representation task, it would be essential to correctly establish the context to evaluate the similarities and differences between the arguments. Following this idea, the definition of a context defines a set of restrictions over the comparison process. As an initial requirement, two arguments can only be compared if they have in common at least one descriptor. In [21], a simple notion of context is proposed where essentially a context corresponds to a set of descriptors. A natural extension of this approach is acknowledging that comparisons among arguments require that different descriptors have a different impact on the similarity analysis, conveying the meaning that some aspects of the arguments are more (or less) relevant for the comparison than others. In this proposal, in the definition of a context, we incorporate to descriptors a relevance weight to achieve this goal, and thus increase the representation capabilities of the argumentative framework.

Definition 11 (Context)**.** Let $\mathcal{D}$ be a set of descriptors, a *context* $\mathbb{C}$ will be represented as $\mathbb{C} = \{(d, w_d) \mid d \in \mathcal{D}, w_d \in [0, 1]\}$, i.e., a context is a set of ordered pairs where $d \in \mathcal{D}$ is a descriptor and $w_d \in [0, 1]$ is the relevance weight associated with d. We denote with $\mathcal{D}_{\mathbb{C}} \subseteq \mathcal{D}$ the set of descriptors mentioned in the context $\mathbb{C}$, i.e., $\mathcal{D}_{\mathbb{C}} = \{d \mid (d, w_d) \in \mathbb{C}\}$.

To exemplify the definition above we continue developing the last example.

Example 4. Returning to *Example* 3 we have the following domain:

$$\mathcal{D} = \{\mathit{weather\_conditions}, \mathit{general\_activity}, \mathit{frame\_of\_mind}, \\ \mathit{food}, \mathit{action\_food}, \mathit{consequence\_activity}\}$$

suppose that we want to compare the arguments in the example regarding weather conditions and their influence over the decision of choosing a general activity to perform.

The context could be instantiated in different ways, perhaps as:

$$\mathbb{C}_1 = \big\{(\mathit{weather\_conditions}, 0.7), (\mathit{general\_activity}, 0.3)\big\}$$

or may be as

$$\mathbb{C}_2 = \big\{(\mathit{weather\_conditions}, 0.1), (\mathit{general\_activity}, 0.8)\big\}.$$

These contexts offer the same descriptors to compare arguments but each context assigns different weights to them: the descriptor *weather_conditions* is the most important in $\mathbb{C}_1$, while the descriptor *general_activity* is the most important in $\mathbb{C}_2$. Suppose that arguments $\langle \mathtt{A}, \delta_{\mathtt{A}} \rangle$ and $\langle \mathtt{B}, \delta_{\mathtt{B}} \rangle$ do not share the descriptor *weather_conditions*, but they share *general_activity* with different values reflecting that it is less important for the comparison under $\mathbb{C}_1$ and more important for the comparison under $\mathbb{C}_2$. In this case, the similarity between such arguments will be lower in the context $\mathbb{C}_1$ and higher in $\mathbb{C}_2$. Thus, the weight associated with a descriptor establishes a *relevance relation* over the elements of the context affecting how the comparison is computed.

In order to keep the definition as general as possible, no conditions are established to set the weight associated with the elements of the context. However, as a broad guide, we will discuss some ideas explored in the literature to carry out such valuations in different domains. In one way, we can analyze the valuations of descriptors from the same point of view as that of social voting in the valued-based argumentation frameworks [9,15,63], where the appraisals mainly depend on the user or the audience participating in the discussion. Thus, we embed the user's, or audience's, preferences in the framework, but no semantic analysis on descriptors is performed. On the other hand, we can say that the descriptors associated with the arguments are not isolated, and they could be related to each other, leading to a semantic network of concepts. Thus, we can propose a mathematical representation where nodes are topics, and the connection between the nodes (arcs) is the semantic connection between them. Then, an initial assessment can be achieved, according to the number of times which are used those descriptors in the discussion. A possible improvement of this idea is to apply a "page rank" technique in the descriptor ontology. Thus, the initial valuation can be strengthened by the semantic linkage between the descriptors through the application of the page ranking algorithm [31].

With the elements defined so far, we can compute the similarity between arguments as formalized below.

4.3. Computing similarity degree between arguments

Some recent works have studied the problem of similarity between arguments. In [62], the authors propose to find similar arguments, or argument facets, by detecting propositions paraphrased or labels that refer to a particular aspect of an argument, limiting to political and social dialogues. Rather than providing a mechanism to decide whether two arguments are similar or not, we propose to evaluate the values of argument's descriptors in a given context $\mathbb{C}$ to specify a *degree of similarity* between the arguments. For this, we examine how well the different descriptors in context $\mathbb{C}$ match: the higher the number of matching values per descriptor is, the higher the similarity degree will be, weighted by the relevance of each descriptor provided in the context definition. We establish that two arguments are incomparable given a context $\mathbb{C}$ when no descriptor associated with them belong to $\mathbb{C}$, note that a pair of arguments can be incomparable in a given $\mathbb{C}$, but could be compared in a different one.

In the following, given an enriched argument $\mathrm{A} = \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle$, we denote with $\mathcal{D}_{\mathrm{A}}$ the set of descriptors that appear in $\delta_{\mathtt{A}}$, i.e., $\mathcal{D}_{\mathrm{A}} = \{d \mid (d, \mathcal{V}^{\mathtt{A}}) \in \delta_{\mathtt{A}}\}$ and with $\mathcal{V}_d^{\mathtt{A}}$ the set of values associated to descriptor $d \in \mathcal{D}_{\mathrm{A}}$.

Definition 12 (Similarity coefficient for a descriptor)**.** Let $\mathbb{A}\mathrm{rg}$ be a set of enriched arguments, $\mathrm{A} = \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle$ and $\mathrm{B} = \langle \mathtt{B}, \delta_{\mathtt{B}} \rangle$ two enriched arguments in $\mathbb{A}\mathrm{rg}$, and $\mathbb{C}$ a context. We define the similarity coefficient for each descriptor $d \in \mathcal{D}_{\mathrm{A}} \cap \mathcal{D}_{\mathrm{B}} \cap \mathcal{D}_{\mathbb{C}}$, denoted $\mathsf{Coef}_d(\mathrm{A}, \mathrm{B})$, as follows:

$$\mathsf{Coef}_d(\mathrm{A}, \mathrm{B}) = \begin{cases} \frac{|\mathcal{V}_d^{\mathtt{A}} \cap \mathcal{V}_d^{\mathtt{B}}|}{|\overline{\mathcal{V}_d^{\mathtt{A}} \cap \mathcal{V}_d^{\mathtt{B}}}|} \cdot w_d & \text{if } |\overline{\mathcal{V}_d^{\mathtt{A}} \cap \mathcal{V}_d^{\mathtt{B}}}| \neq 0, \\ w_d & \text{otherwise.} \end{cases}$$

Definition 13 (Similarity degree between arguments)**.** Let $\mathbb{A}\mathrm{rg}$ be a set of enriched arguments, $\mathrm{A} = \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle$ and $\mathrm{B} = \langle \mathtt{B}, \delta_{\mathtt{B}} \rangle$ two enriched arguments in $\mathbb{A}\mathrm{rg}$, and $\mathbb{C}$ be a context. We define the similarity degree between arguments in a context $\mathbb{C}$, denoted $\mathsf{Sim}_{\mathbb{C}}$, as $\mathsf{Sim}_{\mathbb{C}} : \mathbb{A}\mathrm{rg} \times \mathbb{A}\mathrm{rg} \to [0, 1]$, such that:

$$\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) = \begin{cases} \alpha_n & \text{if } \mathcal{D}_{\mathrm{A}} \cap \mathcal{D}_{\mathrm{B}} \cap \mathcal{D}_{\mathbb{C}} = \{d_1, \dots, d_n\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $\alpha_1 = \mathsf{Coef}_{d_1}(\mathrm{A}, \mathrm{B})$ and $\alpha_i = \odot(\alpha_{i-1}, \mathsf{Coef}_{d_i}(\mathrm{A}, \mathrm{B}))$ with $2 \leqslant i \leqslant n$, and, $\odot$ should be either a *t-norm* satisfying the following properties: commutative, associative, monotony increasing, and having 1 as the neutral element; or $\odot$ should be a *c-norm*, satisfying the following properties: commutative, associative, monotony increasing, and having 0 as the neutral element.

Remark (Similarity coefficient for a descriptor)**.** The coefficient function operates considering the number of descriptors specified in the context. For this reason, the lower bound of the function is zero, while the upper bound is not possible to determine *a priori* but is finite and positive.

Remark (Similarity degree between arguments)**.** The similarity function is operated under a t-norm or c-norm, satisfying the commutative and associative properties. For this reason, the order in which the descriptors in the set $\{d_1, \dots, d_n\}$ are operated does not affect the final similarity measure; therefore, we can assume an arbitrary order to calculate this measure.

Note that $\mathsf{Sim}_{\mathbb{C}}$ is parameterized by the context $\mathbb{C}$ and the operator $\odot$. The rationale behind such a general definition for $\mathsf{Sim}_{\mathbb{C}}$ is to provide the possibility of customizing the operator by taking into consideration the application's requirements, according to the goal of each particular argument comparison.

For instance, it may be required to apply $\mathsf{Sim}_{\mathbb{C}}$ under several different contexts, or with different assumptions of how the combination of values for each descriptor in the context should be done, and these goals can be achieved independently from each other. In the following, sometimes we will not explicitly include one or both parameters whenever they are clear from the text. Furthermore, the proposed similarity function satisfies the following properties, as a consequence of being based on a t-norm or a c-norm function.

Let $\mathbb{C}$ be a context, then for all $\mathrm{A} = \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle$, $\mathrm{B} = \langle \mathtt{B}, \delta_{\mathtt{B}} \rangle \in \mathbb{A}\mathrm{rg}$, it holds that:

- *Positivity*: $\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) \geqslant 0$.
- *Symmetry*: $\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) = \mathsf{Sim}_{\mathbb{C}}(\mathrm{B}, \mathrm{A})$.
- *Maximality*: $\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{A}) \geqslant \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B})$.

Example 5. In our running example, we focus on the value that each of the descriptors takes for every argument, according to the following context:

$$\mathbb{C} = \{(weather\_conditions, 0.5), (general\_activity, 0.3), (frame\_of\_mind, 0.2)\}.$$

In Fig. 2, we summarize the similarity degrees corresponding to the set of enriched arguments shown in Example 3.

	C		
Sim_c	(*weather_conditions, 0.5*)	(*general_activity, 0.3*)	(*frame_of_mind, 0.2*)
Sim_c (A, B) = *max* (0, *max* (0, 0.2) = 0.2	0	$\frac{0}{3} \times 0.3 = 0$	$1 \times 0.2 = 0.2$
Sim_c (B, C) = *max* (0, *max* (0.15, 0.2) = 0.2	0	$\frac{1}{2} \times 0.3 = 0.15$	$\frac{1}{1} \times 0.2 = 0.2$
Sim_c (C, G) = *max* (0, *max* (0.3, 0) = 0.3	0	$\frac{2}{2} \times 0.3 = 0.3$	0
Sim_c (G, F) = *max* (0, *max* (0.2, 0) = 0.2	0	$\frac{2}{3} \times 0.3 = 0.2$	0
Sim_c (F, B) = *max* (0, *max* (0.6, 0) = 0.6	0	$2 \times 0.3 = 0.6$	0
Sim_c (D, C) = *max* (0.5, *max* (0.5, 0) = 0.5	$1 \times 0.5 = 0.5$	$\frac{0}{3} \times 0.3 = 0$	$\frac{0}{3} \times 0.2 = 0$
Sim_c (H, D) = *max* (0, *max* (0.3, 0) = 0.3	0	$1 \times 0.3 = 0.3$	$\frac{0}{2} \times 0.2 = 0$

Fig. 2. Similarity degrees for the set of enriched arguments from Example 3.

In this case, based on the argument descriptors established in the Example 3, an optimistic instantiation of the similarity function, i.e., a maximum t-conorm is used to define $\mathsf{Sim}_{\mathbb{C}}$. Figure 2 shows how the similarity degree between pairs of arguments is computed; For instance, in the sixth row, to calculate $\mathsf{Sim}_{\mathbb{C}}(\mathrm{D}, \mathrm{C})$ we have:

$\mathsf{Coef}_{weather\_conditions}(\mathrm{D}, \mathrm{C}) = 1 \times 0.5 = 0.5$, due to the set of values associated to this descriptor is unique (*rainy_day*) for both arguments.
$\mathsf{Coef}_{general\_activity}(\mathrm{D}, \mathrm{C}) = \frac{0}{3} \times 0.3 = 0$, since the set of values associated to this descriptor are different for the involved arguments (for the argument D the value is *keeping_house* while for the argument C are *go_out* and *shopping*).
$\mathsf{Coef}_{frame\_of\_mind}(\mathrm{D}, \mathrm{C}) = \frac{0}{3} \times 0.2 = 0$, because the set of values associated to this descriptor are different for the involved arguments (for the argument D the value is *depresing* while for the argument C are *happy* and *good_mood*).

Finally, we instantiate the similarity function with the maximum t-conorm as follows:

$$\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) = \begin{cases} \alpha_n & \text{if } \mathcal{D}_\mathrm{A} \cap \mathcal{D}_\mathrm{B} \cap \mathcal{D}_\mathbb{C} = \{d_1, \dots, d_n\} \text{ with } n \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where $\alpha_1 = \mathsf{Coef}_{d_1}(\mathrm{A}, \mathrm{B})$ and $\alpha_i = \max(\alpha_{i-1}, \mathsf{Coef}_{d_i}(\mathrm{A}, \mathrm{B}))$. Then, by successively computing the max between the three values obtained, we have $\mathsf{Sim}_{\mathbb{C}}(\mathrm{D}, \mathrm{C}) = 0.5$.

In the next section, we analyze how this similarity function can be applied to characterize the conflict and support relations in a BAF. In particular, we instantiate the similarity function to present an optimistic (or pessimistic) posture in the interpretation process that we perform over a BAF.

From the elements presented in this section, we address in the following section a mechanism to interpret the support and attack relationships in a BAF. Our leading purpose is to determine the cohesion of a set of supporting arguments and the controversy associated with a set of attacking arguments.

5. A similarity valued argumentation framework

We are now ready to extend the notions of support and attack from abstract argumentation frameworks to enriched arguments. Thus, two enriched arguments $\langle \mathtt{A}, \delta_\mathtt{A} \rangle$ and $\langle \mathtt{B}, \delta_\mathtt{B} \rangle$ are related through a support (attack) relation if and only if the underlying abstract arguments $\mathtt{A}$ and $\mathtt{B}$ are related through a support (attack) relation. Formally, this is as follows:

Definition 14 (Support and attacks between enriched arguments)**.** Let $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a bipolar argumentation framework, and $\mathbb{A}\mathrm{rg}$ be the set of all enriched arguments w.r.t. $\mathtt{Arg}$. Then,

– The attack relation between enriched arguments, denoted as $\mathbb{R}_a$, is defined as $\mathbb{R}_a : \mathbb{A}\mathrm{rg} \times \mathbb{A}\mathrm{rg}$ such that $(\langle \mathtt{A}, \delta_\mathtt{A} \rangle, \langle \mathtt{B}, \delta_\mathtt{B} \rangle) \in \mathbb{R}_a$ iff $(\mathtt{A}, \mathtt{B}) \in \mathtt{R}_a$; and
– the support relation between enriched arguments, denoted as $\mathbb{R}_s$, is defined as $\mathbb{R}_s : \mathbb{A}\mathrm{rg} \times \mathbb{A}\mathrm{rg}$ such that $(\langle \mathtt{A}, \delta_\mathtt{A} \rangle, \langle \mathtt{B}, \delta_\mathtt{B} \rangle) \in \mathbb{R}_s$ iff $(\mathtt{A}, \mathtt{B}) \in \mathtt{R}_s$.

Note that, these notions provide an extension associated with an underlying BAF Θ. Consequently, we use the bipolar argumentation graph associated to Θ to represent the enriched arguments and the relations between them.

In [21], the authors introduce a formalism to compute the similarity between two arguments, as long as they possess *descriptors* in common. The idea behind this proposal is that to determine the similarity degree between entities it is necessary that these entities have common and comparable aspects in the context where the comparison is made. However, it is possible to use the similarity between arguments to analyze the relations between them more effectively, by specifying a *controversy* evaluation of an attack relation, and a *cohesiveness* assessment of a support relation, respectively providing a measure of the strength of conflicts and supports. That is to say that the controversy or cohesion values associated with two arguments which are in an attack or support relationship should be high when these arguments both refer to the same aspect of the debate, in a specific sense. Following these intuitions, we examine these issues in the context of a BAF, introducing similarities between arguments as a device to enrich the argumentation analysis needed to distinguish those entities weakly related from those whose relationship is stronger.

Definition 15 (Cohesion and controversial operators)**.** Let $\Theta = \langle \mathtt{Arg}, \mathrm{R}_a, \mathrm{R}_s \rangle$ be a bipolar argumentation framework defined, $\mathbb{A}\mathrm{rg}$ the set of enriched argument corresponding to arguments in Θ, and $\mathbb{R}_a$ and $\mathbb{R}_s$ the corresponding attack and support relations for $\mathbb{A}\mathrm{rg}$. For any $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ and context $\mathbb{C}$, let $\mathsf{Sim}_{\mathbb{C}}$ be a similarity function for $\mathbb{C}$, then we have:

– The cohesion operator of $\mathbb{S}$, denoted as $\mathsf{Coh}_{\mathbb{C}}(\mathbb{S})$, is defined as $\mathsf{Coh}_{\mathbb{C}}(\mathbb{S}) \geqslant 0$, where:

$$\mathsf{Coh}_{\mathbb{C}}(\mathbb{S}) = \begin{cases} \beta_n & \text{if } \mathbb{R}_s^{\mathbb{S}} = \{(\mathrm{A}_1, \mathrm{B}_1), \dots, (\mathrm{A}_n, \mathrm{B}_n)\} \subseteq \mathbb{R}_s, \\ 0 & \text{otherwise,} \end{cases}$$

where $\beta_1 = \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_1, \mathrm{B}_1)$ and $\beta_i = \bigoplus(\beta_{i-1}, \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i))$ with $2 \leqslant i \leqslant n$.

– The controversy operator of $\mathbb{S}$, denoted as $\mathsf{Cont}_{\mathbb{C}}(\mathbb{S})$, is defined as $\mathsf{Cont}_{\mathbb{C}}(\mathbb{S}) \geqslant 0$, where:

$$\mathsf{Cont}_{\mathbb{C}}(\mathbb{S}) = \begin{cases} \gamma_n & \text{if } \mathbb{R}_a^{\mathbb{S}} = \{(\mathrm{A}_1, \mathrm{B}_1), \dots, (\mathrm{A}_n, \mathrm{B}_n)\} \subseteq \mathbb{R}_a, \\ 0 & \text{otherwise,} \end{cases}$$

where $\gamma_1 = \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_1, \mathrm{B}_1)$ and $\gamma_i = \bigotimes(\gamma_{i-1}, \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i))$ with $2 \leqslant i \leqslant n$.

As with the similarity function $\mathsf{Sim}_{\mathbb{C}}$, depending on the user modeling intentions, the operators $\oplus$ and $\otimes$ can be instantiated in different ways. These operators must satisfy commutativity, monotonicity, and associativity; for example, if we have a pessimistic posture to analyze the cohesiveness associated to a BAF, then $\oplus$ should be instantiated through a t-norm modeling the weak-link principle, while a c-norm implements an optimistic posture modeling a strong-composition principle. Of course, the inverse analysis can be performed over the operator $\otimes$ to compute the controversy value. Note that the use of t-norms is widely recognized in different application domains; for example, Dubois and Prade [30] use t-norms to model the uncertainty associated with subjective evidence in the analysis of legal cases. In another vein, Lukasiewicz and Straccia [58] analyze the efficiency of t-norms to model the uncertainty and the precision of the information in the Semantic Web domain. On the other hand, the c-norm operators are used in different application domains; for example, Grabisch et al. [30] present different ways to perform aggregation of arguments based on user preference through t-conorms. Another example is the work of Krause et al. [58], which introduces a series of criteria to perform aggregation of arguments supporting a particular conclusion in decision-making support systems, taking into account the uncertainty level associated with these arguments. The authors highlight the use of t-conorms as a sensible way to obtain the uncertainty level of a conclusion supported by multiple arguments. Furthermore, in a general direction, Budán et al. in [17,18,65] shows the application of these operations (c-norm and t-norm) within argumentation as a sensitive tool to diverse domains, such as decision support systems, recommendation systems, and legal systems.

In a given context, cohesiveness and controversy values are two distinct outcomes arising from the similarity function. They can be interpreted independently or jointly, depending on the domain in which they are applied. In this paper, cohesiveness represents the degree of support that an argument receives from a set of supporting arguments, while controversy expresses how much a particular set of arguments resists an argument according to pre-established comparison conditions. Thus, in a bipolar analysis, these notions are used, for example, to examine supported attacks where the support and conflict relations play a role together (see Fig. 5). Furthermore, when internal and external coherency is analyzed, we will use both concepts to identify the cohesion and controversy level associated with the argument set (see Fig. 7). Besides, the information provided by the enriched arguments is fundamental in the examination of the argumentative process.

We now define a *S-BAF*, which extends a BAF [10], incorporating the tools to interpret the relationship between the arguments in the argumentation process under a different light. In particular, this framework provides the means to compute the similarity between related arguments to decide which attacks (supports) must be considered in the discussion, given their degree of controversy (cohesion, respectively).

Definition 16 (Similarity-based Bipolar Argumentation Framework)**.** Given a bipolar argumentation framework $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ and a context $\mathbb{C}$, a Similarity-based Bipolar Argumentation Framework (*S-BAF*) is defined as a tuple

$$\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle,$$

where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ is the enriched bipolar argumentation framework, $\mathbb{A}\mathrm{rg}$ is the set of enriched arguments corresponding to arguments in $\mathtt{Arg}$, and $\mathbb{R}_s$ and $\mathbb{R}_a$ are the corresponding support and attack relations for $\mathbb{A}\mathrm{rg}$ provided by Θ; furthermore, $\mathsf{Sim}_{\mathbb{C}}$ is a similarity function for enriched arguments in Φ, and $\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}$ and $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}$ are, respectively, the cohesion operator and controversial operators in Φ, in the context $\mathbb{C}$.

The definition of a *S-BAF* is quite general so that we can use, in theory, different similarity functions for the comparisons among arguments. This proposal is based on the use of Sim as proposed in Definition 13 instantiated as necessary for specific contexts in which the function is applied and the $\odot$ operator. These parameters are left to be set according to the needs of the application user. In the rest of this paper, we assume that a single (but arbitrary) context is fixed beforehand for all comparisons performed in the analysis of the *S-BAF*, i.e., users select the set of descriptors by which they want to analyze all relationships between arguments. From now on, given a context $\mathbb{C}$, the definition of a *S-BAF* Φ will be expanded to $\langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$, unpacking $\overline{\Theta}$ when necessary to its components $\langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$.

It is important to note that, in the argumentation framework, once we determine (or fix) the similarity function that is to be used to compute the similarity between related arguments, we can affirm that the similarity values associated to these relations do not change even if the discussion is extended. That is, if the number of arguments that participate in the discussion increases, the computed similarity values calculated for the original set of arguments do not change giving a fixed similarity function. We formalize this idea in the next proposition.

Proposition 1. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the underlying Enriched BAF, and* $\Phi' = \langle \overline{\Theta}', \mathsf{Sim}'_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}'}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}'} \rangle$ *be an extended* S-BAF *where* $\overline{\Theta}' = \langle \mathbb{A}\mathrm{rg}', \mathbb{R}'_s, \mathbb{R}'_a \rangle$ *is an extended Enriched BAF such that* $\mathbb{A}\mathrm{rg} \subseteq \mathbb{A}\mathrm{rg}'$, $\mathbb{R}_s \subseteq \mathbb{R}'_s$, $\mathbb{R}_a \subseteq \mathbb{R}'_a$, *and the similarity functions are operated under* $\odot$. *We can say that, if* $\mathrm{A}, \mathrm{B} \in \mathbb{A}\mathrm{rg}$, *then* $\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) = \mathsf{Sim}'_{\mathbb{C}}(\mathrm{A}, \mathrm{B})$.

The result in Proposition 1 states that we can analyze an argumentation framework once, and if in the future we want to consider an expansion of it there will not be necessary to recompute all similarity values between the arguments in the previously existent relationships, i.e., as long as the similarity function does not change, it will suffice to compute the new ones. This result potentially gives us a way to work in the argumentation process dynamically and incrementally when necessary.

Under this new framework, we analyze how the attacks between the arguments must be considered in a Enriched BAF taking into account the similarity function defined in the previous section. Next, we discuss a possible way to classify attacks based on similarity, controversy, and cohesion. Informally at first, we use notions such as "high" and "weak" to refer to these degrees but we formally establish their proper meaning later on this section.

- *Direct attack*: An attack between two arguments A and B is evaluated considering the similarity measure associated with them. Thus, if the similarity between them is high (weak), then we can assume that the controversy between these arguments is high (weak) because they refer to related (unrelated) issues (see Fig. 3).
- *Supported attack*: Let $\mathrm{A}_1\ \mathbb{R}_1 \ldots \mathbb{R}_{n-1}\ \mathrm{A}_n$ be a sequence of enriched arguments, with $n \geqslant 3$, such that $\mathbb{R}_i = \mathbb{R}_s$, $1 \leqslant i \leqslant n-2$ and $\mathbb{R}_{n-1} = \mathbb{R}_a$. First, we analyze the cohesion value associated with the support chain that performs the attack (in this case, the chain is $\mathrm{A}_1\ \mathbb{R}_s \ldots \mathbb{R}_s\ \mathrm{A}_{n-1}$). Thus, if the cohesion is strong, the strength associated to the attack produced by the enriched argument A_1 is established according to the strength associated to the direct attack between A_{n-1} and A_n (strong or weak depending on the controversy value). On the other hand, if the cohesion is weak, then the attack produced by the enriched argument A_1 has associated a weak strength since the arguments involved in the support chain do not refer to similar topics (see Fig. 4).
- *Secondary attack*: Let $\mathrm{A}_1\ \mathbb{R}_1 \ldots \mathbb{R}_{n-1}\ \mathrm{A}_n$ be a sequence of enriched arguments, with $n \geqslant 3$, such that $\mathbb{R}_1 = \mathbb{R}_a$ and $\mathbb{R}_i = \mathbb{R}_s$, $2 \leqslant i \leqslant n-1$. First, the controversy associated with the direct attack between the enriched arguments A_1 and A_2 is analyzed based on the similarity between the involved arguments. Then, this attack is distributed considering to the cohesion value associated with the affected supporting chain (in this case the chain $\mathrm{A}_2\ \mathbb{R}_s \ldots \mathbb{R}_s\ \mathrm{A}_n$). Thus, if the cohesion is strong, then the attack from A_1 to A_n can be performed with it corresponding strength (strong or weak depending on the controversy value). On the other hand, if the cohesion is weak, then the attack only can be performed with a weak strength, since the arguments involved in the support chain not refer to similar topics (See Fig. 5).

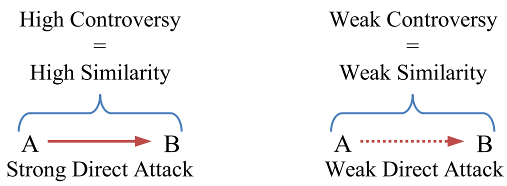


Fig. 3. Analysis of direct attacks under a similarity measure.

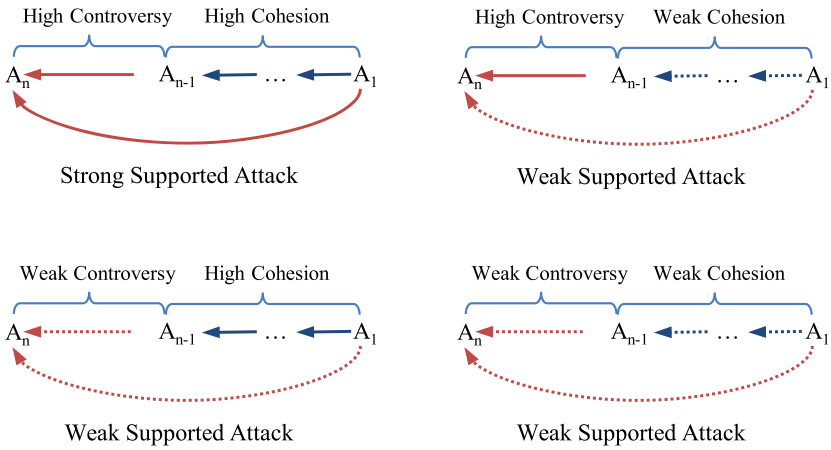


Fig. 4. Analysis of supported attacks under a similarity measure.

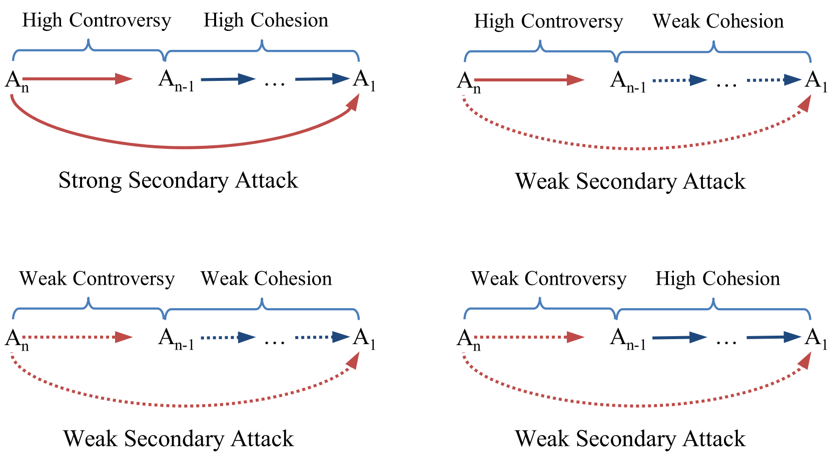


Fig. 5. Analysis of secondary attacks under a similarity measure.

In the analysis presented, we refer to a high or weak cohesion and controversial values. To this end, we can set a threshold that determines the minimal cohesion or controversial values associated with the arguments involved. Formally:

Definition 17 (Attack and support interpretation in *S-BAF*)**.** Let $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ be a *S-BAF* where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ is the enriched bipolar argumentation framework, and $\tau \in [0, 1]$ be a given threshold. Then:

- There is a *strong support* from A to B if $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_s$ and $\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\{\mathrm{A}, \mathrm{B}\}) \geqslant \tau$.
- There is a *weak support* from A to B if $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_s$ and $0 \leqslant \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\{\mathrm{A}, \mathrm{B}\}) < \tau$.
- There is a *strong direct attack* from A to B if $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$ and $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\{\mathrm{A}, \mathrm{B}\}) \geqslant \tau$.
- There is a *weak direct attack* from A to B if $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$ and $0 \leqslant \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\{\mathrm{A}, \mathrm{B}\}) < \tau$.

– There is a *strong supported attack* from A to B if there exists a sequence $A_1\ \mathbb{R}_1 \ldots \mathbb{R}_{n-1}\ A_n$, with $n \geqslant 3$, where $A_1 = A$ and $A_n = B$, such that $\mathbb{R}_i = \mathbb{R}_s$, $1 \leqslant i \leqslant n-2$ and $\mathbb{R}_n = \mathbb{R}_a$, verifying that $\mathsf{Coh}^{\Theta}_{\mathbb{C}}(\{A_1, \ldots, A_{n-1}\}) \geqslant \tau$ and $\mathsf{Cont}^{\Theta}_{\mathbb{C}}(\{A_{n-1}, A_n\}) \geqslant \tau$.
– There is a *weak supported attack* A to B if there exists a sequence $A_1\ \mathbb{R}_1 \ldots \mathbb{R}_{n-1}\ A_n$, with $n \geqslant 3$, where $A_1 = A$ and $A_n = B$, such that $\mathbb{R}_i = \mathbb{R}_s$, $1 \leqslant i \leqslant n-2$ and $\mathbb{R}_{n-1} = \mathbb{R}_a$, verifying that $0 \leqslant \mathsf{Coh}^{\Theta}_{\mathbb{C}}(\{A_1, \ldots, A_{n-1}\}) < \tau$ or $\mathsf{Cont}^{\Theta}_{\mathbb{C}}(\{A_{n-1}, A_n\}) < \tau$.
– There is a *strong secondary attack* A to B if there exists a sequence $A_1\ \mathbb{R}_1 \ldots \mathbb{R}_{n-1}\ A_n$, with $n \geqslant 3$, where $A_1 = A$ and $A_n = B$, such that $\mathbb{R}_1 = \mathbb{R}_a$ and $\mathbb{R}_i = \mathbb{R}_s$, $2 \leqslant i \leqslant n-1$ verifying that $\mathsf{Coh}^{\Theta}_{\mathbb{C}}(\{A_2, \ldots, A_n\}) \geqslant \tau$ and $\mathsf{Cont}^{\Theta}_{\mathbb{C}}(\{A_1, A_2\}) \geqslant \tau$.
– There is a *weak secondary attack* A to B if there exists a sequence $A_1\ \mathbb{R}_1 \ldots \mathbb{R}_{n-1}\ A_n$, with $n \geqslant 3$, where $A_1 = A$ and $A_n = B$, such that $\mathbb{R}_1 = \mathbb{R}_a$ and $\mathbb{R}_i = \mathbb{R}_s$, $2 \leqslant i \leqslant n-1$, verifying that $0 \leqslant \mathsf{Coh}^{\Theta}_{\mathbb{C}}(\{A_2, \ldots, A_n\}) < \tau$ or $\mathsf{Cont}^{\Theta}_{\mathbb{C}}(\{A_1, A_2\}) < \tau$.

Once the attacks are appropriately evaluated, we analyze the coherence of an enriched argument set. As in [10], the internal coherence is captured by the definition of *conflict free set*, and *external coherence* is captured with the notion of *safe* set. In our case, the classical conflict-free notion can be weakened by the existence of weak attacks, which allows a certain degree of tolerance to inconsistent information (see Fig. 6). So, we can specify a strong-conflict-free set (a set with no conflict), a τ-conflict-free set (only admitting weak-conflicts whereas the controversy value associated with this set does not exceed the limit τ), and a weak-conflict-free set (the more general set, since it is possible to admit any weak attacks in the domain). Furthermore, the classical notion of safety can also be weakened considering weak attacks and supports to an external argument. Note that, in Fig. 7, $\mathbb{S}_2'$ and $\mathbb{S}_3'$ are composed by the arguments used to perform a support for an external argument, while $\mathbb{S}_2''$ and $\mathbb{S}_3''$ have just the arguments involved in an attack (supported, secondary, or direct attack) over the external argument. Thus, we can specify a strong-safe set (there not exist arguments in the set attacking and supporting an external argument), τ-safe set (there are no arguments in the set performing both a strong attacking and a strong supporting, i.e., the cohesion value associated to the support chain does not exceed the threshold τ, whereas the controversy value associated to the set does not exceed the threshold τ, and a weak-safe set, i.e., there are no arguments in the set performing both a strong-attack and strong-support over an external argument. Formally:

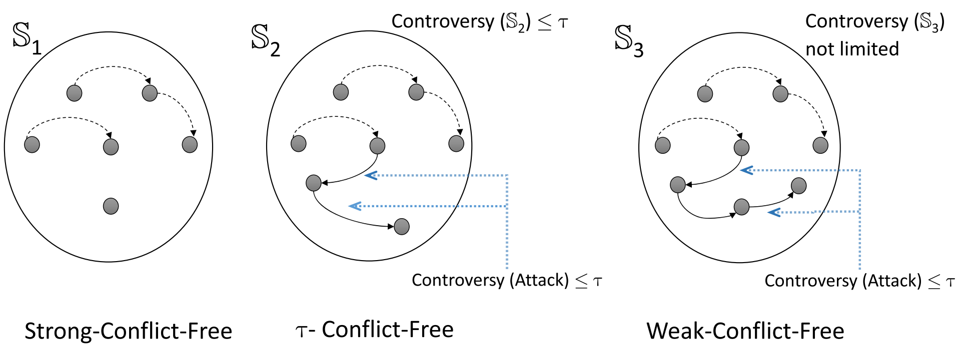


Fig. 6. Interpretation of conflict-free set (conflict-free strength).

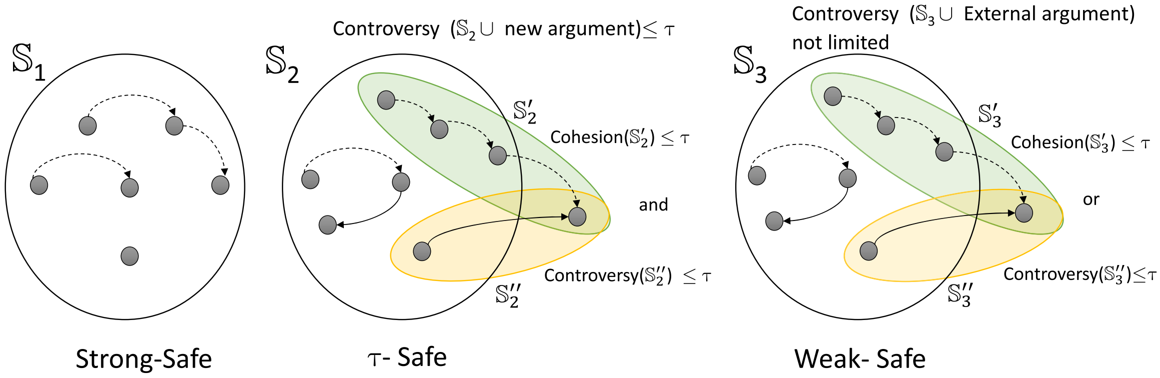


Fig. 7. Interpretation of safe set (safe strength).

Definition 18 (Conflict-freeness and safety property in *S-BAF*)**.** Given a *S-BAF* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ is the enriched bipolar argumentation framework, and $\tau \in [0, 1]$ be a given threshold. Then:

- $\mathbb{S}$ is a *strong-conflict-free* set iff there is no $\mathrm{A}, \mathrm{B} \in \mathbb{S}$ such that there exists a strong or weak attack (direct or supported or secondary attack) from A to B.
- $\mathbb{S}$ is a τ*-conflict-free* set iff there is no $\mathrm{A}, \mathrm{B} \in \mathbb{S}$ such that there exists a strong attack (direct or supported or secondary attack) from A to B and $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\mathbb{S}) > \tau$.
- $\mathbb{S}$ is a *weak-conflict-free* set iff there is no $\mathrm{A}, \mathrm{B} \in \mathbb{S}$ such that there exists a strong attack (direct or supported or secondary attack) from A to B.
- $\mathbb{S}$ is a *strong-safe* set iff there is no $\mathrm{A} \in \mathbb{A}\mathrm{rg}$ and no pair $\mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there exists a strong or weak attack (direct or supported or secondary attack) from B to A, and either there is a sequence of support from C to A, or $\mathrm{A} \in \mathbb{S}$.
- $\mathbb{S}$ is τ*-safe* set iff there is no $\mathrm{A} \in \mathbb{A}\mathrm{rg}$ and no pair $\mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there exists a strong attack (direct or supported or secondary attack) from B to A, $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\mathbb{S} \cup \mathrm{A}) > \tau$, and either there is a sequence of support from C to A such that $\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\mathrm{C}, \ldots, \mathrm{A}) > \tau$, or $\mathrm{A} \in \mathbb{S}$.
- $\mathbb{S}$ is *weak-safe* set iff there is no $\mathrm{A} \in \mathbb{A}\mathrm{rg}$ and no pair $\mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong attack (direct or supported or secondary attack) from B to A and either there is a sequence of support from C to A such that $\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\mathrm{C}, \ldots, \mathrm{A}) > \tau$, or $\mathrm{A} \in \mathbb{S}$.

Next, in Proposition 2, we show the relationships between the strong conflict-freeness and safety versions and the classical ones, while Proposition 3, shows the connection between the safety, conflict-freeness, and closure (under $\mathtt{R}_s$) properties.

Proposition 2. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ *be the underlying bipolar argumentation framework and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then:*

(i) *if* $\mathbb{S}$ *is strong-conflict-free, then* $\Pi_{\Theta}(\mathbb{S})$ *is conflict-free in* Θ*; and*
(ii) *if* $\mathbb{S}$ *is strong-safe, then* $\Pi_{\Theta}(\mathbb{S})$ *is safe in* Θ*,*

where $\Pi_{\Theta}(\mathbb{S}) = \{\mathtt{A} \mid \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle \in \mathbb{S}\}$.

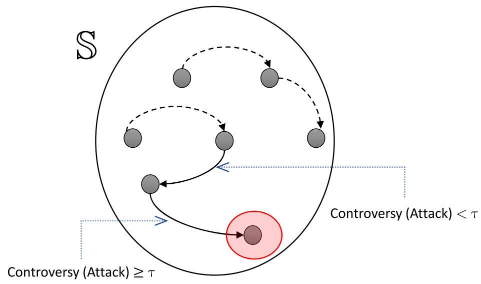


Fig. 8. Reinstatement of arguments with strong attacks in τ and weak-conflict-free sets.

Proposition 3. *Let* $\Phi = \langle \overline{\Theta}, \mathbb{A}\mathrm{rg}, \mathsf{Sim}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then:*

(i) *if* $\mathbb{S}$ *is strong-safe, then* $\mathbb{S}$ *is strong-conflict-free;*
(ii) *if* $\mathbb{S}$ *is* τ*-safe, then* $\mathbb{S}$ *is at least* τ*-conflict-free;*
(iii) *if* $\mathbb{S}$ *is weak-safe, then* $\mathbb{S}$ *is at least weak-conflict-free;*
(iv) *if* $\mathbb{S}$ *is strong-conflict-free and closed for* $\mathbb{R}_s$*, then* $\mathbb{S}$ *is strong-safe;*
(v) *if* $\mathbb{S}$ *is* τ*-conflict-free and closed under* $\mathbb{R}_s$*, then* $\mathbb{S}$ *is at least* τ*-safe; and*
(vi) *if* $\mathbb{S}$ *is weak-conflict-free and closed for* $\mathbb{R}_s$*, then* $\mathbb{S}$ *is at least weak-safe.*

Note that, conflict-freeness and safety sets are expanded from a classical one to be more "flexible" sets. In this sense, we can introduce weak attackers into the argument sets; consequently, these new arguments can be strong attackers of the previous ones (see Fig. 8). Thus, there is no strict inclusion relationship between the different types of conflict-free sets. The intuition is that, if we raise the tolerance to attack up to a threshold τ big enough, strong attackers may appear and eliminate arguments that are weakly defended.

Next, we present an example that will make clear the intuitions of this approach.

Example 6. Continuing with the Example 3 which is represented in Fig. 9, where the similarity between the involved arguments are computed in the Example 5 considering the context $\mathbb{C} = \{(\mathit{weather\_conditions}, 0.5), (\mathit{general\_activity}, 0.3), (\mathit{frame\_of\_mind}, 0.2)\}$. We now introduce for this special case a possible instantiation of the Cohesion and Controversial operators through the Einstein sum c-norm and the Probabilistic sum c-norm respectively, to analyze the relations between the presented arguments.

$$\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\mathbb{S}) = \begin{cases} \beta_n & \text{if } \mathbb{R}_s = \{(\mathrm{A}_1, \mathrm{B}_1), \dots, (\mathrm{A}_n, \mathrm{B}_n)\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $\beta_1 = \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_1, \mathrm{B}_1)$ and $\beta_i = \frac{\beta_{i-1} + \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i)}{1 - \beta_{i-1} \times \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i)}$ with $n \geqslant 2$.

$$\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\mathbb{S}) = \begin{cases} \gamma_n & \text{if } \mathbb{R}_a = \{(\mathrm{A}_1, \mathrm{B}_1), \dots, (\mathrm{A}_n, \mathrm{B}_n)\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $\gamma_1 = \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_1, \mathrm{B}_1)$ and $\gamma_i = \gamma_{i-1} + \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i) - \gamma_{i-1} \times \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i)$ with $n \geqslant 2$.

To facilitate the practical interpretation of the enunciated concepts, Fig. 9 will be useful:

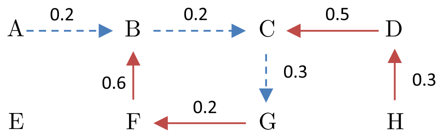


Fig. 9. Representation of attack and support relations in *S-BAF*.

Thus, for this example and considering a setting where a threshold $\tau = 0.4$, we identify the following relations:

H is a weak direct attacker for D, since $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(H, D)\}) = 0.3$;
G is a weak direct attacker for F, since $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(G, F)\}) = 0.2$;
D is a strong direct attacker for C, since $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(D, C)\}) = 0.5$;
C is a weak supported attacker for F, since $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(C, G)\}) = 0.3$;
F is a strong direct attacker for B, since $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(F, B)\}) = 0.6$;
B is a weak supported attacker for F, since $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(B, C), (C, G)\}) = 0.53$, however $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(G, F)\}) = 0.2$;
A is a weak supported attacker for F, since $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(A, B), (B, C)(C, G)\}) = 0.82$, but $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(G, F)\}) = 0.2$;
D is a weak supported attacker for G, since $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(D, C)\}) = 0.5$, however $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(C, G)\}) = 0.3$;
F is a weak secondary attacker for C, since $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(B, C)\}) = 0.2$;
F is a strong secondary attacker for G, since $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(B, C), (C, G)\}) = 0.53$, and $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(F, B)\}) = 0.6$.

Next, we examine the different conflict-free sets that are possible to obtain analyzing the presented framework: The set $\mathbb{S}_1 = \{A, B, C, E, G, H\}$ is strong-conflict-free since the controversy value associated to the set is zero. That is, there are not conflicting arguments in $\mathbb{S}_1$. On another hand, the set $\mathbb{S}_2 = \{A, B, D, E, H, G\}$ is τ-conflict-free since the controversy associated to this set is $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\mathbb{S}_2) = 0.3 < \tau$ (there exists a weak direct attack from H to D with $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(H, D)\}) = 0.3$), while the set $\mathbb{S}_3 = \{A, D, E, F, G, H\}$ is weak-conflict-free with a controversy value associated to $\mathbb{S}_3$ equal to 0.44 (H is a weak direct attacker for D and G is a weak direct attacker for F with $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(G, F)\}) = 0$).

On the other hand, analyzing the external coherence of the sets, we have that: $\mathbb{S}_1$ is a strong-safe set, while $\mathbb{S}_3$ is a weak-safe set since there exist arguments that belong to $\mathbb{S}_3$ supporting and attacking an external argument (there exists a strong direct attack from F to B with the controversy value is equal to 0.6 and a weak support from A to B with a cohesion value equal to 0.2). Also, $\mathbb{S}_2$ is a weak-safe set since there exist arguments that belong to $\mathbb{S}_2$ supporting and attacking an external argument (there exists a strong direct attack from D to C with the controversy value equal to 0.5 and a strong support from A to C through B with a cohesion value equal to 0.41).

Next, we extend the notions of defense for an argument with respect to a set of arguments, where we consider the attack relations introduced in Definition 17 instead.

Definition 19. Let $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ be a set of arguments, and $A \in \mathbb{A}\mathrm{rg}$ an argument. Then:

– $\mathbb{S}$ is a *strong defense* for A iff for all B $\in \mathbb{A}$rg such that if B is a strong or weak attacker (direct or supported or secondary attacker) of A then there exists C $\in \mathbb{S}$ where C is a strong attacker (direct or supported or secondary attacker) of B.
– $\mathbb{S}$ is a *weak defense* for A iff for all B $\in \mathbb{A}$rg such that if B is a strong or weak attacker (direct or supported or secondary attacker) of A then there exists C $\in \mathbb{S}$ where C is a weak attacker (direct or supported or secondary attacker) of B.

We present different definitions for admissibility, from the most general and strong to the most specific and weak. The most general is based on the classical notion of admissibility where only the attack relations are considered (both the strong and the weak ones). Then, we extended this notion taking into account external coherence considering the different attack and support degrees among arguments. Finally, external coherence is strengthened by requiring the closure of the $\mathtt{R}_s$ relation.

Definition 20. Let $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ be a *S-BAF* where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ is the enriched bipolar argumentation framework, and $\mathbb{S} \subseteq \mathbb{A}$rg be a set of enriched arguments. Then:

– $\mathbb{S}$ is *d-strong-admissible* if $\mathbb{S}$ is strong-conflict-free and strong defends all its elements.
– $\mathbb{S}$ is *d-τ-admissible* if $\mathbb{S}$ is τ-conflict-free and there exists a strong or weak defense for all its elements.
– $\mathbb{S}$ is *d-weak-admissible* if $\mathbb{S}$ is weak-conflict-free and there exists a strong or weak defense for all its elements, or $\mathbb{S}$ is strong-conflict-free and weak defends all its elements.
– $\mathbb{S}$ is *s-strong-admissible* if $\mathbb{S}$ is strong-safe and strong defends all its elements.
– $\mathbb{S}$ is *s-τ-admissible* if $\mathbb{S}$ is τ-safe and there exists a strong or weak defense for all its elements.
– $\mathbb{S}$ is *s-weak-admissible* if $\mathbb{S}$ is weak-safe and there exists a strong or weak defense for all its elements, or $\mathbb{S}$ is strong-safe and weak defends all its elements.
– $\mathbb{S}$ is *c-strong-admissible* if $\mathbb{S}$ strong-conflict-free, closed for $\mathbb{R}_s$ and strong defends all its elements.
– $\mathbb{S}$ is *c-τ-admissible* if $\mathbb{S}$ τ-conflict-free, closed for $\mathbb{R}_s$ and there exists a strong or weak defense for all its elements.
– $\mathbb{S}$ is *c-weak-admissible* if $\mathbb{S}$ weak-conflict-free, closed for $\mathbb{R}_s$ and there exists a strong or weak defense for all its elements, or $\mathbb{S}$ strong-conflict-free, closed for $\mathbb{R}_s$ and weak defends all its elements.

In this manner, admissibility becomes a characteristic of a set of arguments that can be evaluated from different perspectives. The most restrictive admissible sets are those that do not admit conflicts and that defend all their elements with values of controversy greater than the given threshold. A more flexible admissibility property is one in which a certain level of controversy associated with the set, limited by a threshold, is acceptable. In this case, the defense related to the arguments can oscillate between strong and weak. Finally, the most flexible set is one that allows the incorporation of conflicts where the controversy associated with them is strictly less than the threshold; i.e., the controversy is analyzed in each conflict between arguments. Note that, in the two last cases, the defense related to the arguments can oscillate between strong and weak.

Next, in Proposition 4, we show the relationships between the strong versions of admissibility and the classical ones. Furthermore, in Proposition 5, we identify how the conflict-freeness, safety, and closure property are related in the admissibility notion.

Proposition 4. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ *be the underlying bipolar argumentation framework and* $\mathbb{S} \subseteq \mathbb{A}$rg *be a set of enriched arguments. Then:*

(i) *if* $\mathbb{S}$ *is d-strong-admissible, then* $\Pi_{\Theta}(\mathbb{S})$ *is d-admissible in* Θ*;*
(ii) *if* $\mathbb{S}$ *is s-strong-admissible, then* $\Pi_{\Theta}(\mathbb{S})$ *is s-admissible in* Θ*; and*
(iii) *if* $\mathbb{S}$ *is c-strong-admissible, then* $\Pi_{\Theta}(\mathbb{S})$ *is c-admissible in* Θ*,*

where $\Pi_{\Theta}(\mathbb{S}) = \{\mathtt{A} \mid \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle \in \mathbb{S}\}$.

Proposition 5. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then:*

(i) *if* $\mathbb{S}$ *is c-strong-admissible, then* $\mathbb{S}$ *is s-strong-admissible;*
(ii) *if* $\mathbb{S}$ *is s-strong-admissible, then* $\mathbb{S}$ *is d-strong-admissible;*
(iii) *if* $\mathbb{S}$ *is c-*τ*-admissible, then* $\mathbb{S}$ *is at least s-*τ*-admissible;*
(iv) *if* $\mathbb{S}$ *is s-*τ*-admissible, then* $\mathbb{S}$ *is at least d-*τ*-admissible;*
(v) *if* $\mathbb{S}$ *is c-weak-admissible, then* $\mathbb{S}$ *is at least s-weak-admissible; and*
(vi) *if* $\mathbb{S}$ *is s-weak-admissible, then* $\mathbb{S}$ *is at least d-weak-admissible.*

From the notions of coherence (internal and external) and admissibility, it is possible to introduce different acceptability semantics. First, we define *strong-stable*, τ*-stable*, and *weak-stable* extensions. The *strong-stable* extension asks for a strong-conflict-free set and the existence of a strong attack for every element that does not belong to the extension. Meanwhile a *weak-stable* extension would a be maximal weak-conflict-free set, and the attack produced from the extension to each element that does not belong to it must be strong (direct, supported, or secondary attack) or, alternatively, be a strong-conflict-free set for which there must exist at least one element that does not belong to the set and it is only weak attacked (direct, supported, or secondary attack) by the set. In other words, a strong-stable extension guarantees inconsistency with all external arguments. A τ*-stable* extension is reasonably destabilized since it allows some degree of inconsistency as long as it does not exceed the determined threshold.

Definition 21. Let $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ be a *S-BAF* where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ is the enriched bipolar argumentation framework, and $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ be a set of enriched arguments. Then:

– $\mathbb{S}$ is a *strong-stable extension* of Φ if $\mathbb{S}$ is strong-conflict-free and for any A $\notin \mathbb{S}$, there is a strong attack (direct or supported or secondary attack) of A in $\mathbb{S}$.
– $\mathbb{S}$ is a τ*-stable extension* of Φ if $\mathbb{S}$ is a maximal τ-conflict-free such that for any A $\notin \mathbb{S}$ there is a strong or weak attack (direct or supported or secondary attack) of A in $\mathbb{S}$.
– $\mathbb{S}$ is a *weak-stable extension* of Φ if $\mathbb{S}$ is the maximal weak-conflict-free such that for any A $\notin \mathbb{S}$ there is a strong attack (direct or supported or secondary attack) of A in $\mathbb{S}$, or $\mathbb{S}$ is the maximal strong-conflict-free such that for any A $\notin \mathbb{S}$ there is an attack (direct or supported or secondary attack) of A in $\mathbb{S}$ and at least one argument A $\notin \mathbb{S}$ is only weak attacked by $\mathbb{S}$.

In general, an extension will be stable when it can ensure its internal and external coherence and the defense of its elements; this may have different implications as we explain in what follows. First, it is clear that the strong-stable extension is equivalent to a *BAF*'s stable extension. Since the controversy of the set cannot exceed the τ-value, in the τ-stable extension, it may be null (when the value of τ is very close to zero, or it is zero). If this is the case, there must be at least a weak attack over each external element. Being the conditions more flexible, it is possible that there exist no internal attacks in the weak-stable extension, in which case the set must weakly attack each external element. However, if the extension has internal attacks, where these individual attacks do not exceed the τ-value, it is sufficient that the set strongly attacks each external element. Next, in Proposition 6, we show the relationships between the strong versions of stable extension and the classical one. Furthermore, in Proposition 8 we establish the case in which a stable extension combined with a safety property results in a set that is closed under support, and the circumstances in which a stable extension combined with a closed under support condition result in a safety set.

Proposition 6. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ *be the underlying bipolar argumentation framework and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then, if* $\mathbb{S}$ *is strong-stable extension, then* $\Pi_\Theta(\mathbb{S}) = \{\mathtt{A} \mid \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle \in \mathbb{S}\}$ *is a stable extension in* Θ.

Next, in Proposition 7, we present the property of uniqueness associated to the strong-stable extensions as presented by Cayrol and Lagasquie-Schiex [22] for BAF.

Proposition 7. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ *be the underlying bipolar argumentation framework and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. We have that, if there exists a strong-stable extension* $\mathbb{S}$*, then* $\mathbb{S}$ *is unique.*

Proposition 8. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then:*

(i) *if* $\mathbb{S}$ *is a strong-stable extension, then* $\mathbb{S}$ *is strong-safe if and only if* $\mathbb{S}$ *is closed under* $\mathbb{R}_s$*; and*
(ii) *if* $\mathbb{S}$ *is* τ*-stable and closed under* $\mathbb{R}_s$*, then* $\mathbb{S}$ *is at least* τ*-safe;*
(iii) *if* $\mathbb{S}$ *is weak-stable and closed under* $\mathbb{R}_s$*, then* $\mathbb{S}$ *is at least weak-safe.*

In the following we propose a more fine-grained definition of preferred extensions.

Definition 22. Let $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ be a *S-BAF* where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ is the enriched bipolar argumentation framework, and $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ be a set of enriched arguments. Then:

- $\mathbb{S}$ is a *d-strong-preferred (resp. s-strong-preferred, c-strong-preferred) extension* of Φ if $\mathbb{S}$ is maximal (for set-inclusion) among the d-strong-admissible (resp. s-strong-admissible, c-strong-admissible) subsets of $\mathbb{A}\mathrm{rg}$.
- $\mathbb{S}$ is a *d-*τ*-preferred (resp. s-*τ*-preferred, c-*τ*-preferred) extension* of Φ if $\mathbb{S}$ is maximal (for set-inclusion) among the d-τ-admissible (resp. s-τ-admissible, c-τ-admissible) subsets of $\mathbb{A}\mathrm{rg}$.
- $\mathbb{S}$ is a *d-weak-preferred (resp. s-weak-preferred, c-weak-preferred) extension* of Φ if $\mathbb{S}$ is maximal (for set-inclusion) among the d-weak-admissible (resp. s-weak-admissible, c-weak-admissible) subsets of $\mathbb{A}\mathrm{rg}$.

Next, in Proposition 9, we show the relationships between the strong versions of preferred extensions and the classical ones, while in Corollay 1 we identify the conditions under which the strong-preferred extensions are equal to the classical preferred extensions. This result is relevant given that represents a bridge between both formalisms.

Proposition 9. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ *be the underlying bipolar argumentation frame-*

work, and $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then, if* $\mathbb{S}$ *is a d-strong-preferred (resp. s-strong-preferred, c-strong-preferred) extension, then* $\Pi_\Theta(\mathbb{S}) = \{\mathtt{A} \mid \langle \mathtt{A}, \delta_\mathtt{A}\rangle \in \mathbb{S}\}$ *is a subset of a d-preferred (resp. s-preferred, c-preferred) extension* $\mathtt{S}$ *in* Θ.

Note that, to ensure that there is an equivalence between a strong-preferred extension and a classical preferred extension, the relationships between the arguments in *S-BAF* must all be higher than the threshold. This condition is imposed mainly by the characterization of the notion of defense in *S-BAF*. That is, suppose that in *BAF*, there is a conflict-free set that defends all its arguments; in this scenario, we cannot differentiate the class of defenders that the same ones possess, while in *S-BAF* a strong defense is required.

Corollary 1. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_\mathbb{C}, \mathsf{Coh}_\mathbb{C}^{\overline{\Theta}}, \mathsf{Cont}_\mathbb{C}^{\overline{\Theta}}\rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a\rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s\rangle$ *be the underlying bipolar argumentation framework,* τ *be a threshold,* $\mathsf{Coh}_\mathbb{C}^{\overline{\Theta}}(\mathbb{A}\mathrm{rg}) > \tau$*, and* $\mathsf{Cont}_\mathbb{C}^{\overline{\Theta}}(\mathbb{A}\mathrm{rg}) > \tau$*. Under these conditions, if* $\mathbb{S}$ *is d-strong-preferred (resp. s-strong-preferred, c-strong-preferred) extension, then* $\Pi_\Theta(\mathbb{S}) = \{\mathtt{A} \mid \langle \mathtt{A}, \delta_\mathtt{A}\rangle \in \mathbb{S}\}$ *is a d-preferred (resp. s-preferred, c-preferred) extension in* Θ.

In Proposition 10, we identify how the conflict-freeness, safety, and closure property are related to the preferred extensions.

Proposition 10. *Let* $\Phi = \langle \Theta, \mathsf{Sim}_\mathbb{C}, \mathsf{Coh}_\mathbb{C}^{\overline{\Theta}}, \mathsf{Cont}_\mathbb{C}^{\overline{\Theta}}\rangle$ *be a* S-BAF *where* $\Theta = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a\rangle$ *is the enriched bipolar argumentation framework, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then:*

(i) *if* $\mathbb{S}$ *is a c-strong-preferred extension, then* $\mathbb{S}$ *is a s-strong-preferred extension;*
(ii) *if* $\mathbb{S}$ *is a s-strong-preferred extension, then* $\mathbb{S}$ *is a d-strong-preferred extension;*
(iii) *if* $\mathbb{S}$ *is a c-*τ*-preferred extension, then* $\mathbb{S}$ *is at least a s-*τ*-preferred extension;*
(iv) *if* $\mathbb{S}$ *is a s-*τ*-preferred extension, then* $\mathbb{S}$ *is at least a d-*τ*-preferred extension;*
(v) *if* $\mathbb{S}$ *is a c-weak-preferred extension, then* $\mathbb{S}$ *is a at least s-weak-preferred extension; and*
(vi) *if* $\mathbb{S}$ *is a s-weak-preferred extension, then* $\mathbb{S}$ *is at least a d-weak-preferred extension.*

Next, in Proposition 11 , we identify the relation between the preferred and stable extensions in *S-BAF*.

Proposition 11. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_\mathbb{C}, \mathsf{Coh}_\mathbb{C}^{\overline{\Theta}}, \mathsf{Cont}_\mathbb{C}^{\overline{\Theta}}\rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a\rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s\rangle$ *be the underlying bipolar argumentation framework, and* $\mathbb{S}, \mathbb{S}' \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then:*

(i) *if* $\mathbb{S}$ *is a c-strong-preferred (c-*τ*-preferred or c-weak-preferred extension) extension and* $\mathbb{S}'$ *is a strong-stable (*τ*-stable or weak-stable, respectively) extension, then* $\mathbb{S} \subseteq \mathbb{S}'$*;*
(ii) *if* $\mathbb{S}$ *is a s-strong-preferred (s-*τ*-preferred or s-weak-preferred) extension and* $\mathbb{S}'$ *is a strong-stable (* τ*-stable or weak-stable, respectively) extension, then* $\mathbb{S} \subseteq \mathbb{S}'$*; and*
(iii) *if* $\mathbb{S}$ *is the strong-stable extension and satisfy that* $\mathbb{S}$ *is strong-safe, then* $\mathbb{S}$ *is the unique c-strong-preferred and s-strong-preferred extension.*

Example 7. Continuing with Example 5, we now analyze the extension that characterize the bipolar argumentation framework. Hence, considering the set $\mathbb{S}_1 = \{\mathrm{E, A, B, C, G, H}\}$ we can say that $\mathbb{S}_1$ is a maximal strong-conflict-free set that weak defends the arguments C and B (there exists a weak direct attack from H to D and there exists a weak direct attack from G to F). Then, $\mathbb{S}_1$ is d-weak-admissible. Furthermore, $\mathbb{S}_1$ is s-weak-admissible, since it is strong-safe and contains weak defenses for its elements. Finally, $\mathbb{S}_1$ is c-weak-admissible, since it is closed under $\mathtt{R}_s$ and contains weak defenses for its elements. In consequence, $\mathbb{S}_1$ is a weak-stable extension, a d-weak-preferred extension, a s-weak-preferred extension, and a c-weak-preferred extension.

On the other hand, $\mathbb{S}_2 = \{A, B, D, E, H, G\}$ is a maximal τ-conflict-free set that weak defends the argument B (there exists a weak direct attack from G to F). So $\mathbb{S}_2$ is d-τ-admissible. However, $\mathbb{S}_2$ is not c-τ-admissible because it is not closed for $\mathtt{R}_s$. Also, $\mathbb{S}_2$ is s-weak-admissible since it is a weak-safe set that weak defends all its elements. Analyzing these characteristics, we have that $\mathbb{S}_2$ is a τ-stable extension, a d-τ-preferred extension, and a s-weak-preferred extension.

About the set $\mathbb{S}_3 = \{A, D, E, F, G, H\}$ we can deduce that it is a maximal weak-conflict free set that strong attacks all the external arguments. Thus, $\mathbb{S}_3$ is a weak-stable extension. Besides, in this case, it is assumed that the set strong defends all its elements, therefore is a d-weak-admissible and a s-weak-admissible extension. However, $\mathbb{S}_3$ is not closed under $\mathtt{R}_s$, whereby is not c-weak-admissible.

We have introduced a *Similarity-based Bipolar Argumentation Framework*, considering the context of the comparison between arguments, based on a set of descriptors that are common to the arguments which are being analyzed. In this way, we use a tool to enrich the representation of the relationships between the arguments, being able to determine and represent similarities between them, distinguishing among arguments weakly related from those whose relationship is stronger. In this direction, we determined a *cohesion value* between supporting arguments and a *controversy value* between conflicting arguments, as measures of the arguments relationships quality. Based on this analysis, we improve the acceptability process considering a threshold that specifies how permissive are we about the quality of relationships among arguments. More specifically, we can specify how cohesive the supporting arguments must be and how much controversy it is possible to admit in the accepted arguments set. In the argumentation domain, we are looking for those sets of arguments that possess a strong cohesive and a low controversy position. In this work, the notions of cohesion and controversy associated with an acceptable set of arguments are considered independently, despite working jointly. Indeed, only a threshold is taken into account to refine the argument relations and to perform the acceptability process. However, the study of the relation between these two concepts in our acceptability semantics is an interesting aspect that we will study in future works. Thus, we can refine the family of semantics introduced in this paper characterizing the acceptable arguments in the following way: acceptable arguments possess a higher degree of cohesiveness than of controversy, and on the other end of the spectrum, those arguments that possess a lower degree of cohesiveness than controversy are rejected. In the following section, we present a concrete example applying our formalism.

6. A case study

Now, let us examine the following scenario, in which Stephanie, concerned about the effects on health, wants to investigate the possible benefits coming from ingesting antioxidants supplements. Naturally, it is a simplified view of the problem but with enough elements as to show the use of the framework. Quoting from *MedlinePlus* (see footnote below):

> *"Antioxidants are human-made or natural substances that may prevent or delay some types of cell damage. Antioxidants are found in many foods, including fruits and vegetables. They are also available as dietary supplements."*

To decide whether it is advisable to consume these physio-chemicals, she examines the following information from the nutritional domain consulting different web pages.[1]

A Antioxidants may protect cells from free radical damage, improving the immune system. So, they are a healthy choice to incorporate to a diet.
B The dietary antioxidants can be damaging to your health if they are consumed for long periods, and may interact negatively with certain medications.
C If you incorporate antioxidants to your diet, you lower the risk of infections. Especially, vitamin 'A' which is important to improve the immune system.
D Increasing antioxidant intake is essential for optimum health, especially in today's polluted world because the body cannot keep up with antioxidant production. It is recommendable to consume frozen vegetables.
E As functional foods, dietary antioxidants help in the control of human diseases, protecting cells from free radical damage and reducing oxidative stress.
F Vitamin 'A' intake might increase cholesterol and raise the chance of vitamin 'A' poisoning. Therefore, this vitamin can affect the general immune system.
G Dietary antioxidants have been claimed to be the magic bullets for keeping a healthy living mostly without interfering with any medication. These physio-chemicals are found extensively in fruits and vegetables, particularly brightly colored varieties.

These examples describe the knowledge of a particular domain as a set of arguments. Additionally, we need to consider the logical interactions between them; then, we can analyze the set to decide which arguments survive in the light of these relationships. To do so, as part of our reasoning mechanisms, we can determine how strong the relations are, based on the arguments' similarities. In the proposed framework, the interaction between arguments can be given as supports (e.g., D $\mathbb{R}_s$ C) or as conflicts (e.g., E $\mathbb{R}_a$ B), and these relationships can be evaluated recognizing the context that provides the intended semantics for the arguments comparison process. Next, we will describe the scenario presented before in terms of support or attack relations, as evaluated in a *BAF*: Suppose that in these arguments, which are extracted from a webpage, Stephanie identifies the following aspects referred to *options* for the intake of the *antioxidant substance*, *sources* in which it is found, the *impact* of antioxidants on health, benefits, and disadvantages of its consumption. Thus, it is possible to determine the descriptors essential to these arguments as follows:

$$\delta_{\mathrm{A}} = \big\{\big(substance, \{antoxidants\}\big), \big(health\_impact, \{cells\_radical\_damage, inmune\_system\}\big)\big\},$$

$$\delta_{\mathrm{B}} = \big\{\big(substance, \{antoxidants\}\big), \big(health\_impact, \{state\_health, drug\_interactions\}\big)\big\},$$

$$\delta_{\mathrm{C}} = \big\{\big(substance, \{antoxidants, vitamin\_A\}\big), \big(health\_impact, \{infection\_risk, inmune\_system\}\big)\big\},$$

$$\delta_{\mathrm{D}} = \big\{\big(substance, \{antoxidants\}\big), \big(health\_impact, \{state\_health\}\big), \big(external\_factors, \{pollution\}\big), \big(source\_substance, \{vegetables\}\big)\big\},$$

$$\delta_{\mathrm{E}} = \big\{\big(substance, \{antoxidants\}\big), \big(health\_impact, \{cells\_radical\_damage, oxidative\_stress\}\big)\big\},$$

[1]For instance: https://medlineplus.gov/antioxidants.html, https://www.verywellfit.com/what-are-the-benefits-of-antioxidants-2507083, https://www.nutrex-hawaii.com/blogs/learn/benefits-of-antioxidants, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3249911/, https://www.webmd.com/vitamins/ai/ingredientmono-964/vitamin-a.

$\mathbb{A}\mathrm{rg} = \{A, B, C, D, E, F, G\}$
$\mathbb{R}_s = \{(C, A), (D, C), (E, A), (E, D), (D, G)\}$
$\mathbb{R}_a = \{(B, A), (B, D), (E, B), (F, C), (G, B)\}$

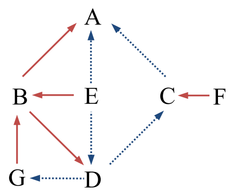


Fig. 10. Bipolar Argumentation model.

$$\delta_F = \{(substance, \{vitamin\_A\}), (health\_impact, \{immune\_system, cholesterol, poisoning\})\},$$

$$\delta_G = \{(substance, \{antoxidants\}), (health\_impact, \{state\_health, drug\_interactions\}), (source\_substance, \{vegetables\})\}.$$

Furthermore, consider the following context which Stephanie chooses to represent the issues that she considers essential to establish an adequate comparison. This context is ordered by a relevance degree that represents her preferences.

$$\mathbb{C} = \{(substance, 0.3), (health\_impact, 0.6), (external\_factors, 0.1)\}.$$

Once we have specified the arguments, the associated descriptor, and the context, we instantiate a similarity function, a cohesion operator, a controversy operator, and a threshold to analyze the bipolar argumentation framework depicted in Fig. 10. In this case, we establish a threshold $\tau = 0.45$ that represents a tolerant posture, linked to the similarity requirements between the arguments, in the specific application domain. Moreover, we use an optimistic instantiation of the similarity function employing the *probabilistic sum*, where we consider the similarity coefficient for each descriptor as it is presented in Definition 12, as follows:

$$\mathsf{Sim}_{\mathbb{C}}(A, B) = \begin{cases} \alpha_n & \text{if } \mathcal{D}_A \cap \mathcal{D}_B \cap \mathcal{D}_{\mathbb{C}} = \{d_1, \dots, d_n\} \text{ with } n \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where $\alpha_1 = \mathsf{Coef}_{d_1}(A, B)$ and $\alpha_i = \alpha_{i-1} + \mathsf{Coef}_{d_i}(A, B) - \alpha_{i-1} \times \mathsf{Coef}_{d_i}(A, B)$.

Thus, for instance, to calculate $\mathsf{Sim}_{\mathbb{C}}(A, B)$ we have:

- $\mathsf{Coef}_{substance}(A, B) = \frac{|\mathcal{V}_d^A \cap \mathcal{V}_d^B|}{|\mathcal{V}_d^A \cap \mathcal{V}_d^B|} \times 0.3 = 1 \times 0.3 = 0.3$, since the set of values associated to this descriptor is unique (*antoxidants*) for both arguments.
- $\mathsf{Coef}_{health\_impact}(A, B) = \frac{0}{4} \times 0.6 = 0$, considering that the set of values associated to this descriptor are different for the involved arguments (for the argument A the value are *cells_radical_damage* and *inmune_system* while for the argument B are *state_health* and *drug_interactions*).
- $\mathsf{Coef}_{external\_factors}(A, B) = 0$, because none of these arguments refer to this descriptor (the arguments A and B have associated the descriptors *substance* and *health_impact* but not the descriptor *external_factors*).

Then, by successively computing the *probabilistic sum* between the three values obtained, we have $\mathsf{Sim}_{\mathbb{C}}(A, B) = 0.3$. Following the method proposed, Fig. 11 describes the similarity degree between the arguments presented in this specific argumentation domain. Next, in Fig. 12, we present the bipolar argumentation graph where each relation is labeled with the corresponding similarity function in order

	$\mathbb{C}$		
$\mathsf{Sim}_{\mathbb{C}}$	(*substance*, 0.3)	(*health_impact*, 0.6)	(*external_factors*, 0.1)
$\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) = 0.3$ State 1: $0.3 + 0 - 0.3 \times 0 = 0.3$ State 2: $0.3 + 0 - 0.3 \times 0 = 0.3$	$1 \times 0.3 = 0.3$	$\frac{0}{4} \times 0.6 = 0$	0
$\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{C}) = 0.51$ State 1: $0.3 + 0.3 - 0.3 \times 0.3 = 0.51$ State 2: $0.51 + 0 - 0.51 \times 0 = 0.51$	$\frac{1}{1} \times 0.3 = 0.3$	$\frac{1}{2} \times 0.6 = 0.3$	0
$\mathsf{Sim}_{\mathbb{C}}(\mathrm{D}, \mathrm{C}) = 0.3$ State 1: $0.3 + 0 - 0.3 \times 0 = 0.3$ State 2: $0.3 + 0 - 0.3 \times 0 = 0.3$	$\frac{1}{1} \times 0.3 = 0.3$	$\frac{0}{3} \times 0.6 = 0$	0
$\mathsf{Sim}_{\mathbb{C}}(\mathrm{B}, \mathrm{D}) = 0.72$ State 1: $0.3 + 0.6 - 0.3 \times 0.6 = 0.72$ State 2: $0.72 + 0 - 0.72 \times 0 = 0.72$	$1 \times 0.3 = 0.3$	$\frac{1}{1} \times 0.6 = 0.6$	0
$\mathsf{Sim}_{\mathbb{C}}(\mathrm{E}, \mathrm{A}) = 0.4$ State 1: $0.3 + 0.15 - 0.3 \times 0.15 = 0.4$ State 2: $0.4 + 0 - 0.4 \times 0 = 0.4$	$1 \times 0.3 = 0.3$	$\frac{1}{2} \times 0.3 = 0.15$	0
$\mathsf{Sim}_{\mathbb{C}}(\mathrm{E}, \mathrm{B}) = 0.3$ State 1: $0.3 + 0 - 0.3 \times 0 = 0.3$ State 2: $0.3 + 0 - 0.3 \times 0 = 0.3$	$1 \times 0.3 = 0.3$	$\frac{0}{4} \times 0.6 = 0$	0
$\mathsf{Sim}_{\mathbb{C}}(\mathrm{E}, \mathrm{D}) = 0.3$ State 1: $0.3 + 0 - 0.3 \times 0 = 0.3$ State 2: $0.3 + 0 - 0.3 \times 0 = 0.3$	$1 \times 0.3 = 0.3$	$\frac{0}{3} \times 0.6 = 0$	0
$\mathsf{Sim}_{\mathbb{C}}(\mathrm{F}, \mathrm{C}) = 0.44$ State 1: $0.3 + 0.2 - 0.3 \times 0.2 = 0.44$ State 2: $0.44 + 0 - 0.44 \times 0 = 0.44$	$1 \times 0.3 = 0.3$	$\frac{1}{3} \times 0.6 = 0.2$	0
$\mathsf{Sim}_{\mathbb{C}}(\mathrm{D}, \mathrm{G}) = 0.75$ State 1: $0.3 + 0.6 - 0.3 \times 0.6 = 0.72$ State 2: $0.72 + 0.1 - 0.72 \times 0.1 = 0.75$	$1 \times 0.3 = 0.3$	$\frac{1}{1} \times 0.6 = 0.6$	$\frac{1}{1} \times 0.1 = 0.1$
$\mathsf{Sim}_{\mathbb{C}}(\mathrm{G}, \mathrm{B}) = 0.72$ State 1: $0.3 + 0.6 - 0.3 \times 0.6 = 0.72$ State 2: $0.72 + 0 - 0.72 \times 0 = 0.72$	$1 \times 0.3 = 0.3$	0.6	0

Fig. 11. Similarity degrees between considered arguments.

to perform a refined analysis. To do that, first we instantiate the cohesion operator through a Product t-norm, while the controversial operator is instantiated with the probabilistic sum t-conorm.

$$\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\mathbb{S}) = \begin{cases} \beta_n & \text{if } \mathbb{R}_s = \{(\mathrm{A}_1, \mathrm{B}_1), \ldots, (\mathrm{A}_n, \mathrm{B}_n)\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $\beta_1 = \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_1, \mathrm{B}_1)$ and $\beta_i = \beta_{i-1} \times \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i)$ with $n \geqslant 2$.

$$\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\mathbb{S}) = \begin{cases} \gamma_n & \text{if } \mathbb{R}_a = \{(\mathrm{A}_1, \mathrm{B}_1), \ldots, (\mathrm{A}_n, \mathrm{B}_n)\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $\gamma_1 = \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_1, \mathrm{B}_1)$ and $\gamma_i = \gamma_{i-1} + \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i) - \gamma_{i-1} \times \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i)$ with $n \geqslant 2$.

Thus, for this example, we identify the following relations considering the threshold $\tau = 0.45$:

B is a weak direct attack for A, since $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\{(\mathrm{B}, \mathrm{A})\}) = 0.3$;
B is a strong direct attack for D, since $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\{(\mathrm{B}, \mathrm{D})\}) = 0.72$;
E is a weak direct attack for B, since $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\{(\mathrm{E}, \mathrm{B})\}) = 0.3$;

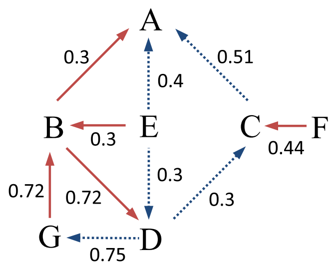


Fig. 12. S-BAF argumentation model.

F is a weak direct attack for C, since $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(F, C)\}) = 0.44$;
G is a strong direct attack for B, since $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(G, B)\}) = 0.72$;
F is a weak secondary attack for A, since $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(F, C)\}) = 0.44$ and $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(C, A)\}) = 0.51$;
B is a strong secondary attack for G, since $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(B, D)\}) = 0.72$ and $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(D, G)\}) = 0.75$;
D is a strong supported attack for B, since $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(D, G)\}) = 0.75$ and $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(G, B)\}) = 0.72$;
E is a weak supported attack for B, since $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\{(E, D), (D, G)\}) = 0.225$ and $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\{(G, B)\}) = 0.72$.

Next, we will describe how the acceptability process over a bipolar argumentative framework can be improved incorporating the extra knowledge obtained so far. To do that, we will analyze the maximal sets with respect to the inclusion operator, then we will consider the following sets:

– $\mathbb{S}_1 = \{F, E, G, D\}$, is a maximal strong-conflict-free set. Furthermore, $\mathbb{S}_1$ is a τ-safe set since there exists a weak direct attack from F to C with a controversy value of 0.44 and a weak support from D to C with a cohesion value of 0.3. We can observe that $\mathbb{S}_1$ is also a conflict-free set, as expressed in the item (i) of Proposition 2.
– $\mathbb{S}_2 = \{F, E, G, D, A, C\}$, is a maximal τ-conflict free set which satisfy the closure under $\mathtt{R}_s$. Note that it is not possible to add controversy to the set without exceeding τ. Thus, we can establish that the set is τ-safe, verified the point v) of the Proposition 3: "If $\mathbb{S}$ is τ-conflict-free and closed under $\mathtt{R}_s$, then $\mathbb{S}$ is at least τ-safe". Furthermore, due to the set is τ-safe, also is a τ-conflict-free set, as the item (ii) of Proposition 3 affirm.
– $\mathbb{S}_3 = \{A, B, C, E, F\}$, is a maximal weak-conflict-free set, since the controversy of the set is $0.73 > \tau$, but the controversy of each attack in $\mathbb{S}_3$ is less than τ and it is not possible to add individual attacks with a controversy $< \tau$. Besides, it is the maximal weak-safe set since exists $B \in \mathbb{S}_3$ that strong direct attacks $D \notin \mathbb{S}_3$ with a controversy value of 0.72 and exists a support from $E \in \mathbb{S}_3$ to $D \notin \mathbb{S}_3$ with a cohesion value of 0.3. So, this is consistent as expressed in the item (iii) of Proposition 3: "If $\mathbb{S}$ is weak-safe, then $\mathbb{S}$ is at least weak-conflict-free".

Regarding admissibility notion and considering the characteristics previously analyzed of each set, we have that:

– $\mathbb{S}_1$ is *d-strong-admissible*, since G strong direct attacks B, which attacks D instead. In this way, $\mathbb{S}_1$ strong defends all its elements. Besides, and considering the item (i) of Proposition 4, the set is d-admissible because is the maximal conflict-free set and defends all its elements. Also, the set is *s-τ-admissible*. On the other hand, $\mathbb{S}_1$ is not closed under $\mathtt{R}_s$, since there exists $E\ \mathtt{R}_s\ A$, and $A \notin \mathbb{S}_1$.

- $\mathbb{S}_2$ is *d-τ-admissible, s-τ-admissible*, and *c-τ-admissible* since, in addition to the characteristics analyzed above, G strong direct attacks B, which attacks A and D instead. So, $\mathbb{S}_2$ strong defends all its elements. In this real example, the items (iii) and (iv) of Proposition 5 are satisfied.
- $\mathbb{S}_3$ is a *d-weak-admissible* set, since B is the only element attacked by an external element G but, at the same time, B strong secondary attacks G. Besides, $\mathbb{S}_3$ is *s-weak-admissible* but it is not c-weak-admissible because it is not closed for $\mathtt{R}_s$, since there exists E $\mathtt{R}_s$ D, and D $\notin \mathbb{S}_3$. Given that these two conditions about admissibility are met in the set, the item (vi) of Proposition 5 is satisfied.

Analyzing the acceptability notions and considering the characteristics previously analyzed of each set we have that:

- $\mathbb{S}_1$ is a *strong-stable extension*, since the set is strong-conflict-free and it attacks each external element; besides, $\mathbb{S}_1$ is *d-strong-preferred* because it is the maximal d-strong-admissible set, and $\mathbb{S}_1$ is a *s-τ-preferred* due to it is the maximal s-τ-admissible set. From this analysis, and based on the Proposition 6, we can deduce that $\mathbb{S}_1$ is a stable extension. On the other hand, the analysis of item (iv) of Proposition 10 is especially important. We know that $\mathbb{S}_1$ is *d-strong-preferred* but it is a *maximal τ-safe* set. In this specific case, the set is a *d-strong-preferred* extension as a particular case of a *d-τ-preferred extension* since the value of τ is zero.
- $\mathbb{S}_2$ is a *τ-stable extension*, given that the set is τ-conflict-free and it attacks B which is the only external element. So, by (ii) of Proposition 8 we have that the set is closed for $\mathtt{R}_s$ and τ-safe. Also, $\mathbb{S}_2$ is *d-τ-preferred*, *s-τ-preferred* and *c-τ-preferred*. In this way, the items (iii) and (iv) of Proposition 10 are met in this practical example.
- $\mathbb{S}_3$ attacks each external element to the set, we have that the set is *a weak-stable, a d-weak-preferred, and s-weak-preferred extension* of Φ. Thus, the relation postulated in item (vi) of Proposition 10 is met in the set.

Note that this simple example illustrates the usefulness of a mechanism that allows introducing a degree of flexibility when analyzing the admissibility of arguments. Using the measure of similarity to express the support cohesion, the controversy of an attack, and the behavior of both measures combined in indirect attacks, it is possible to consider in the decision process those arguments that would have been discarded from the perspective of a classical *BAF*.

7. Related work

Several meaningful research efforts have motivated the ideas put together in the research introduced here. We have already described some of the works that are closely related to ours in Section 3, but there is additional literature that should be mentioned. Below, we will analyze this corpus divided into three main categories: the notion of similarity, the degradation or strengthening of the relationships between arguments, and the admission of inconsistency in the semantics of acceptable arguments.

7.1. Similarity in argumentation

The notion of *similarity* has been widely studied in terms of its meaning and usage [29,38,39,43,68]. The pioneering works address the treatment of the similarity as mathematical proportions to represent common behaviors and experimental effects in scientific models, to obtain patterns of causal relations in the phenomenal observed [43]. In [73], Sowa used an estimation function using and conceptual graphs, to find the differences between the arguments. On the other hand, in a refinement of Hesse's idea, Walton [78] points out that it is not easy to clearly define the comparison between arguments as this requires interpreting the similarities and differences between them at various levels. Cecchi et al. in [25] used a binary relation to characterize and formalize the behavior of a preference criterion among arguments; as an approximation of the similarity between them. In [74] the authors proposed a process for comparing two sets of words or entities to find semantic similarities between them using ontologies. The process of comparison is based on BOW (*Bag-Of-Word*) format, and used the *cosine similarity measures* to represent each entity by a set of weighted terms that describes it. In a similar direction, in [71], the authors presented a summary of the different metrics that can be used to determine the similarity between concepts over ontologies; for example, the authors mention *definitions-based measures* such as *Lesk Algorithm*, which compares two concepts according to the number of common words in their definitions. They use *structure-based measures* as Rada Distance based on ontology's graph representation, where the metric used to estimate the similarity between two concepts is the minimum number of edges connecting them.

More recently, in [61,62] the authors studied the use of paraphrased phrases in the summaries that provide websites, based on labels that represent essential aspects of arguments or *argument facet*. The authors used a regression model in machine learning to introduce the arguments and predict the similarity between them using a scalar value. Although the term *argument facet* is equivalent to the *context* in our work, the focus on [61,62] is different from our propose, since our principal contribution is at the conceptual level, regardless of the various techniques that can be used. Furthermore, both postures are complementary: in [42] the authors presented a review over different techniques to evaluate the similar textual semantic and proposed a *Align-and-Penalize Approach*, where two sentences are compared and the penalization is applied over syntactic contradictions or terms not aligned. In [37], the authors used techniques based on the manipulation of distances in graphs, to determine the similarity between text documents in the biomedical domain. It is a practical approach where the values of the descriptors or concepts needed to measure the semantic similarity, correspond to a vocabulary included in Systematized Nomenclature of Medicine-Clinical Terminology (SNOMED CT); Medical Subject Headings (MeSH); and the Unified Medical Language System (UMLS, [4]). This work demonstrates the importance of considering the similarity between arguments in specific domains. However, it is a restricted example based on specific biomedical concepts. In [11], the authors explore several similarity measures between logical arguments and define a very general function as a similarity measure; then, the authors define a set of principles that a similarity measure should satisfy. In this case, the arguments are expressed as logical entities denoted by pairs (support, conclusion), where the syntactic similarity between two arguments is calculated using an extension of the traditional Jaccard measure [47], comparing the elements of their correspondent pairs. This novel approach expressed arguments as logical entities, in a different form as the arguments considered in our proposal, which make, for instance, postulates not being directly translatable for our similarity functions.

7.2. *Relation refinement in argumentation*

Following a related idea that expands the representational capabilities of abstract argumentation frameworks, Martinez et al. [59,60] and later Dunne et al. in [34] proposed various refinements of the attack relation assigning *weight* to each attack indicating the relative force of this attack. In [24], Cayrol et al. convincingly held that argumentation is based on the exchange of interacting arguments and their valuation, leading to the adoption of the most acceptable ones and proposing the concept of "graduality" in the selection process of the best arguments to render progressive levels of acceptability. Furthermore, in a series of works [5–7,13], Amgoud et al. introduce a closely related line of research, where the authors define principles that a particular semantics should satisfy in a bipolar setting. Such principles are useful for defining reasonable semantics, for a better understanding of the design choices or foundations of each semantics, and for comparing pairs of semantics. Furthermore, the authors propose the definition of new gradual semantics for the subclass of non-maximal acyclic bipolar graphs, showing that it satisfies a set of principles. In [67], Potyka proposes a continuous dynamical system as a well-suited tool to analyze cyclic and acyclic bipolar argumentation frameworks, arriving at a convergence state. Towards this end, the author gives the conditions under successive procedures that can be transformed into well-defined dynamical systems; furthermore, the model satisfies a set of axiomatic properties that complement the existing approaches. In contrast with our work, this approach includes the possibility of treating cyclic bipolar argumentation frameworks; however, the author proposed a special and unique propagating function, where the valuations given by this function may not always represent real-world behavior. On another work, Hunter in [45] argues that when constructing an argument graph from informal arguments, where arguments are described in free text, it is often evident that there is uncertainty about whether some of the attacks hold. This situation might be produced because there is some expressed doubt that an attack holds or because there is some imprecision in the language used in the arguments. In this direction, Hunter assigns an uncertainty measure to the attacks in the argumentation framework; to do this assignment, the set of the spanning subgraphs of an argument graph is analyzed as a sample space, where a spanning subgraph contains all the arguments and a subset of the attacks of the argument graph. Finally, using the probability distribution over subgraphs, the probability that a set of arguments be admissible can be determined.

Our formalism shares the same goal, with particular attention over a well-defined notion of similarity defined between arguments that participate in an argumentative discussion. These similarity functions can be instantiated in different ways, and each one of them embodying a specific viewpoint, shifting the model from one perspective to another. Furthermore, we improve the argumentation framework analyzing the effectiveness of the support and conflict relations, considering the similarity measure associated with the participating arguments. Additionally, we present a new family of semantics refining the classic ones.

7.3. Tolerance to inconsistency in argumentation

An interesting investigation is presented by Bertossi et al. in [16], where the authors highlighted the need for inconsistency tolerance in order to create more robust and more intelligent computing systems. Inconsistency tolerance is being introduced on foundational technologies for identifying and analyzing inconsistency in information, for representing and reasoning with inconsistent information, for resolving inconsistencies in information, and for merging inconsistent information. In this direction, Dunne et al. in [33,34] propose a natural extension of Dung's well-known model of argument systems in which attacks are associated with a weight, indicating the relative strength of the attack. An important point of the system is the use of an inconsistency tolerance threshold, which allowed attacks to be discarded when the threshold was not exceeded. In this way, it was possible to perform a more refined scan of the framework giving useful solutions when conventional (unweighted) argument systems have none. Furthermore, Hunter and Konieczny in [46] present a review of the measures of information and contradiction, studying some potential practical applications. Specifically, they analyze two ideas: the importance of the conflict is reflected by the number of formulae in the knowledge base involved in deriving the contradiction (the more formulae needed, the less important the conflict), and the importance of the conflict is described by the number of atoms on which we have contradictory information. Thus, the notion of weights presented by Dunne et al. in [33,34] can be interpreted considering these intuitions. Additionally, Arieli in [14] presents a new kind of semantics for abstract argumentation frameworks, in which conflicting arguments can be accepted. The rationality behind such semantics is that, in real-world applications, there are situations in which contradictory arguments coexist in the same theory; moreover, the author correctly argues that the removal of contradictory indications in such theories may imply a loss of information and may lead to erroneous conclusions. Thus, in their framework they propose to: introduce self-referring argumentation and avoid information loss that may be caused by the conflict-freeness requirement (then, for instance, it may be better to accept extensions with a small fragment of conflicting arguments than, say, obtaining the empty extension), and refine the undecided case in standard labeling systems. The last point reflects (at least) two totally different situations: one case is that the reasoner abstains from having an opinion about an argument because there are no indications whether this argument should be accepted or rejected; another case that may cause a neutral opinion is that there are simultaneous considerations for and against accepting a specific argument. These two cases should be distinguishable since their outcomes may be different. Our research shares the same intuitions, relaxing the conditions imposed by classic formalisms to consider a certain tolerance to conflicting arguments to obtain resolutions to particular problematics, despite containing certain inconsistency levels. In our case, the support and conflict relation are analyzed, impacting on the notions of internal and external coherence associated with a set of arguments.

8. Conclusions and future works

In this work, we presented a novel mechanism for determining the similarity between arguments, based on labels or *descriptors* which represent aspects that an argument refers to. This mechanism involves a comparison process guided by a *context*, which determines the set of descriptors and a relevance relation among them. With these elements, we proposed a similarity function between arguments, which we later use to measure the *controversy* of attacks and the *cohesion* of supports in Bipolar Argumentation Frameworks. These valuations, applied to abstract argumentation frameworks allowed us to define different notions of argument sets acceptability, which is useful in determining how strong the support is for an attacking argument; this is relevant in domains where it is necessary to consider arguments that have weak opinions against but may be dismissed under existing argumentative approaches. Our proposal allows for a more fine-grained analysis among arguments relationships. It is important to note that Natural Language Processing techniques are beyond the scope of this research, but future advances in the state of the art of the subject will help generate more robust implementations of frameworks similar to the one presented.

Future work presents different possibilities, such as the development of an implementation of *S-BAFs*by using the existing DeLP [35,36,52] system as a basis.[2] The resulting implementation will be applied to different domains that require modeling decision support systems associated with context restrictions that model the users' preferences. Furthermore, we are working to generalize the function of similarity in such a way as to contemplate more broad domains. Another area of work to explore is the refinement of the formalization of a language of descriptors, more comprehensive than a set of descriptors. Formalizing such language can be especially useful when the argument mining techniques returns,

[2] See http://lidia.cs.uns.edu.ar/delp.

for example, values as *go_out* and *not_go_out*. However, it is highly dependent on the results from the argument mining, exceeding the treatment of similarity in itself. On the other hand, the similarity measure companied by the functions of cohesiveness and controversy presented will be evaluated as tools to automate the detection of support and attack between arguments. This work requires interaction with qualified experts, able to determine how successful relationships can be automatically found.

Appendix. Proofs

Proposition 1. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the underlying Enriched BAF, and* $\Phi' = \langle \overline{\Theta}', \mathsf{Sim}'_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}'}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}'} \rangle$ *be an extended* S-BAF *where* $\overline{\Theta}' = \langle \mathbb{A}\mathrm{rg}', \mathbb{R}'_s, \mathbb{R}'_a \rangle$ *is an extended Enriched BAF such that* $\mathbb{A}\mathrm{rg} \subseteq \mathbb{A}\mathrm{rg}'$, $\mathbb{R}_s \subseteq \mathbb{R}'_s$, $\mathbb{R}_a \subseteq \mathbb{R}'_a$, *and the similarity functions are operated under* $\odot$. *We can say that, if* $\mathrm{A}, \mathrm{B} \in \mathbb{A}\mathrm{rg}$, *then* $\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) = \mathsf{Sim}'_{\mathbb{C}}(\mathrm{A}, \mathrm{B})$.

Proof. By hypothesis, $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ is a *S-BAF* where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$, $\Phi' = \langle \overline{\Theta}', \mathsf{Sim}'_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}'}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}'} \rangle$ is an extended *S-BAF* where $\overline{\Theta}' = \langle \mathbb{A}\mathrm{rg}', \mathbb{R}'_s, \mathbb{R}'_a \rangle$, such that $\mathbb{A}\mathrm{rg} \subseteq \mathbb{A}\mathrm{rg}'$, $\mathbb{R}_s \subseteq \mathbb{R}'_s$, $\mathbb{R}_a \subseteq \mathbb{R}'_a$ and the similarity functions are operated under $\odot$. Since each argument comes with a set of descriptors and a mapping from these descriptors to values in a corresponding application domain (see Section 4.1) and furthermore, since by Definition 13 the similarity degree between two arguments given a context $\mathbb{C}$ is computed from the descriptors that these arguments have in common, we can conclude that $\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) = \mathsf{Sim}'_{\mathbb{C}}(\mathrm{A}, \mathrm{B})$ for all $\mathrm{A}, \mathrm{B} \in \mathbb{A}\mathrm{rg} \subseteq \mathbb{A}\mathrm{rg}'$. □

Proposition 2. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ *be the underlying bipolar argumentation framework and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then:*

(i) *if* $\mathbb{S}$ *is strong-conflict-free, then* $\Pi_\Theta(\mathbb{S})$ *is conflict-free in* Θ*; and*
(ii) *if* $\mathbb{S}$ *is strong-safe, then* $\Pi_\Theta(\mathbb{S})$ *is safe in* Θ*,*

where $\Pi_\Theta(\mathbb{S}) = \{\mathtt{A} \mid \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle \in \mathbb{S}\}$.

Proof. This demonstration will be held in two parts:

(i) *If* $\mathbb{S}$ *is strong-conflict-free, then* $\Pi_\Theta(\mathbb{S})$ *is conflict-free*. Suppose that $\Pi_\Theta(\mathbb{S}) \subseteq \mathtt{Arg}$ is not a conflict-free set. Hence, there exist two arguments $\mathtt{A}, \mathtt{B} \in \mathtt{S}$ such that $\mathtt{A}$ direct, supported or secondary attacks $\mathtt{B}$. Since, we have a projection from Θ to $\overline{\Theta}$ (see Definition 9 and Definition 14) and $\mathbb{S}$ is strong-conflict-free, then there do not exist two arguments $\mathrm{A}, \mathrm{B} \in \mathbb{S}$ such that there is a strong or weak attack (direct, supported, or secondary attack) from A to B, which contradicts the assumption.

(ii) *If* $\mathbb{S}$ *is strong-safe, then* $\Pi_\Theta(\mathbb{S})$ *is safe*. Suppose that $\Pi_\Theta(\mathbb{S}) \subseteq \mathtt{Arg}$ is not a safe set. Hence, there are three arguments $\mathtt{A} \in \mathtt{Arg}$ and $\mathtt{B}, \mathtt{C} \in \mathtt{S}$ such that there is an attack (direct, supported, or secondary attack) from $\mathtt{B}$ to $\mathtt{A}$, and either there is a sequence of support from $\mathtt{C}$ to $\mathtt{A}$ or $\mathtt{A} \in \mathtt{S}$. Since, we have a projection from Θ to $\overline{\Theta}$ (see Definition 9 and Definition 14) and $\mathbb{S}$ is strong-safe, then there do not exist three arguments $\mathrm{A} \in \mathbb{A}\mathrm{rg}$ and $\mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong or weak attack (direct, supported, or secondary attack) from B to A, and either there is a sequence of support from C to A, or $\mathrm{A} \in \mathbb{S}$, leading us to a contradiction. □

Proposition 3. *Let* $\Phi = \langle \overline{\Theta}, \mathbb{A}\mathrm{rg}, \mathsf{Sim}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then:*

(i) *if $\mathbb{S}$ is strong-safe, then $\mathbb{S}$ is strong-conflict-free;*
(ii) *if $\mathbb{S}$ is τ-safe, then $\mathbb{S}$ is at least τ-conflict-free;*
(iii) *if $\mathbb{S}$ is weak-safe, then $\mathbb{S}$ is at least weak-conflict-free;*
(iv) *if $\mathbb{S}$ is strong-conflict-free and closed for $\mathbb{R}_s$, then $\mathbb{S}$ is strong-safe;*
(v) *if $\mathbb{S}$ is τ-conflict-free and closed under $\mathbb{R}_s$, then $\mathbb{S}$ is at least τ-safe; and*
(vi) *if $\mathbb{S}$ is weak-conflict-free and closed for $\mathbb{R}_s$, then $\mathbb{S}$ is at least weak-safe.*

Proof. We separate the proof in six parts:

(i) *If $\mathbb{S}$ is strong-safe, then $\mathbb{S}$ is strong-conflict-free.* Suppose that $\mathbb{S}$ is not strong-conflict-free. Then, $\exists \mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong or weak attack (direct, supported, or secondary attack) from B to C. By hypothesis, $\mathbb{S}$ is strong-safe. Thus, by Definition 17, $\nexists \mathrm{A} \in \mathbb{A}\mathrm{rg}$ and $\nexists \mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong or weak attack (direct, supported, or secondary attack) from B to A, and either there is a sequence of support from C to A, or $\mathrm{A} \in \mathbb{S}$. This leads us to a contradiction; any argument that belongs to $\mathbb{S}$ or that is supported by $\mathbb{S}$ can not be attacked by $\mathbb{S}$ by the strong-safe definition.

(ii) *If $\mathbb{S}$ is τ-safe, then $\mathbb{S}$ is at least τ-conflict-free.* Suppose that $\mathbb{S}$ is not τ-conflict-free. Then, $\exists \mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong attack (direct, supported, or secondary attack) from B to C or $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\mathbb{S}) > \tau$. By hypothesis, $\mathbb{S}$ is τ-safe. Thus, by Definition 17, $\nexists \mathrm{A} \in \mathbb{A}\mathrm{rg}$ and $\nexists \mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong attack (direct, supported, or secondary attack) from B to A, $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\mathbb{S} \cup \mathrm{A}) > \tau$, and either there is a sequence of support from C to A such that $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\mathrm{C}, \dots, \mathrm{A}) > \tau$, or $\mathrm{A} \in \mathbb{S}$. This lead us to a contradiction, since any argument that belongs to $\mathbb{S}$ can not be strong attacked by $\mathbb{S}$, it only can be weak attacked by $\mathbb{S}$ under the condition $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\mathbb{S}) \leqslant \tau$, which is satisfied.

(iii) *If $\mathbb{S}$ is weak-safe, then $\mathbb{S}$ is at least weak-conflict-free.* Suppose that $\mathbb{S}$ is not weak-conflict-free. Then, $\exists \mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong attack (direct, supported, or secondary attack) from B to C. By hypothesis, $\mathbb{S}$ is weak-safe. Thus, by Definition 17, $\nexists \mathrm{A} \in \mathbb{A}\mathrm{rg}$ and $\nexists \mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong attack (direct, supported, or secondary attack) from B to A, and either there is a sequence of support from C to A such that $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\mathrm{C}, \dots, \mathrm{A}) > \tau$, or $\mathrm{A} \in \mathbb{S}$. This leads us to a contradiction, since any argument that belongs to $\mathbb{S}$ can not be strong attacked by $\mathbb{S}$.

(iv) *If $\mathbb{S}$ is strong-conflict-free and closed for $\mathbb{R}_s$, then $\mathbb{S}$ is strong-safe.* Suppose that $\mathbb{S}$ is not strong-safe. Thus, $\exists \mathrm{A} \in \mathbb{A}\mathrm{rg}$ and $\exists \mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong or weak attack (direct or supported or secondary attack) from B to A, and either there is a sequence of support from C to A, or $\mathrm{A} \in \mathbb{S}$. By hypothesis, $\mathbb{S}$ is strong-conflict-free. Thus, by Definition 17, $\nexists \mathrm{B}, \mathrm{A} \in \mathbb{S}$ such that there is a strong or weak attack (direct or supported or secondary attack) from B to A. Furthermore, the closure property establishes that $\forall \mathrm{A} \in \mathbb{S}$, $\forall \mathrm{B} \in \mathbb{A}\mathrm{rg}$ if $\mathrm{A}\ \mathbb{R}_s\ \mathrm{B}$ then $\mathrm{B} \in \mathbb{S}$. This leads us to a contradiction, since any argument supported by $\mathbb{S}$ belongs to $\mathbb{S}$ and it is strong-conflict-free.

(v) *If $\mathbb{S}$ is τ-conflict free and closed for $\mathbb{R}_s$, then $\mathbb{S}$ is at least τ-safe.* Suppose that $\mathbb{S}$ is not τ-safe. Thus, $\exists \mathrm{A} \in \mathbb{A}\mathrm{rg}$ and $\exists \mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong attack (direct or supported or secondary attack) from B to A, $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\mathbb{S} \cup \mathrm{A}) > \tau$, and either there is a sequence of support from C to A such that $\mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}(\mathrm{C}, \dots, \mathrm{A}) > \tau$, or $\mathrm{A} \in \mathbb{S}$. By hypothesis, $\mathbb{S}$ is τ-conflict free. Thus, by Definition 17, $\nexists \mathrm{A}, \mathrm{B} \in \mathbb{S}$ such that there is a strong attack (direct or supported or secondary attack) from A to B with $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\mathbb{S}) > \tau$, and $\forall \mathrm{A} \in \mathbb{S}$. Furthermore, the closure property establishes that $\forall \mathrm{B} \in \mathbb{A}\mathrm{rg}$ if $\mathrm{A}\ \mathbb{R}_s\ \mathrm{B}$ then $\mathrm{B} \in \mathbb{S}$. This leads us to a contradiction, since any argument supported by $\mathbb{S}$ belongs to $\mathbb{S}$ and it is τ-conflict-free satisfying that $\mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}}(\mathbb{S}) \leqslant \tau$.

(vi) *If $\mathbb{S}$ is weak-conflict free and closed for $\mathbb{R}_s$, then $\mathbb{S}$ is at least weak-safe.* Suppose that $\mathbb{S}$ is not weak-safe. Thus, $\exists \mathrm{A} \in \mathbb{A}\mathrm{rg}$ and $\exists \mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong attack (direct or supported or secondary attack) from B to A and either there is a sequence of support from C to A such that $\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\mathrm{C}, \dots, \mathrm{A}) > \tau$, or $\mathrm{A} \in \mathbb{S}$. By hypothesis, $\mathbb{S}$ is weak-conflict free. Thus, by Definition 17, $\nexists \mathrm{A}, \mathrm{B} \in \mathbb{S}$ such that there is a strong attack (direct or supported or secondary attack) from A to B. Furthermore, the closure property establishes that $\forall \mathrm{A} \in \mathbb{S}, \forall \mathrm{B} \in \mathbb{A}\mathrm{rg}$ if $\mathrm{A}\ \mathbb{R}_s\ \mathrm{B}$ then $\mathrm{B} \in \mathbb{S}$. This leads us to a contradiction, since any argument supported by $\mathbb{S}$ belongs to $\mathbb{S}$ and it is weak-conflict-free allowing only weak attacks. $\square$

Proposition 4. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ *be the underlying bipolar argumentation framework and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then:*

(i) *if* $\mathbb{S}$ *is d-strong-admissible, then* $\Pi_\Theta(\mathbb{S})$ *is d-admissible in* Θ*;*
(ii) *if* $\mathbb{S}$ *is s-strong-admissible, then* $\Pi_\Theta(\mathbb{S})$ *is s-admissible in* Θ*; and*
(iii) *if* $\mathbb{S}$ *is c-strong-admissible, then* $\Pi_\Theta(\mathbb{S})$ *is c-admissible in* Θ*,*

where $\Pi_\Theta(\mathbb{S}) = \{\mathtt{A} \mid \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle \in \mathbb{S}\}$.

Proof. We separate the proof in three parts:

(i) *If* $\mathbb{S}$ *is d-strong-admissible, then* $\Pi_\Theta(\mathbb{S})$ *is d-admissible in* Θ. Suppose that $\Pi_\Theta(\mathbb{S})$ is not d-admissible in Θ. Hence, $\exists \mathtt{A}, \mathtt{B} \in \mathtt{S}$ such that $\mathtt{A}$ attacks $\mathtt{B}$, or $\mathtt{A}$ is attacked by an argument $\mathtt{C} \notin \mathtt{S}$. Then, $\mathtt{A}$ is not defended by $\mathtt{S}$ (there does not exist an argument in $\mathtt{S}$ attacking $\mathtt{C}$). First, we know that through Definition 9 and Definition 14 we have a projection from Θ to $\overline{\Theta}$. Furthermore, by hypothesis, we know that $\mathbb{S}$ is d-strong-admissible. Thus, by Definition 19, $\mathbb{S}$ is strong-conflict-free and strong defends all its elements. Then, $\nexists \mathrm{A}, \mathrm{B} \in \mathbb{S}$ such that there is a strong or weak attack (direct or supported or secondary attack) from A to B. Furthermore, $\forall \mathrm{A} \in \mathbb{S}$ if $\forall \mathrm{B} \in \mathbb{A}\mathrm{rg}$ such that B is a strong or weak attacker (direct or supported or secondary attacker) of A, then $\exists \mathrm{C} \in \mathbb{S}$ where C is a strong attacker (direct or supported or secondary attacker) of B. Leading us to a contradiction.

(ii) *If* $\mathbb{S}$ *is s-strong-admissible, then* $\Pi_\Theta(\mathbb{S})$ *is s-admissible in* Θ. Suppose that $\Pi_\Theta(\mathbb{S})$ is not s-admissible in Θ. Hence, an argument $\mathtt{A} \in \mathtt{Arg}$, is supported for a sequence of arguments $\mathtt{B}_1, \mathtt{B}_2, \dots, \mathtt{B}_n$ that all belong to $\mathtt{S}$ or $\mathtt{A} \in \mathtt{S}$, and there exists an attack (direct or supported or secondary attack) from an argument $\mathtt{C} \in \mathtt{S}$ to $\mathtt{A}$. First, we know that through Definition 9 and Definition 14 we have a projection from Θ to $\overline{\Theta}$. Furthermore, by hypothesis, $\mathbb{S}$ is s-strong-admissible. Thus, by Definition 19, $\mathbb{S}$ is strong-safe and strong defends all its elements. Then, $\nexists \mathrm{A} \in \mathbb{A}\mathrm{rg}$ and $\nexists \mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong or weak attack (direct or supported or secondary attack) from B to A, and either there is a sequence of support from C to A where the cohesion associated to such sequence exceeds the threshold τ, or $\mathrm{A} \in \mathbb{S}$. Furthermore, $\forall \mathrm{A} \in \mathbb{S}$ if $\forall \mathrm{B} \in \mathbb{A}\mathrm{rg}$ such that B is a strong or weak attacker (direct, supported or secondary attacker) of A, then that $\exists \mathrm{C} \in \mathbb{S}$ where C is a strong attacker (direct or supported or secondary attacker) of B. Leading us to a contradiction.

(iii) *If* $\mathbb{S}$ *is c-strong-admissible, then* $\Pi_\Theta(\mathbb{S})$ *is c-admissible in* Θ. Suppose that $\Pi_\Theta(\mathbb{S})$ is not c-admissible in Θ. Hence, there exist two arguments $\mathtt{A}$ and $\mathtt{B}$ in $\mathtt{S}$ such that they are in conflict, or $\mathtt{A}$ is attacked by an argument $\mathtt{C}$ that does not belong to $\mathtt{S}$, where $\mathtt{A}$ is not defended by $\mathtt{S}$ (there are not arguments in $\mathtt{S}$ attacking $\mathtt{C}$), or $\mathtt{A}$ support $\mathtt{B}$ where $\mathtt{B}$ does not belong to $\mathtt{S}$. First, we know that through Definition 9 and Definition 14 we have a projection from Θ to $\overline{\Theta}$. Furthermore, by hypothesis, $\mathbb{S}$ is c-strong-admissible. Thus, by Definition 19, $\mathbb{S}$ is strong-conflict-free, closed by $\mathtt{R}_s$ and strong defends all of its elements. Then, $\nexists \mathrm{A}, \mathrm{B} \in \mathbb{S}$ such that there is a strong or weak attack (direct or supported or secondary attack) from A to B. Furthermore, $\forall \mathrm{A} \in \mathbb{S}$ if $\forall \mathrm{B} \in \mathbb{A}\mathrm{rg}$ such that B is a strong or weak attacker (direct, supported or secondary attacker) of A, then $\exists \mathrm{C} \in \mathbb{S}$ where C is a strong attacker (direct or supported or secondary attacker) of B. In addition, $\forall \mathrm{A} \in \mathbb{S}$, $\forall \mathrm{B} \in \mathbb{A}\mathrm{rg}$ if A $\mathbb{R}_s$ B then $\mathrm{B} \in \mathbb{S}$. Leading us to a contradiction. □

Proposition 5. *Let* $\Phi = \langle \overline{\Theta}, \mathbb{A}\mathrm{rg}, \mathsf{Sim}, \mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}, \mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of arguments. Then:*

(i) *if* $\mathbb{S}$ *is c-strong-admissible, then* $\mathbb{S}$ *is s-strong-admissible;*
(ii) *if* $\mathbb{S}$ *is s-strong-admissible, then* $\mathbb{S}$ *is d-strong-admissible;*
(iii) *if* $\mathbb{S}$ *is c-*τ*-admissible, then* $\mathbb{S}$ *is at least s-*τ*-admissible;*
(iv) *if* $\mathbb{S}$ *is s-*τ*-admissible, then* $\mathbb{S}$ *is at least a d-*τ*-admissible;*
(v) *if* $\mathbb{S}$ *is c-weak-admissible, then* $\mathbb{S}$ *is at least s-weak-admissible; and*
(vi) *if* $\mathbb{S}$ *is s-weak-admissible, then* $\mathbb{S}$ *is at least a d-weak-admissible.*

Proof. We separate the proof in six parts:

(i) *If* $\mathbb{S}$ *is c-strong-admissible, then* $\mathbb{S}$ *is s-strong-admissible.* If $\mathbb{S}$ is c-strong-admissible, then $\mathbb{S}$ is strong-conflict-free, closed by $\mathtt{R}_s$ and strong defends all its elements. Furthermore, by Proposition 3(iv), all strong-conflict-free set closed by $\mathtt{R}_s$ is a strong-safe set. Thus, $\mathbb{S}$ is a strong-safe set that strong defends all its elements, corresponding to the notion of s-strong-admissible.

(ii) *If* $\mathbb{S}$ *is s-strong-admissible, then* $\mathbb{S}$ *is d-strong-admissible.* If $\mathbb{S}$ is s-strong-admissible, then $\mathbb{S}$ is strong-safe and strong defends all its elements. Furthermore, by Proposition 3(i), all strong-safe set is a strong-conflict-free set. Thus, $\mathbb{S}$ is a strong-conflict-free set that strong defends all its elements, corresponding to the notion of d-strong-admissible.

(iii) *If* $\mathbb{S}$ *is c-*τ*-admissible, then* $\mathbb{S}$ *is at least s-*τ*-admissible.* If $\mathbb{S}$ is c-τ-admissible, then $\mathbb{S}$ is τ-conflict-free, closed by $\mathtt{R}_s$ and there exists a strong or weak defense for all its elements. Furthermore, by Proposition 3(v), all τ-conflict-free set closed by $\mathtt{R}_s$ is at least a τ-safe set. Thus, $\mathbb{S}$ is a τ-safe set where there exists a strong or weak defense for all its elements, corresponding to the notion of s-τ-admissible.

(iv) *If* $\mathbb{S}$ *is s-*τ*-admissible, then* $\mathbb{S}$ *is at least a d-*τ*-admissible.* If $\mathbb{S}$ is s-τ-admissible, then $\mathbb{S}$ is τ-safe and there exists a strong or weak defense for all its elements. Furthermore, by Proposition 2(ii), all τ-safe is at least a τ-conflict-free set. Thus, $\mathbb{S}$ is a τ-conflict-free set where there exists a strong or weak defense for all its elements, corresponding to the notion of d-τ-admissible.

(v) *If* $\mathbb{S}$ *is c-weak-admissible, then* $\mathbb{S}$ *is at least s-weak-admissible.* If $\mathbb{S}$ is c-weak-admissible, then $\mathbb{S}$ is weak-conflict-free, closed by $\mathtt{R}_s$ and weak defends all its elements. Furthermore, by Proposition 3(vi), all weak-conflict-free set closed by $\mathtt{R}_s$ is at least a weak-safe set. Thus, $\mathbb{S}$ is a weak-safe set that weak defends all its elements, corresponding to the notion of s-weak-admissible.

(vi) *If* $\mathbb{S}$ *is s-weak-admissible, then* $\mathbb{S}$ *is at least a d-weak-admissible.* If $\mathbb{S}$ is s-weak-admissible, then $\mathbb{S}$ is weak-safe and weak defends all its elements. Furthermore, by Proposition 3(iii), all weak-safe is at least a weak-conflict-free set. Thus, $\mathbb{S}$ is a weak-conflict-free set that weak defends all its elements, corresponding to the notion of d-weak-admissible. □

Proposition 6. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}^{\overline{\Theta}}_{\mathbb{C}}, \mathsf{Cont}^{\overline{\Theta}}_{\mathbb{C}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ *be the underlying bipolar argumentation framework and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then, if* $\mathbb{S}$ *is strong-stable extension, then* $\Pi_\Theta(\mathbb{S}) = \{\mathtt{A} \mid \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle \in \mathbb{S}\}$ *is a stable extension in* Θ*.*

Proof. *If* $\mathbb{S}$ *is strong-stable extension, then* $\Pi_\Theta(\mathbb{S})$ *is a stable extension in* Θ*.* Suppose that $\Pi_\Theta(\mathbb{S})$ is not a stable extension in Θ. Hence, $\Pi_\Theta(\mathbb{S})$ is not conflict-free or there does not exist an argument $\mathtt{B} \in \Pi_\Theta(\mathbb{S})$

such that $\mathtt{B}$ attacks $\mathtt{A}$. First, we know that through Definition 9 and Definition 14 we have a projection from Θ to $\overline{\Theta}$. Furthermore, by hypothesis, $\mathbb{S}$ is strong-stable extension. Then, by Definition 21, $\mathbb{S}$ is strong-conflict-free and for all $\mathrm{A} \notin \mathbb{S}$, there is a strong or weak attack (direct or supported or secondary attack) of A in $\mathbb{S}$. This leads us to a contradiction, by Proposition 2 a set that is strong-conflict-free is also conflict-free, and for all $\mathrm{A} \notin \mathbb{S}$ there is a strong or weak attack (direct or supported or secondary attack) of A in $\mathbb{S}$. $\square$

Proposition 7. *Let* $\Phi = \langle\overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}\rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle\mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a\rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle\mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s\rangle$ *be the underlying bipolar argumentation framework and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then, if there exists a strong-stable extension* $\mathbb{S}$*, then* $\mathbb{S}$ *is unique.*

Proof. *If there exists a strong-stable extension* $\mathbb{S}$*, then* $\mathbb{S}$ *is unique.* Suppose that $\mathbb{S}$ and $\mathbb{S}'$ are strong-stable extensions of Θ. Then, by Definition 21, we know that $\mathbb{S}$ and $\mathbb{S}'$ are strong-conflict-free and we have that: for all $\mathbb{A} \notin \mathbb{S}$ there exists a strong attack (direct, supported or secondary attack) from $\mathbb{S}$ to $\mathbb{A}$ and for all $\mathbb{B} \notin \mathbb{S}'$ there exists a strong attack (direct, supported or secondary attack) from $\mathbb{S}'$ to $\mathbb{B}$. Hence, if $\mathbb{S} \neq \mathbb{S}'$, then there exists an argument $\mathbb{C} \notin \mathbb{S}$ and $\mathbb{C} \in \mathbb{S}'$. Thus, exists an strong attack (direct, supported or secondary attack) from $\mathbb{S}$ to $\mathbb{C}$; however, this is a contradiction because $\mathbb{S}'$ is strong-conflict-free set. $\square$

Proposition 8. *Let* $\Phi = \langle\overline{\Theta}, \mathbb{A}\mathrm{rg}, \mathsf{Sim}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}\rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle\mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a\rangle$ *is the enriched bipolar argumentation framework, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of arguments. Then:*

(i) *if* $\mathbb{S}$ *is a strong-stable extension, then* $\mathbb{S}$ *is strong-safe if and only if* $\mathbb{S}$ *is closed under* $\mathbb{R}_s$*; and*
(ii) *if* $\mathbb{S}$ *is a* τ*-stable extension closed under* $\mathbb{R}_s$*, then* $\mathbb{S}$ *is at least a* τ*-safe;*
(iii) *if* $\mathbb{S}$ *is a weak-stable extension closed under* $\mathbb{R}_s$*, then* $\mathbb{S}$ *is at least a weak-safe.*

Proof. We separate the proof in three parts:

(i) *If* $\mathbb{S}$ *is a strong-stable extension, then* $\mathbb{S}$ *is strong-safe if and only if* $\mathbb{S}$ *is closed under* $\mathbb{R}_s$.

($\Rightarrow$) Suppose that $\mathbb{S}$ is not closed under $\mathbb{R}_s$. Hence, there exists an argument $\mathrm{A} \in \mathbb{S}$ that supports another argument $\mathrm{B} \notin \mathbb{S}$. By hypothesis, $\mathbb{S}$ is strong-stable extension which is strong-safe. Then, by Definition 21, $\forall \mathrm{A} \notin \mathbb{S}$ there is a strong or weak attack (direct or supported or secondary attack) of A in $\mathbb{S}$ where A is not supported by $\mathbb{S}$ satisfying the strong-safe condition. This leads us to a contradiction.

($\Leftarrow$) By hypothesis, $\mathbb{S}$ is strong-stable. Then, by Definition 21, $\mathbb{S}$ is strong-conflict-free and $\forall \mathrm{A} \notin \mathbb{S}$, there is a weak or strong or weak attack (direct or supported or secondary attack) of A in $\mathbb{S}$. In addition, we have that $\mathbb{S}$ is closed under $\mathbb{R}_s$. Thus, $\mathbb{S}$ is strong-conflict-free and closed under $\mathbb{R}_s$. Hence, by Proposition 3(iv), $\mathbb{S}$ is strong-safe.

(ii) *If* $\mathbb{S}$ *is a* τ*-stable extension closed under* $\mathbb{R}_s$*, then* $\mathbb{S}$ *is at least* τ*-safe.* By hypothesis, $\mathbb{S}$ is τ-stable extension. Then, by Definition 21, $\mathbb{S}$ is τ-conflict-free and $\forall \mathrm{A} \notin \mathbb{S}$, there is a strong or weak attack (direct, supported, or secondary attack) of A in $\mathbb{S}$. In addition, we have that $\mathbb{S}$ is closed under $\mathbb{R}_s$. Thus, $\mathbb{S}$ is τ-conflict-free and closed under $\mathbb{R}_s$. Hence, by Proposition 3(v), $\mathbb{S}$ is at least τ-safe.

(iii) *If* $\mathbb{S}$ *is a weak-stable extension closed under* $\mathbb{R}_s$*, then* $\mathbb{S}$ *is at least weak-safe.* By hypothesis, $\mathbb{S}$ is weak-stable extension. Then, by Definition 21, $\mathbb{S}$ is weak-conflict-free and $\forall \mathrm{A} \notin \mathbb{S}$ there is a strong attack (direct, supported, or secondary attack) of A in $\mathbb{S}$, or $\mathbb{S}$ is strong-conflict-free such that $\forall \mathrm{A} \notin \mathbb{S}$ there is an attack (direct, supported, secondary attack) of A in $\mathbb{S}$ and at least one argument $\mathrm{A} \notin \mathbb{S}$ is only weak attacked by $\mathbb{S}$. In addition, we have that $\mathbb{S}$ is closed under $\mathbb{R}_s$. Thus, $\mathbb{S}$ is weak-conflict-free and closed under $\mathbb{R}_s$ or $\mathbb{S}$ is strong-conflict-free and closed under $\mathbb{R}_s$. Hence, by Proposition 3(vi) $\mathbb{S}$ is at least weak-safe or by Proposition 3(iv) $\mathbb{S}$ is strong-safe. $\square$

Proposition 9. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ *be the underlying bipolar argumentation framework, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then, if* $\mathbb{S}$ *is a d-strong-preferred (resp. s-strong-preferred, c-strong-preferred) extension, then* $\Pi_\Theta(\mathbb{S}) = \{\mathtt{A} \mid \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle \in \mathbb{S}\}$ *is a subset of a d-preferred (resp. s-preferred, c-preferred) extension* $\mathtt{S}$ *in* Θ.

Proof. *If* $\mathbb{S}$ *is d-strong-preferred (resp. s-strong-preferred, c-strong-preferred) extension, then* $\Pi_\Theta(\mathbb{S})$ *is a subset of a d-preferred (resp. s-preferred, c-preferred) extension in* Θ. By hypothesis, $\mathbb{S}$ is d-strong-preferred (resp. s-strong-preferred, c-strong-preferred) extension. Then, by Definition 22, $\mathbb{S}$ is a maximal (for set-inclusion) among the d-strong-admissible (resp. s-strong-admissible, c-strong-admissible) subsets of $\mathbb{A}\mathrm{rg}$. Furthermore, by Proposition 4, we know that if $\mathbb{S}$ is a d-strong-admissible (resp. s-strong-preferred, c-strong-preferred) set, then $\Pi_\Theta(\mathbb{S})$ is a d-admissible (resp. s-strong-admissible, c-strong-admissible) set in Θ. Thus, only we need to prove that $\Pi_\Theta(\mathbb{S}) \subseteq \mathtt{S}'$. Suppose that $\Pi_\Theta(\mathbb{S}) \nsubseteq \mathtt{S}'$. Hence, there exists an argument $\mathtt{A}$ that belongs to $\Pi_\Theta(\mathbb{S})$ but not $\mathtt{S}'$. Then, $\mathbb{A}$ is strong defended by $\mathbb{S}$ or it has not attackers in $\mathbb{A}\mathrm{rg}$. This leads us to a contradiction, since by Definition 8, $\mathtt{S}'$ is a maximal (for set-inclusion) among the d-admissible extension. Thus, $\mathtt{S}'$ is conflict-free and defends all its elements. In case that A has not attacker, A must belong to $\mathtt{S}'$, while if A is strong defended by $\mathbb{S}$ in $\overline{\Theta}$ it is defended by $\mathtt{S}'$ in Θ since $\mathtt{S}'$ contain all kind of defenders (strong and weak defense). $\square$

Proposition 10. *Let* $\Phi = \langle \overline{\Theta}, \mathbb{A}\mathrm{rg}, \mathsf{Sim}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rg}$ *be a set of enriched arguments. Then:*

(i) *if* $\mathbb{S}$ *is a c-strong-preferred extension, then* $\mathbb{S}$ *is a s-strong-preferred extension;*
(ii) *if* $\mathbb{S}$ *is a s-strong-preferred extension, then* $\mathbb{S}$ *is a d-strong-preferred extension;*
(iii) *if* $\mathbb{S}$ *is a c-*τ*-preferred extension, then* $\mathbb{S}$ *is at least a s-*τ*-preferred extension;*
(iv) *if* $\mathbb{S}$ *is a s-*τ*-preferred extension, then* $\mathbb{S}$ *is at least a d-*τ*-preferred extension;*
(v) *if* $\mathbb{S}$ *is a c-weak-preferred extension, then* $\mathbb{S}$ *is a at least s-weak-preferred extension; and*
(vi) *if* $\mathbb{S}$ *is a s-weak-preferred extension, then* $\mathbb{S}$ *is at least a d-weak-preferred extension.*

Proof. We separate the proof in six parts:

(i) *If* $\mathbb{S}$ *is a c-strong-preferred extension, then* $\mathbb{S}$ *is a s-strong-preferred extension.* If $\mathbb{S}$ is a c-strong-preferred extension, then $\mathbb{S}$ is maximal (for set-inclusion) among the c-strong-admissible subsets of $\mathbb{A}\mathrm{rg}$. By Proposition 5(i), a c-strong-admissible set is also a s-strong-admissible set. Thus, only we need to prove that $\mathbb{S}$ is the maximal s-strong-admissible set. Suppose that $\mathbb{S}$ is not the maximal s-strong-admissible set. Hence, there exists an argument A $\notin \mathbb{S}$ such that there does not exist a weak or strong attack (direct, secondary, or supported attack) from $\mathbb{S}$ to A and either there does not exist a sequence of support from $\mathbb{S}$ to A. In addition, A is defended by $\mathbb{S}$. Contradiction, $\mathbb{S}$ is a maximal c-strong-admissible set, then $\mathbb{S}$ is the maximal strong-conflict-free set closed under $\mathtt{R}_s$. Thus, A must belong to $\mathbb{S}$.

(ii) *If* $\mathbb{S}$ *is a s-strong-preferred extension, then* $\mathbb{S}$ *is a d-strong-preferred extension.* If $\mathbb{S}$ is a s-strong-preferred extension, then $\mathbb{S}$ is maximal (for set-inclusion) among the s-strong-admissible subsets of $\mathbb{A}\mathrm{rg}$. By Proposition 5(ii), a s-strong-admissible set is also a d-strong-admissible set. Thus, only we need to prove that $\mathbb{S}$ is the maximal d-strong-admissible set. Suppose that $\mathbb{S}$ is not the maximal d-strong-admissible set. Hence, there exists an argument A $\notin \mathbb{S}$ such that there does not exist an attack from $\mathbb{S}$ to

A and is defended by $\mathbb{S}$. Contradiction, $\mathbb{S}$ is s-strong-admissible set, then $\mathbb{S}$ is the maximal strong-safe set. Thus, if there does not exist a weak or strong attack (direct, secondary, or supported attack) from $\mathbb{S}$ to A and either there does not exist a sequence of support from $\mathbb{S}$ to A, then A must belong to $\mathbb{S}$.

(iii) *If $\mathbb{S}$ is a c-τ-preferred extension, then $\mathbb{S}$ is at least a s-τ-preferred extension.* If $\mathbb{S}$ is a c-τ-preferred extension, then $\mathbb{S}$ is maximal (for set-inclusion) among the c-τ-admissible subsets of $\mathbb{A}\text{rg}$. By Proposition 5(iii), a c-τ-admissible set is at least a s-τ-admissible set. Thus, only we need to prove that $\mathbb{S}$ is the maximal s-τ-admissible set. Suppose that $\mathbb{S}$ is not the maximal s-τ-admissible set. Hence, there exists an argument A $\notin \mathbb{S}$ such that there does not exist a weak or strong attack (direct, secondary, or supported attack) from $\mathbb{S}$ to A with $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\mathbb{S} \cup \mathrm{A}) > \tau$ and either there does not exist a sequence of support from $\mathbb{S}$ to A with $\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\mathrm{C}, \dots, \mathrm{A}) > \tau$. In addition, A is defended by $\mathbb{S}$. Contradiction, $\mathbb{S}$ is a maximal c-τ-admissible set, then $\mathbb{S}$ is the maximal τ-conflict-free set closed under $\mathtt{R}_s$. Thus, A must belong to $\mathbb{S}$.

(iv) *If $\mathbb{S}$ is a s-τ-preferred extension, then $\mathbb{S}$ is at least a d-τ-preferred extension.* If $\mathbb{S}$ is a s-τ-preferred extension, then $\mathbb{S}$ is maximal (for set-inclusion) among the s-τ-admissible subsets of $\mathbb{A}\text{rg}$. By Proposition 5(iv), a s-τ-admissible set is at least a d-τ-admissible set. Thus, only we need to prove that $\mathbb{S}$ is the maximal d-τ-admissible set. Suppose that $\mathbb{S}$ is not the maximal d-τ-admissible set. Hence, there exists an argument A $\notin \mathbb{S}$ such that there does not exist an attack from $\mathbb{S}$ to A with $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\mathbb{S} \cup \mathrm{A}) > \tau$ and is defended by $\mathbb{S}$. Contradiction, $\mathbb{S}$ is s-τ-admissible set, then $\mathbb{S}$ is the maximal τ-safe set. Thus, if there does not exist a weak or strong attack (direct, secondary, or supported attack) from $\mathbb{S}$ to A with $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}(\mathbb{S} \cup \mathrm{A}) > \tau$ and either there does not exist a sequence of support from $\mathbb{S}$ to A with $\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\mathrm{C}, \dots, \mathrm{A}) > \tau$, then A must belong to $\mathbb{S}$.

(v) *If $\mathbb{S}$ is a c-weak-preferred extension, then $\mathbb{S}$ is at least a s-weak-preferred extension.* If $\mathbb{S}$ is a c-weak-preferred extension, then $\mathbb{S}$ is maximal (for set-inclusion) among the c-weak-admissible subsets of $\mathbb{A}\text{rg}$. By Proposition 5(v), a c-weak-admissible set is at least a s-weak-admissible set. Thus, only we need to prove that $\mathbb{S}$ is the maximal s-weak-admissible set. Suppose that $\mathbb{S}$ is not the maximal s-weak-admissible set. Hence, there exists an argument A $\notin \mathbb{S}$ such that there does not exist a strong attack (direct, secondary, or supported attack) from $\mathbb{S}$ to A and either there does not exist a sequence of support from $\mathbb{S}$ to A with $\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\mathrm{C}, \dots, \mathrm{A}) > \tau$. In addition, A is defended by $\mathbb{S}$. Contradiction, $\mathbb{S}$ is a maximal c-weak-admissible set, then $\mathbb{S}$ is the maximal weak-conflict-free set closed under $\mathtt{R}_s$. Thus, A must belong to $\mathbb{S}$.

(vi) *If $\mathbb{S}$ is a s-weak-preferred extension, then $\mathbb{S}$ is at least a d-weak-preferred extension.* If $\mathbb{S}$ is a s-weak-preferred extension, then $\mathbb{S}$ is maximal (for set-inclusion) among the s-weak-admissible subsets of $\mathbb{A}\text{rg}$. By Proposition 5(vi), all s-weak-admissible set is at least a d-weak-admissible set. Thus, only we need to prove that $\mathbb{S}$ is the maximal d-weak-admissible set. Suppose that $\mathbb{S}$ is not the maximal d-weak-admissible set. Hence, there exists an argument A $\notin \mathbb{S}$ such that there does not exist an attack from $\mathbb{S}$ to A and is defended by $\mathbb{S}$. Contradiction, $\mathbb{S}$ is s-weak-admissible set, then $\mathbb{S}$ is the maximal weak-safe set. Thus, if there does not exist a strong attack (direct, secondary, or supported attack) from $\mathbb{S}$ to A and either there does not exist a sequence of support from $\mathbb{S}$ to A with $\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}(\mathrm{C}, \dots, \mathrm{A}) > \tau$, then A must belong to $\mathbb{S}$. $\square$

Proposition 11. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$ *be a* S-BAF *where* $\overline{\Theta} = \langle \mathbb{A}\text{rg}, \mathbb{R}_s, \mathbb{R}_a \rangle$ *is the enriched bipolar argumentation framework,* $\Theta = \langle \mathtt{Arg}, \mathtt{R}_a, \mathtt{R}_s \rangle$ *be the underlying bipolar argumentation framework, and* $\mathbb{S}, \mathbb{S}' \subseteq \mathbb{A}\text{rg}$ *be a set of enriched arguments. Then:*

(i) *if $\mathbb{S}$ is a c-strong-preferred (c-τ-preferred or c-weak-preferred extension) extension and $\mathbb{S}'$ is a strong-stable (τ-stable or weak-stable, respectively) extension, then $\mathbb{S} \subseteq \mathbb{S}'$;*

(ii) *if $\mathbb{S}$ is a s-strong-preferred (s-τ-preferred or s-weak-preferred) extension and $\mathbb{S}'$ is a strong-stable (τ-stable or weak-stable, respectively) extension, then $\mathbb{S} \subseteq \mathbb{S}'$; and*

(iii) *if $\mathbb{S}$ is the strong-stable extension and satisfy that $\mathbb{S}$ is strong-safe, then $\mathbb{S}$ is the unique c-strong-preferred and s-strong-preferred extension.*

Proof. We separate the proof in two parts:

(i) If $\mathbb{S}$ is a c-strong-preferred, then by Definition 22 we know that $\mathbb{S}$ is the maximal among the c-strong-admissible subset of $\mathbb{A}$rg. Thus, by Definition 20 we have that $\mathbb{S}$ is the maximal among the strong-conflict-free, closed under $\mathbb{R}_s$ and strong defends all its elements. Furthermore, by hypothesis we know that $\mathbb{S}'$ is a strong-stable extension. Thus, by Definition 21 we have that $\mathbb{S}'$ is the maximal strong-conflict-free and for any $A \notin \mathbb{S}'$ there is a strong attack (direct, supported, or secondary attack) from $\mathbb{S}'$ to A. Suppose that $\mathbb{S} \nsubseteq \mathbb{S}'$, then there exists an argument $A \in \mathbb{S}$ such that $A \notin \mathbb{S}'$. Thus, there exists a strong attack (direct, supported, or secondary attack) from $B \in \mathbb{S}'$ to A. Now, if $B \in \mathbb{S}$, we lead to a contradiction since $\mathbb{S}$ is the maximal strong-conflict-free set. While if $B \notin \mathbb{S}$ and $A \in \mathbb{S}$, then there exists an argument $C \in \mathbb{S}$ that strong defends A from B (there exists a strong attack from C to B). Now, if $C \in \mathbb{S}'$, we lead to a contradiction since $\mathbb{S}'$ is the maximal strong-conflict-free set. While, if $C \notin \mathbb{S}'$, then there exists a strong attack (direct, supported, or secondary attack) from $D \in \mathbb{S}'$ to C. Continuing with this analysis, we can conclude one of the following contradictions: there is no defense for an attacked argument in $\mathbb{S}$ or there does not exist an attacker for an external argument of $\mathbb{S}'$. Thus, we can deduce that $\mathbb{S} \subseteq \mathbb{S}'$. The proof for the other two relations are analogous.

(ii) If $\mathbb{S}$ is a s-strong-preferred, then by Definition 22 we know that $\mathbb{S}$ is the maximal among the s-strong-admissible subset of $\mathbb{A}$rg. Thus, by Definition 20 we have that $\mathbb{S}$ is the maximal among the strong-safe and strong defends all its elements. Furthermore, by hypothesis we know that $\mathbb{S}'$ is a strong-stable extension. Thus, by Definition 21 we have that $\mathbb{S}'$ is the maximal strong-conflict-free and for any $A \notin \mathbb{S}'$ there is a strong attack (direct, supported, or secondary attack) from $\mathbb{S}'$ to A. Suppose that $\mathbb{S} \nsubseteq \mathbb{S}'$, then there exists an argument $A \in \mathbb{S}$ such that $A \notin \mathbb{S}'$. Thus, there exists a strong attack (direct, supported, or secondary attack) from $B \in \mathbb{S}'$ to A. Now, if $B \in \mathbb{S}$, we lead to a contradiction since $\mathbb{S}$ is the maximal strong-safe set and by Proposition 3 all strong-safe set is also a strong-conflict-free set. While if $B \notin \mathbb{S}$ and $A \in \mathbb{S}$, then there exists an argument $C \in \mathbb{S}$ that strong defends A from B (there exists a strong attack from C to B). Now, if $C \in \mathbb{S}'$, we lead to a contradiction since $\mathbb{S}'$ is the maximal strong-conflict-free set. While, if $C \notin \mathbb{S}'$, then there exists a strong attack (direct, supported, or secondary attack) from $D \in \mathbb{S}'$ to C. Continuing with this analysis, we can conclude one of the following contradictions: there is no defense for an attacked argument in $\mathbb{S}$ or there does not exist an attacker for an external argument of $\mathbb{S}'$. Thus, we can deduce that $\mathbb{S} \subseteq \mathbb{S}'$. The proof for the other two relations are analogous.

(iii) If $\mathbb{S}$ is the strong-stable extension and satisfy that $\mathbb{S}$ is strong-safe, then $\mathbb{S}$ is the unique c-strong-preferred and s-strong-preferred extension. We separate the proof in two parts:

(a) If $\mathbb{S}$ is the strong-stable extension and satisfy that $\mathbb{S}$ is strong-safe, then $\mathbb{S}$ is the unique c-strong-preferred extension. First, by Proposition 7, we know that the strong-stable extension is unique. By Hypothesis, we have that $\mathbb{S}$ is the strong-stable extension. Then, by Definition 21, $\mathbb{S}$ is the maximal strong-conflict-free and for all argument $A \notin \mathbb{S}$ there exists a strong attack (direct, supported, or secondary attack) from $\mathbb{S}$ to A. Furthermore, $\mathbb{S}$ is strong-safe, then by Definition 18 we know that there is no $A \in \mathbb{A}$rg and no pair $B, C \in \mathbb{S}$ such that there exists a strong or weak attack (direct, supported, or secondary attack) from B to A, and either there is a sequence of support from C to A, or $A \in \mathbb{S}$. Thus, if an argument $A \in \mathbb{S}$, and it supports another argument $B \in \mathbb{A}$rg, B then it must belong to $\mathbb{S}$. Suppose that B $\notin \mathbb{S}$, then B is attacked and supported at the same time for $\mathbb{S}$. But $\mathbb{S}$ is strong-safe. This lead us to a contradiction. Thus, $\mathbb{S}$ is strong-conflict-free and closed under $\mathbb{R}_s$. Also, it attacks all the argument that do not belong to $\mathbb{S}$. Thus, $\mathbb{S}$ strong defends all its elements. Then, we can conclude that $\mathbb{S}$ is the c-strong-preferred extension.

(b) If $\mathbb{S}$ is the strong-stable extension and satisfy that $\mathbb{S}$ is strong-safe, then $\mathbb{S}$ is the unique s-strong-preferred extension. By (a) we know that if $\mathbb{S}$ is the strong-stable extension and satisfy that $\mathbb{S}$ is strong-safe, then $\mathbb{S}$ is the unique c-strong-preferred extension. Furthermore, by Proposition 10 we can say that the c-strong-preferred extension is also a s-strong-preferred extension. □

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