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A multi-criteria approach for evaluating major league baseball batting performance

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Abstract. The evaluation of player performance typically involves a number of criteria representing various aspects of performance that are of interest. Pareto optimality and weighted aggregation are useful tools to simultaneously evaluate players with respect to the multiple criteria. In particular, the Pareto approach allows trade-offs among the criteria to be compared, does not require specifications of weighting schemes, and is not sensitive to the scaling of the criteria. The Pareto optimal players can be scored according to their ranks or according to their distance from the global optimum for informative comparisons of performance or for evaluating trade-offs among the criteria. These multi-criteria approaches are defined and illustrated for evaluating batting performance of Major League Baseball players.

Keywords: Baseball, MANOVA, pareto optimal, player ranking, sabermetric, weighted aggregation

1. Introduction

Albert (2010) defines *sabermetrics* as the science of learning about baseball through objective evidence. Grabiner (2014) indicates that the basic goal of sabermetrics is to evaluate past player performance and to predict future performance of player contributions to their teams. The information can be useful for determining who wins season awards and when determining the value of making a certain trade. The *sabermetrician* looks to contribute to this field through creating new statistics to better assess player performance (Albert, 2010). Often, these new statistics are aggregations or combinations of existing statistics.

This paper describes a multi-criteria approach in which the sabermetrician can evaluate and rank player performance using a simultaneous evaluation of multiple criteria. Two popular approaches are adopted from the multiple optimization literature: (1) Pareto optimization discussed by Marler and Arora (2004) and (2) Weighted aggregation discussed by Ngatchou et al. (2005). Pareto optimal solutions are those which are not *dominated* or which cannot be bettered with respect to all of the criteria under consideration. Once Pareto solutions are determined, the sabermatrician can examine trade-offs between the criteria and identify the collection of players that cannot be beat with respect to the specified criteria. Weighted aggregation develops linear combinations of the optimization criteria which can then be used to rank or score the players. Weighted aggregation can also be helpful for characterizing the performance of those players that are Pareto optimal. As a result, the proposed approach has the following advantages:

1. Allows for simple and informative simultaneous comparisons of numerous players in terms of multiple performance criteria.
2. Avoids having to combine multiple criteria into single metrics based upon complex specifications for weighting and scaling of the criteria.
3. Allows the trade-offs among the criteria to be compared.

Koop (2002) recognizes that evaluation of baseball players is a difficult task since "baseball is

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fundamentally a multiple-output sport”. This author uses frontier models to create an aggregator of the multiple outputs pertaining to hitting in Major League Baseball (MLB). A Bayesian modeling approach is then implemented to estimate player efficiencies. The proposed approach in this paper does not rely on complicated modeling of aggregated outputs. Rather, the multiple hitting criteria are evaluated directly with observed or predicted data using Pareto optimality. Weighted aggregations, in the form of ranks, are then used to characterize the Pareto optimal players. Efficiencies are also calculated directly from the best possible performance with respect to each of the criteria. The proposed approach would be useful to sabermetricians, general managers, and fantasy baseball players for assessing player performance with respect to multiple criteria.

2. Multi-Criteria Optimization

Suppose there is interest in evaluating player performance according to c criteria for a particular collection of players $\aleph$. Let $f_i(x)$ denote a *criterion value* for $i = 1, 2, \ldots, c$ involving a player $x \in \aleph$. Furthermore, suppose each $f_i(x) \equiv -g_i(x)$ is to be maximized. As in Ardakani and Wulff (2013), the multi-criteria setting can be stated as the optimization problem

$$\text{Maximize } \mathbf{f}(x) = (f_1(x), f_2(x), \ldots, f_s(x))' \quad \text{subject to } x \in \aleph, \tag{1}$$

where $\mathbf{f}(x)$ denotes the *criterion vector* evaluated at x. The *utopia point* corresponds to the criterion vector associated with the player who simultaneously maximizes all criterion values. That is, x^u is a utopia point provided $f_i(x^u) \geq f_i(x)$ for all $x \in \aleph$, for all $i = 1, 2, \ldots, c$. For $c \geq 2$, the utopia point rarely exists due to conflicts of simultaneously maximizing all the criteria. However, it is hoped that players can be identified that are ‘close’ to the utopia point. For an application to baseball hitters, the utopia point could be better referred to as *Batman* since such a player would be a mythical batting superhero.

2.1. Pareto optimality

Consider the problem of finding a solution (player) according to (1). While Batman may not exist, a collection of players may be identified that cannot be bettered, or dominated, according to the c criteria.

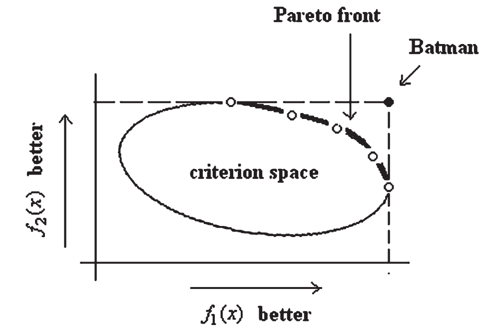


Fig. 1. Illustration of the Pareto front and utopia point (Batman) in a two-criteria maximization problem.

A player $x^* \in \aleph$ is said to *dominate* another player x provided the following two conditions are met:

$$\begin{aligned} &1.\ f_i(x^*) \geq f_i(x) \text{ for all } i \in 1, 2, \ldots, c, \\ &2.\ f_i(x^*) > f_i(x) \text{ for all at least one } i \in 1, 2, \ldots, c, \end{aligned} \tag{2}$$

A player is said to be *Pareto optimal* provided they are not dominated by any other player in $\aleph$. The set of Pareto optimal players constitutes the *Pareto optimal set* (POS). The corresponding criteria vectors $\mathbf{f}(x)$ comprise the *Pareto front* (PF). An overview of Pareto-related concepts is available in Coello Coello et al. (2007).

By definition, the Pareto optimal set is defined as those players that are not Pareto dominated in the sense of (2). Thus, the POS can be generated by first forming the complement POS^c, which is the set of dominated players. This is done in the following steps:

1. evaluate $\mathbf{f}(x)$ for all $x \in \aleph$, (3)
2. initialize POS^c to be $\varnothing$ and perform the following for each $z \in \aleph$
   a. check if condition (2) is satisfied with $x^* \in \aleph$ such that $x^* \neq z$,
   b. if (2) is satisfied, add z to POS^c, otherwise do not add z to POS^c.

The steps in (3) identify the set POS^c, so that $\text{POS}=(\text{POS}^c)^c$ is the set of non-dominated solutions or the Pareto optimal set. The Pareto front consists of $\mathbf{f}(x)$ for all $x \in \text{POS}$.

Figure 1 shows two performance criteria values f_1 and f_2 so that $c = 2$. Batman corresponds to that player who simultaneously maximizes both criteria. The Pareto optimal set consists of the players lying on the boundary of the criterion space closest to Batman. This boundary forms the Pareto front. There are two points which lie on an axis. These players maximize either f_1 or f_2. Note that none of the Pareto optimal players can bettered in one criterion without deteriorating in the other criterion. If the two criteria are highly positively correlated, then the Pareto front will likely consist of only a few players. Otherwise, the criteria conflict and the Pareto front will then likely consist of numerous players.

2.2. Scoring

A popular approach for finding an optimal solution to (1) is *weighted aggregation* as identified by Ngatchou et al. (2005) or the *weighting method* as identified by Ardakani and Wulff (2013). In this approach, all c criteria are combined to form a single objective function. The optimization problem is thus reduced to one function from which a batter can be scored and an optimal batter can be identified from this *score*. In particular, the formulation for weighted aggregation can be expressed as

$$\text{Maximize } h(x) = \sum_{i=1}^{c} w_i \, |k_i(x)| \ \text{ subject to } x \in \aleph, \tag{4}$$

where $k_i(x)$ is a function of the criterion value and the collection $\{w_i\}$ are weights for the contribution of criterion i. Often, the weights satisfy $w_i \geq 0$ and $\sum_{i=1}^{s} w_i = 1$. There are many approaches for obtaining a single objective function (Ardakani and Wulff, 2013). The expression in (4) can be extended using the weighted p-norm method with powers p greater than or equal to 1 in which $|k_i(x)|$ is replaced by $|k_i(x)|^p$ (Marler and Arora, 2004). Using (4), players are then ranked by the values of h, and the best player is the one that maximizes h A disadvantage of weighted aggregation is that it depends upon the selected weights $\{w_i\}$, and the selection of the weights depends upon the scaling of the criteria. The task of selecting the weights, and interpreting h, is made easier when the criteria are placed on comparable scales. However, specifying a meaningful scale can be difficult. These disadvantages are not present with the Pareto approach since POS and PF do not depend upon weights or the scaling of the criteria.

A naive approach is to let $k_i(x) = f_i(x)$. The sabermetrician can prespecify the weights according to their preferences, and rank players accordingly. However, the criteria could be on different scales which leads to the scaling problems mentioned previously. One approach to deal with differences in scales, and which is consistent with player rankings, is to let $k_i(x) = -r_i(x)$ where $r_i(x)$ denotes the rank of player $x \in \aleph$. with respect to criterion i. Then weights can be assigned in relation to the importance that is to be placed upon the rankings for criterion i. However, the use of rankings can mask differences of magnitude within the criteria values between players. In this study, players with ties in their criterion values are assigned the maximum of the ranks.

It is also possible to use (4) to assess the distance a player is from Batman, or from the hypothetical hitter x^u who maximizes each criterion value separately. Marler and Arora (2004) recommend that the criteria be on the same scale before measuring distance. In particular, these authors consider the two scalings:

$$k_i(x) = \frac{f_i(x)}{f_i(x^u)}, \tag{5}$$

$$k_i(x) = \frac{f_i(x) - f_i(x^l)}{f_i(x^u) - f_i(x^l)}. \tag{6}$$

Equation (5) is the *ratio* between a player and Batman for criterion i. Equation (6) represents the *desirability* between a player and Batman for criterion i where x^l denotes the hypothetical player with lowest criterion value in each of the criterion. This is the opposite of Batman, or the Joker. Thus, (6) compares the difference of a player from Joker relative to the difference of Batman from Joker for criteria i. As previously mentioned, equation (4) can be generalized using a power p to represent a weighted p-norm metric. Equation (4) assumes $p = 1$ which corresponds to the 1-norm or sum norm. Thus, (5) can be interpreted as the sum norm distance a player is to 0 relative to Batman. Equation (6) can be interpreted as the sum norm distance a player is to Joker relative to Batman.

As previously mentioned, a disadvantage of weighted aggregation is that it depends upon the selected weights $\{w_i\}$. An experienced sabermetrician would pre-specify weights according to the specific objectives of the player performance evaluation. If there is no justified apriori rationale for specifying the weights, then equal weights could be used with $w_i = 1/c$ for $i = 1, \ldots, c$. Equal weighting amounts to finding the average of the criteria.

In this study, there was no rationale for pre-specifying the weights. Thus, the weights were determined objectively using *exploratory factor analysis* (EFA). EFA hypothesizes a model in which the criteria are a linear combination of unobserved factors and coefficients in this model, or *loadings*, that are estimated to approximately reproduce the covariance matrix among the criteria (Rencher, 2012, pp. 435–441). A single factor model is hypothesized in this study, where that factor represents player performance. The loadings are estimated using maximum likelihood (Rencher, 2012, pp. 452), and then standardized to obtain the weights $\{w_i\}$ used in (4). The loading equals the covariance between the corresponding criterion and the factor representing player performance (Rencher, 2012, pp. 440). Thus, the higher the loading, the higher the weight for that criterion since it is most linearly related to player performance. The weights from the EFA approach are used for both the scoring in (4) and scored ratio to Batman in (5).

3. Primary Pareto Optimal Set

MLB hitting data for 2016 is taken from *baseball-guru.com* which has annual hitting data in EXCEL files under the player forecast section. Abbreviations for various performance variables are given in Table 1 for convenience. As in Koop (2002), pitchers and hitters with few AB are removed from the dataset. The removal of pitchers results in 620 hitters. Hitters with fewer than 60 AB are also removed to focus on full time hitters and to eliminate possible anomalies. The cut off of 60 AB is close to the first quartile of AB for the 620 hitters (65 AB). The final data set consists of $n = 473$ hitters. The hitting measures considered here are given by the six performance variables (y) in Table 1. These are the traditional well-known statistics to assess offensive performance, and are listed on popular baseball websites such as *mlb.com*. These statistics measure various aspects of offensive performance, including the ability of a hitter to get on base, generate runs, and hit for power. These criteria are also used in specific aggregations to formulate several sabermetrics. To avoid concerns with using the counting statistics (R, H, HR, RBI), these values are scaled by AB as recommended by Grabiner (2014).

Table 2 shows the correlation matrix for the 2016 hitting performance measures for players in the candidate set $\aleph$. All correlations are positive. SLG is moderately correlated with all the hitting measures, including OBP. HRr is correlated with RBIr as expected. The highest correlation is 0.78 which is observed between AVG and OBP as well as HRr and SLG. These correlations are moderate and would result in only mild concerns about multicollinearity according to Kutner et al. (2004, pp. 406–410). It is expected that statistics that are highly correlated will be coherent or produce similar rankings of player performance. Thus, the presence of these correlations is not an impediment to this multi-criteria approach.

Table 1

Baseball abbreviations for variable names (n), performance variables (y), sabermetric variables (s), predictor variables (x)

Variable	Description
n_1 = AB	At Bat
n_2 = BB	Base on Balls
n_3 = CS	Caught Stealing
n_4 = H	Hits
n_5 = HR	Home Runs
n_6 = MVP	Most Valuable Player
n_7 = PA	Plate Appearance
n_8 = R	Runs Scored
n_9 = RBI	Runs Batted In
n_{10} = ROY	Rookie of the Year
n_{11} = SB	Stolen Base
n_{12} = SF	Sacrifices
n_{13} = SS	Silver Slugger
n_{14} = TB	Total Bases
y_1 = Rr = R/AB	Runs per At Bat
y_2 = HRr = HR/AB	Home Runs per At Bat
y_3 = RBIr = RBI/AB	Runs batted In per At Bat
y_4 = AVG = H/AB	Batting Average
y_5 = OBP = (H + BB + HBP)/ (AB + BB + HBP + SF)	On Base Percentage
y_6 = SLG = TB/AB	Slugging Percentage
s_1 = wOBA	Weighted On-base Average
s_2 = wRC+	Weighted Runs Created Plus
s_3 = WAR	Wins Above Replacement
x_1 = G	Games Played In
x_2 = AB	At Bats
x_3 = AGE	Player Age
$x_{4,j4}$ = TEAM (reference = COL)	Team of Player
$x_{5,j5}$ = FP1 (reference = 1B)	Player Fielding Position
$x_{6,j6}$ = BATS (reference = R)	Batting Side of Plate

Table 2

Correlation matrix for the six hitting performance measures

	Rr	HRr	RBIr	AVG	OBP	SLG
Rr	1	0.3963	0.3485	0.4920	0.5918	0.6037
HRr	0.3963	1	0.7609	0.1384	0.2919	0.7769
RBIr	0.3485	0.7609	1	0.3567	0.4227	0.7595
AVG	0.4920	0.1384	0.3567	1	0.7791	0.6899
OBP	0.5918	0.2919	0.4227	0.7791	1	0.6593
SLG	0.6037	0.7769	0.7595	0.6899	0.6593	1

The Pareto optimal set of 2016 MLB hitters with respect to these six performance criteria is shown in Table 3. This collection of 19 hitters lie on the primary Pareto optimal set (POS1) since they are non-dominated according to (2) and as such cannot be bettered by any other hitter in the candidate set with respect to these criteria. Within the Pareto optimal set, players are ranked according to their scored rankings across the six performance criteria using

Table 3
Pareto optimal hitters, Pareto Front shown as ranks, scored rankings, scored ratios to Batman, and comments related to the evaluation of player performance

name	team	pos	bats	age	games	ab	Rr.R	HRr.R	RBIr.R	AVG.R	OBP.R	SLG.R	s.rank	r.rank	s.Batman	comment
Sanchez	NYA	C	R	23	53	201	35	[1]	4	41	26	[1]	16.02	[1]	0.9018	AL rookie 2
Arenado	COL	3B	R	25	160	618	10	21	2	53	44	4	20.74	2	0.8266	NL MVP 5, SS
Trout	LAA	OF	R	24	159	549	[1]	73	20	16	[1]	13	21.36	3	0.8311	AL MVP, SS
Ortiz	BOS	1B	L	40	151	537	125	12	[1]	16	6	2	23.45	4	0.8645	Best Hitter, AL MVP 6, SS
Votto	CIN	1B	L	32	158	556	15	77	35	6	2	13	25.2	5	0.8017	NL MVP 7
Cabrera	DET	1B	R	33	158	595	85	30	24	12	10	8	26.14	6	0.7963	AL MVP 9, SS
Bryant	CHN	3B	R	24	155	603	3	28	47	58	19	10	26.92	7	0.7985	NL MVP
Murphy	WAS	2B	L	31	142	531	46	101	7	2	13	3	27.71	8	0.8069	NL MVP 2, SS
Donaldson	TOR	3B	R	30	155	577	2	29	39	85	8	14	28.87	9	0.8076	AL MVP 4
Freeman	ATL	1B	L	26	158	589	30	47	92	34	6	5	34.93	10	0.7769	NL MVP 6
Braun	MIL	OF	R	32	135	511	78	42	32	27	37	17	36.84	12	0.7631	NL MVP 24
Rizzo	CHN	1B	L	26	155	583	68	61	13	58	18	16	37.02	13	0.7676	NL MVP 4, SS
Blackmon	COL	OF	L	29	143	578	6	83	140	8	22	11	45.67	15	0.7605	NL MVP 26, SS
Story	COL	SS	R	23	97	372	19	9	9	131	128	7	46.11	16	0.7978	NL Rookie 4
Encarnacion	TOR	1B	R	33	160	601	49	13	3	183	82	26	55.05	20	0.7844	AL MVP 14
Altuve	HOU	2B	R	26	161	640	37	174	107	4	10	24	60.33	24	0.7362	AL MVP 3
Turner	WAS	OF	R	23	73	307	32	130	197	3	46	7	68.96	29	0.7345	NL Rookie 2
Joyce	PIT	OF	L	31	140	231	5	52	23	289	7	109	81.83	36	0.7449	Free Agent
LeMahieu	COL	2B	R	27	146	552	9	329	239	[1]	4	66	113.09	58	0.6928	NL MVP 15

the weights obtained from EFA. The weights determined from EFA for the 2016 data are 0.14 for Rr, 0.17 for HRr, 0.17 for RBIr, 0.15 for AVG, 0.15 for OBP, and 0.22 for SLG. The mythical Batman consists of a mixture of Sanchez (HRr, SLG), Trout (Rr, OBP), Ortiz (RBIr), and LeMahieu (AVG). All batters have a scored ratio to Batman using (5) and the EFA weights which ranges from 0.69 (LeMahieu) to 0.90 (Sanchez) for players in POS1.

POS1 identifies many well-known hitters in MLB for 2016. In fact, POS1 identifies the 2016 hitting award winners including the AL MVP, NL MVP, Silver Sluggers, vote-getters for MVP, and vote-getters for ROY. As previously mentioned, this is one of the objectives of sabermetrics. Sanchez had an incredible rookie season and received the lowest scored rank of all hitters given his production in Rr, RBIr, SLG. He is also the closest to Batman. Ortiz, a veteran hitter who had a fantastic year, received the Best Hitter award which is well deserved given that he is the third closest to Batman and had more AB than Sanchez. Four COL players are in POS1 (Arenado, Blackmon, Story, LeMahieu) where they each rank highly in a couple of the performance measures. While these are good hitters, increased performance might be expected at a hitter friendly park such as Coors Field. Joyce, a free agent who plays in OAK for 2017, might be an unexpected hitter to be included in POS1. However, he ranks fifth in Rr. Of those who rank higher, Trout has lower HRr, Donaldson has lower RBIr, Bryant has lower OBP, and DeShields (Table 4) has lower values in all other categories. By the definition in (2), Joyce is non-dominated and so is included in POS1.

The multi-criteria approach is also helpful for learning about trade-offs among the criteria. Figure 2 shows plots involving just two criteria which can be compared to the idealized plot in Fig. 1. The criteria HRr and SLG have the second highest correlation (Table 2) which is evident by the linear relationship shown in Fig. 2 (b). Based upon just these two criteria, a reduced POS (POSr) would just consist of Sanchez, who is also the reduced Batman. Nevertheless, other POS1 hitters are rather scattered throughout these two criteria. The criteria OBP and SLG have the sixth highest correlation (Table 2) and the linear relationship can be seen in Fig. 2 (d). Now, the POSr from just these two criteria would consist of Sanchez, Ortiz, and Trout. The POS1 hitters are all located in the upper right of the plot. There are indications that SLG and OBP are redundant statistics in identifying these best hitters. The criteria Rr and RBIr have the third smallest correlation (Table 2) as is evident by the larger cloud of points in Fig. 2 (a). The POSr consists of a longer front than that in Fig. 2 (d) which contains Ortiz, Arenado, and Trout. Even though the POS1 hitters are all located in the upper right of the plot, the criteria values are also more spread out than they are in Fig. 2 (d). The criteria HRr and AVG have the smallest correlation (Table 2) as might be

Table 4
Secondary Pareto optimal hitters, Secondary Pareto Front shown as ranks, scored rankings, scored ratios to Batman, and comments related to the evaluation of player performance

name	team	posl	bats	age	games	ab	Rr.R	HRr.R	RBIr.R	AVG.R	OBP.R	SLG.R	s.rank	r.rank	s.Batman	comment
Cruz	SEA	OF	R	35	155	589	62	8	30	71	60	[9]	36.77	[11]	0.7859	AL MVP 15
Betts	BOS	OF	R	23	158	672	16	109	50	11	65	19	44.85	[14]	0.7503	AL MVP 2, SS
Beltre	TEX	3B	R	37	153	583	98	61	29	39	65	31	51.44	[17]	0.7425	AL MVP 7
Cano	SEA	2B	L	33	161	655	57	41	83	44	88	23	53.92	[18]	0.7428	AL MVP 8
Cespedes	NYN	OF	R	30	132	479	105	27	27	101	67	25	54.58	[19]	0.7527	NL MVP 8, SS
Diaz	STL	SS	R	25	111	404	24	134	75	39	33	37	57.83	21	0.7231	NL ROY 5
Ramirez	BOS	1B	R	32	147	549	123	63	[5]	75	57	46	58.70	22	0.7453	
Goldschmidt	ARI	1B	R	28	158	579	13	137	64	45	[3]	71	58.81	23	0.7359	AL MVP 11
Rodriguez	PIT	1B	R	31	140	300	58	39	14	141	107	37	62.47	25	0.7436	
Machado	BAL	3B	R	23	157	640	52	46	107	53	112	23	63.10	26	0.7316	AL MVP 5
Martinez	DET	OF	R	28	120	460	108	96	118	24	37	18	64.61	27	0.7185	
Dozier	MIN	2B	R	29	155	615	36	15	74	152	128	15	65.47	28	0.7514	AL MVP 13
Seager	SEA	3B	L	28	158	597	113	82	60	107	49	57	75.90	31	0.7081	AL MVP 12, SS
Naquin	CLE	OF	L	25	116	321	66	123	179	47	28	32	78.87	32	0.6981	AL ROY 3
Toles	LAN	OF	L	24	48	105	17	253	97	17	37	46	80.10	33	0.7012	
Carpenter	STL	3B	L	30	129	473	33	118	137	134	20	46	81.19	34	0.7011	
Ho Kang	PIT	3B	R	29	103	318	153	22	8	224	96	34	82.00	37	0.7418	
Seager	LAN	SS	L	22	157	627	42	136	261	21	46	35	91.12	41	0.6856	NL ROY
Yelich	MIA	OF	L	24	155	578	185	178	44	44	27	78	91.45	42	0.6851	NL MVP 19, SS
Santana	CLE	1B	B	30	158	582	96	44	109	203	57	60	91.65	43	0.7032	
Schimpf	SDN	2B	L	28	89	276	28	10	16	395	140	23	93.65	45	0.7513	
Bradley	BOS	OF	L	26	156	558	38	107	88	158	104	74	94.05	46	0.6928	
Trumbo	BAL	OF	R	30	159	613	92	[2]	33	218	253	23	94.54	47	0.7479	AL SS
Kinsler	DET	2B	R	34	153	618	7	113	177	68	121	76	95.35	48	0.6945	
Healy	OAK	3B	R	24	72	269	193	93	156	27	149	29	102.13	51	0.6850	
Pearce	BAL	1B	R	33	85	264	201	86	187	68	31	69	104.58	53	0.6772	
Bour	MIA	1B	L	28	90	280	262	70	21	177	55	92	107.19	54	0.6957	
Davis	OAK	OF	R	28	150	555	94	3	18	265	303	29	108.31	55	0.7414	
Harper	WAS	OF	L	23	147	506	44	100	43	283	16	155	109.42	56	0.6904	
Napoli	CLE	1B	R	34	150	557	48	34	25	306	161	105	109.90	57	0.7089	
Dahl	COL	OF	L	22	63	222	8	228	298	16	82	53	116.90	67	0.6727	
Rosales	SDN	3B	R	33	105	214	31	35	65	351	216	66	120.91	72	0.7019	
Bautista	TOR	OF	R	35	116	423	70	79	68	333	55	132	122.03	74	0.6811	
Carter	MIL	1B	R	29	160	549	95	6	41	374	225	57	123.68	78	0.7164	
Ross	CHN	C	R	39	67	166	137	37	10	351	88	148	125.58	80	0.6989	
Segura	ARI	2B	L	26	153	637	72	230	326	9	49	57	125.84	81	0.6535	NL MVP 13
Vargas	MIN	1B	B	25	47	152	21	23	191	346	161	53	127.03	83	0.6969	
Davis	BAL	1B	L	30	157	566	26	19	115	378	169	115	133.77	95	0.6913	
Grandal	LAN	C	B	27	126	390	254	14	17	354	141	86	134.00	96	0.7012	NL MVP 22
Pedroia	BOS	2B	R	32	154	633	45	293	248	11	37	138	135.83	102	0.6409	
Swanson	ATL	SS	R	22	38	129	83	297	190	34	24	154	136.99	103	0.6372	
Fowler	CHN	OF	B	30	125	456	12	254	308	115	16	142	148.11	117	0.6391	
Hazelbaker	STL	OF	L	28	114	200	25	39	146	331	333	83	152.81	126	0.6752	
Rivera	NYN	2B	R	27	33	105	409	253	97	[5]	128	91	156.73	134	0.6358	
Maybin	DET	OF	R	29	94	349	11	399	224	16	24	209	159.43	138	0.6279	
Zunino	SEA	C	R	25	55	164	400	7	12	420	245	96	180.10	155	0.6745	
Recker	ATL	C	R	32	33	90	458	308	55	107	13	175	182.33	158	0.5976	
Peraza	CIN	SS	R	22	72	241	367	391	312	8	112	231	239.71	243	0.5553	
DeShields	TEX	OF	R	23	74	182	[4]	310	428	417	414	419	342.85	375	0.4991	

expected given the differences between power hitters and base hitters. Due to this conflict, the PF shown in Fig. 2 (b) is the longest, and the POSr consists of the six hitters LeMahieu (more of a base hitter), Murphy, Votto, Cabrera, Ortiz, and Sanchez (more of a power hitter). The criteria HRr and AVG demonstrate the most conflict among any pair of these criteria.

4. Secondary Pareto Optimal Set

Another tier of hitters can be identified in a secondary Pareto optimal set (POS2). The second-tier players are non-dominated according to (2) using a restricted candidate set in which the POS1 players are first removed, or $\aleph_2 = \aleph - \text{POS1}$. The POS2 in this

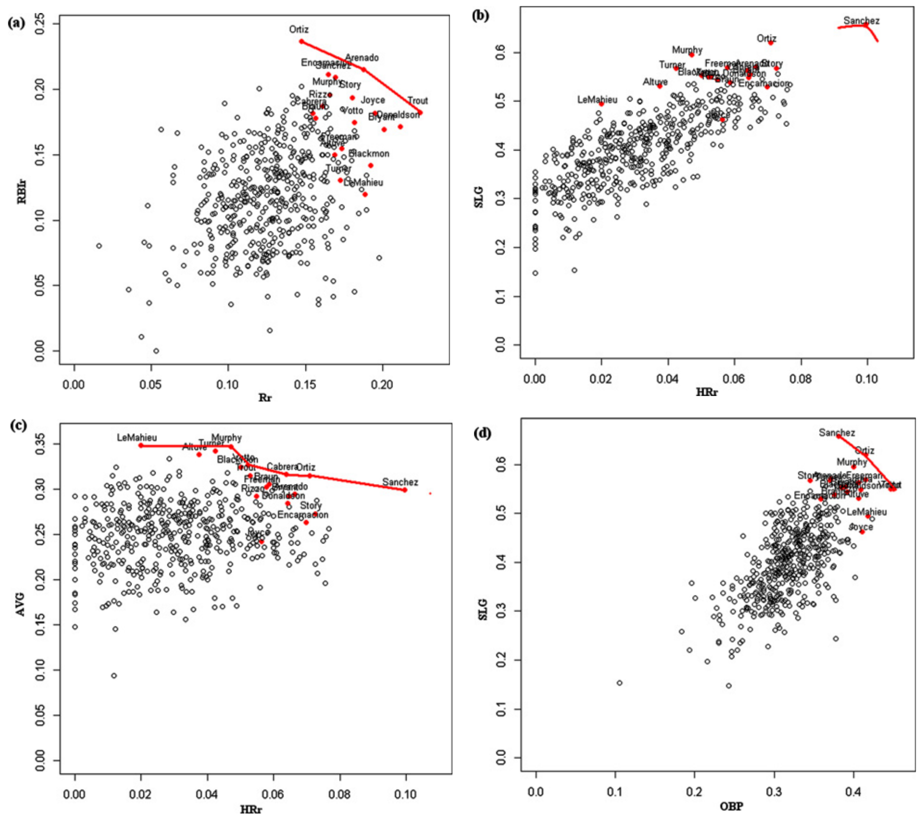


Fig. 2. Plots of the Pareto front involving (a) Rr and RBIr, (b) HRr and SLG, (c) HRr and AVG, (d) OBP and SLG.

section is developed from the same MLB hitting data considered in section 3 for the 2016 season which consists of the $n = 473$ hitters, but without the 19 hitters on POS1 identified in Section 3.

POS2 is shown in Table 4 and consists of 49 well known hitters where the scored ratio to Batman ranges up to 0.79 (Cruz) and down to 0.50 (DeShields). POS 2 also identifies several award winners which is consistent with one of the objectives of sabermetrics. POS2 contains the NL ROY (Seager), the AL ROY 3 (Naquin), and the NL ROY 5 (Diaz). It also contains 5 Silver Slugger award winners (Betts, Cespedes, Seager, Yelich, Trumbo) along with various other hitters who received MVP votes. POS2 notably also contains a number of well-known hitters including Beltre (2004, 2010, 2011, 2014 SS), Goldschmidt (2013, 2015 NL MVP 2 and SS), Cano (2005 AL ROY 2 and 2006, 2010, 2011, 2012, 2013 SS), Machado (2015 AL MVP 4 and 2016 AL MVP 5), Harper (2012 NL ROY, 2015 NL MVP), Bautista (2010, 2011, 2014 SS), and Pedroia (2007 AL ROY, 2008 AL MVP and SS). These batters had good hitting seasons in 2016, but may not have received the same accolades as they did in previous seasons.

A few players have scored ratio to Batman greater than 0.75 (Cruz, Betts), and a few more have scored rankings in the top 19 (Beltre, Cano, Cespedes). POS2 contains players who have Rr ranked 4 (DeShields), HRr ranked 2 (Trumbo), RBIr ranked 5 (Ramirez), AVG ranked 5 (Rivera), OBP ranked 3 (Goldschmidt), and SLG ranked 9 (Cruz). While POS2 contains good hitters according to some of these metrics, these players are dominated by players in POS1 in terms of the other metrics. For example, Goldschmidt performs well in OBP, but is dominated by Trout and Votto. Trumbo hit the most HR in 2016 (47) which produces the second highest HRr (0.0767), but Sanchez has higher HRr (0.0995), and Sanchez dominates Trumbo in all other categories.

5. Predicted Pareto Optimal Set

It is expected that particular hitters have an advantage to be Pareto optimal due to hitter friendly parks, team affiliations, or through regular playing time.

Certain fielding positions are also reserved for the bigger and better hitters. A predicted Pareto optimal set (PPOS) can be constructed from multivariate predictions that are adjusted for these variables. The Multivariate Analysis of Variance (MANOVA) (Rencher and Christensen, 2012) is a tool that can be used to assess the predictor variable contributions and to obtain the predicted criteria values. Pareto optimality and weighted aggregation is then applied to these multivariate predicted values and the resulting uncertainty is propagated through the analyses using repeated sampling or the parametric bootstrap (Efron and Tibshirani, 1993, Section 6.5) from the predictive distribution.

The same MLB hitting data for 2016 is used here as that in sections 3 and 4. However, only qualifying hitters are included who meet the minimum plate appearance (PA) requirement of 502. For validation, MLB hitting data for 2017 is taken from *baseball-guru.com*. There are 146 qualifying hitters for the 2016 season and 144 qualifying hitters for the 2017 season. Predictions are based upon the data from the 2016 season. The predictor variables (x variables) considered here are listed in Table 1. Factors, such as Team, Fielding Position, and Bats, include indicator variables for each level, except for the reference level (Kutner et al., 2004, pp. 313–324). The multiple criteria, or multiple response variables (y variables) are listed in Table 1 which can be denoted as the matrix $\mathbf{Y}$. The partial Wilks' Lambda test is conducted for the six predictors and for quadratic terms involving Age, G, AB to identify statistically important predictors. The test results are shown in Table 5 for the reduced model containing the statistically important predictors. Based upon these statistical test results, the predictions are based upon TEAM, FP1, BATS, AB, G, and G^2 where the latter term denotes the quadratic trend in games played. Table 6 gives the estimated coefficients ($\hat{\mathbf{B}}$) from which the predicted hitting performance values are obtained as $\hat{\mathbf{Y}} = \mathbf{X}\hat{\mathbf{B}}$ where $\mathbf{X}$ is the design matrix containing the values of the predictors for all of the qualifying hitters.

It is also necessary to account for uncertainty in the predictions of future performance. This is accomplished using a parametric bootstrap where the performance criteria are obtained using replicate draws (REP) for each player i (y_i^{REP}) from the MANOVA predictive distribution

$$\text{Normal}(\hat{y}_i, [1 + x_i'(\mathbf{X}'\mathbf{X})^{-1}x_i]\mathbf{S}), \qquad (7)$$

where $\hat{y}_i$ is the predicted criteria for player i, $\boldsymbol{x}_i$ contains the covariate values for player i, and $\mathbf{S}$ is the estimated covariance matrix from MANOVA (Rencher and Christensen, 2012, pp. 370– 371). The predictive distribution in (7) accounts for the uncertainty in the predictions and the uncertainty in the responses (performance criteria). A total of 500 replicate draws are taken from (7). For each draw, players on the primary predicted Pareto optimal set (PPOS1) are identified and their scored ranking of the criteria is calculated. The weights for the scoring are based upon EFA for the 2016 data. Uncertainty is then assessed by examining the proportion of replicate draws in which a player in included in PPOS1 (pPPOS1) and a 90% percentile interval is calculated for the scored ranking of the replicate draws.

Table 5
Partial MANOVA tests of the statistically important predictor variables using the Wilks' Lambda test

predictor	df	test stat	approx F	num Df	den Df	Pr(>F)
TEAM	29	0.109	1.59	174	603	<0.0001
FP1	5	0.500	2.56	30	406	<0.0001
BATS	2	0.661	3.87	12	202	<0.0001
AB	1	0.536	14.59	6	101	<0.0001
G	1	0.783	14.59	6	101	0.0003
G^2	1	0.766	5.15	6	101	0.0001

Validation of the predictions is performed using the data from the 2017 season. Table 7 shows the 19 players who are actually Pareto optimal (POS1) for the 2017 season. Uncertainty in the predictions is shown in the percent of replicate draws in which these players are predicted to be Pareto optimal (pPPOS1) and the 5%, 50%, 95% percentiles of the scored rankings from the replicate draws using the predictive distribution in (7). The criteria pPPOS1 provides a summary on a 0-1 scale where higher percentages denote a higher probability of that player being PPOS1. The ranking of the players in terms of pPPOS1 (pPPOS1r) is also shown in Table 7. The percentile intervals can be quite wide demonstrating large variability in the scored rankings across all of the criteria.

Four of the players in Table 7 (Judge, Stanton, Zimmerman, and Freese) are not qualifiers in the 2016 season and so do not have predictions. Goldschmidt, Blackmon, and Arenado are on the actual POS1 for 2017 and are in the top 10 in terms of pPPOS1 based upon the 2016 predictions. The percentile intervals for these players contain low scored rankings. In particular, Arenado is predicted to be POS1 in nearly 80% of the replicate draws and 90 percent of his scored rankings are between 1 and 44. Arenado and Blackmon likely have high predictions since they play for COL. Goldschmidt and Rendon are POS1

Table 6
MANOVA regression coefficient estimates

predictor	Rr	HRr	RBIr	AVG	OBP	SLG
(Intercept)	1.3121	0.4311	0.6549	0.4809	2.0755	0.4453
TEAM ARI	–0.0178	0.0003	–0.0199	–0.0277	–0.0250	–0.0146
TEAM ATL	–0.0392	–0.0151	–0.0371	–0.0347	–0.0423	–0.0459
TEAM BAL	–0.0353	0.0041	–0.0335	–0.0580	–0.0746	–0.0399
TEAM BOS	–0.0163	–0.0037	–0.0052	–0.0224	–0.0238	–0.0188
TEAM CHA	–0.0545	–0.0148	–0.0375	–0.0451	–0.0550	–0.0548
TEAM CHN	–0.0098	–0.0020	–0.0100	–0.0409	–0.0154	–0.0160
TEAM CIN	–0.0252	–0.0040	–0.0235	–0.0343	–0.0461	–0.0545
TEAM CLE	–0.0193	–0.0067	–0.0253	–0.0434	–0.0420	–0.0353
TEAM DET	–0.0284	–0.0013	–0.0276	–0.0265	–0.0348	–0.0436
TEAM HOU	–0.0303	–0.0118	–0.0350	–0.0341	–0.0338	–0.0455
TEAM KCA	–0.0585	–0.0109	–0.0449	–0.0573	–0.0803	–0.0617
TEAM LAA	–0.0226	–0.0141	–0.0283	–0.0189	–0.0185	–0.0552
TEAM LAN	–0.0275	–0.0094	–0.0383	–0.0297	–0.0376	–0.0293
TEAM MIA	–0.0498	–0.0203	–0.0438	–0.0221	–0.0362	–0.0557
TEAM MIL	–0.0250	0.0069	–0.0172	–0.0377	–0.0351	–0.0449
TEAM MIN	–0.0193	0.0012	–0.0360	–0.0559	–0.0307	–0.0365
TEAM NYA	–0.0431	–0.0120	–0.0446	–0.0383	–0.0492	–0.0550
TEAM NYN	–0.0227	0.0193	0.0024	–0.0374	–0.0290	–0.0204
TEAM OAK	–0.0382	–0.0027	–0.0330	–0.0413	–0.0659	–0.0502
TEAM PHI	–0.0559	–0.0152	–0.0567	–0.0406	–0.0538	–0.0576
TEAM PIT	–0.0346	–0.0130	–0.0261	–0.0293	–0.0322	–0.0433
TEAM SDN	–0.0133	–0.0139	–0.0464	–0.0621	–0.0697	–0.0384
TEAM SEA	–0.0322	0.0057	–0.0184	–0.0355	–0.0432	–0.0326
TEAM SFN	–0.0335	–0.0157	–0.0244	–0.0379	–0.0357	–0.0413
TEAM STL	–0.0307	–0.0114	–0.0282	–0.0252	–0.0193	0.0059
TEAM TBA	–0.0497	0.0033	–0.0254	–0.0481	–0.0660	–0.0488
TEAM TEX	–0.0301	0.0014	–0.0122	–0.0232	–0.0406	–0.0333
TEAM TOR	–0.0268	0.0037	–0.0108	–0.0460	–0.0397	–0.0494
TEAM WAS	–0.0183	0.0042	0.0044	–0.0302	–0.0168	–0.0126
FP1 2B	0.0073	–0.0158	–0.0341	–0.0010	–0.0186	–0.0352
FP1 3B	0.0088	–0.0062	–0.0191	–0.0061	–0.0132	–0.0131
FP1 C	0.0010	–0.0073	–0.0180	0.0077	–0.0010	–0.0047
FP1 OF	0.0091	–0.0075	–0.0260	–0.0135	–0.0199	–0.0295
FP1 SS	–0.0085	–0.0189	–0.0394	–0.0132	–0.0324	–0.0643
BATS B	–0.0012	–0.0043	–0.0130	0.0116	0.0105	0.0050
BATS L	0.0014	–0.0035	–0.0095	0.0005	0.0087	0.0025
AB	0.0001	0.0001	0.0000	0.0004	0.0001	0.0006
G	–0.0169	–0.0059	–0.0078	–0.0039	–0.0245	–0.0288
G^2	0.000059	0.000022	0.000030	0.000009	0.000085	0.000098

for 2017 and also have pPPOS1 values greater than 0.3. These predictions are helpful since these players are not POS1 for 2016 as shown in Table 3. Cruz and Altuve are POS1 for 2017 and these players do have a relatively high pPPOS1 of 0.158, and 0.222, respectively.

As would be expected, not all players on POS1 for 2017 are predicted well based upon just the 2016 performance data. Ozuna, Murphy, Turner, Gonzalez, Davis, Alonso had surprising seasons in 2017, even though pPPOS1 for Gonzalez and Murphy is about 0.1. The scored rankings of Ozuna and Trout for 2017 do not fall within the 90% percentile interval based upon the predictions. The pPPOS1 values are 0.156 for Trout and 0.182 for Votto which do not rank overly high even though they are POS1 for 2017. The prediction model penalizes Trout in terms of R, HR, SLG and penalizes Votto in terms of AVG, OBP, SLG, so that these players perform better than expected according to the predictions. On the other hand, there are some players in the top 10 of pPPOS1 from the 2016 predictions who are not POS1 for 2017. Cabrera of DET (pPPOS1 = 0.372), Cano of SEA (pPPOS1 = 0.338), Gonzalez of COL (pPPOS1 = 0.412) each spent time on the disabled list in 2017. Such injuries may have impacted their 2017 season and some of these players did not perform as well as expected. Betts of BOS has a high pPPOS1 of

Table 7

Pareto optimal hitters along with covariate values, ranks of the performance criteria, and scored ratios to Batman for the 2017 season. Prediction information from the 2016 season is included using the proportion of time the player is predicted to be Pareto optimal (pPPOS1), player ranking in terms of pPPOS1 (pPPOS1r), and the 5%, 50%, 90% percentile values of the scored rankings from the parametric bootstrap. Comments are included to characterize the POS results

nameLast	TEAM	FP1	BATS	AGE	G	AB	Rr.R	HRr.R	RBIr.R	AVG.R	OBP.R	SLG.R	s.rank	r.rank	s.Batman	pPPOSl	pPPOSlr	5%	50%	95%	comment
Trout	LAA	OF	R	26	114	402	2	4	21	17	8	2	8.72	1	0.8357	0.156	43	10.00	64.00	122.00	POS1
Judge	NYA	OF	R	25	155	542	[1]	2	5	46	9	3	10.24	2	0.8736	NA	NA	NA	NA	NA	NA
Votto	CIN	1B	L	34	162	559	11	17	22	7	7	9	12.25	3	0.7718	0.182	31	5.00	41.50	113.95	POS1
Goldschmidt	ARI	1B	R	30	155	558	5	16	2	29	12	11	12.33	4	0.7887	0.370	[9]	2.00	20.00	82.95	PPOS1, POS2
Stanton	MIA	OF	R	28	159	597	6	[1]	[1]	51	23	[1]	12.50	5	0.8624	NA	NA	NA	NA	NA	NA
Blackmon	COL	OF	L	31	159	644	4	28	40	2	17	4	15.85	6	0.7635	0.410	[5]	2.00	26.50	91.00	POS1
Freeman	ATL	1B	L	28	117	440	9	18	41	15	11	6	16.51	7	0.7474	0.152	48	7.00	50.00	120.95	POS1
Arenado	COL	3B	R	26	159	606	33	23	3	13	29	6	16.66	8	0.7636	0.798	[1]	1.00	4.00	43.95	PPOS1, POS1
Zimmerman	WAS	1B	R	33	144	524	25	10	6	22	52	10	19.52	9	0.7638	NA	NA	NA	NA	NA	NA
Cruz	SEA	OF	R	37	155	556	37	8	4	40	28	13	20.28	10	0.7562	0.158	41	6.05	54.00	120.95	POS1, PPOS2
Ozuna	MIA	OF	R	27	159	613	64	24	7	11	31	15	23.83	11	0.7329	0.020	115	37.05	107.50	143.00	
Murphy	WAS	2B	L	32	144	534	20	69	24	5	19	17	25.95	14	0.7015	0.098	59	10.05	67.00	130.00	
Rendon	WAS	3B	R	27	147	508	47	52	10	23	16	21	27.59	15	0.7086	0.302	17	3.00	28.00	91.95	PROS2
Altuve	HOU	2B	R	27	153	590	10	77	79	111	4	16	32.19	20	0.7783	0.222	25	6.00	40.00	114.95	POS1
Gonzalez	HOU	OF	B	28	134	455	73	47	9	22	28	24	32.52	22	0.7002	0.114	54	11.05	61.00	124.95	
Turner	LAN	3B	R	33	130	457	52	62	48	5	10	24	33.51	24	0.6797	0.074	70	13.00	72.00	133.00	
Davis	OAK	OF	R	30	153	566	44	6	13	117	6	26	33.56	25	0.8158	0.042	92	20.00	81.00	136.95	POS2
Alonso	SEA	1B	L	30	142	451	46	21	61	83	2	41	42.15	32	0.7555	0.036	100	23.05	89.00	140.00	traded
Freese	PIT	3B	R	34	130	426	141	126	105	88	[1]	138	102.72	114	0.5888	NA	NA	NA	NA	NA	NA

0.554, but had a disappointing 2017 season, particularly in terms of AVG and OBP. Encarnacion (CLE) and Rizzo (CHN) also have high pPPOS1 values of 0.468 and 0.386, respectively. While they do not appear on POS1 for 2017, they do appear on POS2 for 2017. Thus, their performance was good, even though it may not have been as good as predicted.

6. Pareto Optimal Set with Other Criteria

It is important to recognize that the proposed multi-criteria approach can be easily implemented with any collection of criteria that a manager believes best characterizes player performance. This is the first advantage of the proposed multi-criteria approach mentioned in section 1. However, selection of the criteria is critical in that it must represent the player performance characteristics that are of specific interest to the performance assessment. A few considerations are presented below when it comes to thinking about the multiple criteria and how they compare to some popular sabermetrics. The multi-criteria approach is also demonstrated in this section using wOBA, wRC+, and WAR.

The multiple criteria previously considered are the widely available traditional measures of batting performance (R, HR, RBI, H, OBP, SLG). There are concerns about these traditional hitting statistics. Even though Grabiner (2014) mildly endorses AVG, it has been criticized by some, such as Albert (2010), for not addressing other ways to get on base, and for not distinguishing the type of hit. The statistics R and RBI have been criticized because they depend upon other factors that may not directly reflect player contribution such as scoring off the hit of another batter or needing to have runners on base (Grabiner 2014). Albert (2010) presents strong arguments that better measures of hitting are OBP, SLG, and OPS. On the other hand, rather than focusing on arguments that a statistic is flawed, it can be informative to recognize that statistics measure different aspects of hitting that cannot be captured in a single sabermetric. As stated by Grabiner (2014), “Batting average does fairly well because it counts hits, but it ignores power and walks, which are also important.” That point does not necessarily mean AVG is flawed, but that it measures a different aspect of hitting than does OBP or SLG. An advantage of the multi-objective approach is in its ability to work with a multitude of statistics that account for different aspects of player performance and for its ability to evaluate the trade-offs among these different aspects. This is the third advantage of the multi-criteria approach that is mentioned Section 1.

Concerns about traditional hitting statistics could lead a sabermatrician to apply the proposed approach with a different set of hitting performance measures. Some popular sabermetric statistics can be formulated from (4). In particular, this includes the following:

$$
\begin{aligned}
&\text{On-Base Plus Slugging (OPS)=OBP+SLG,} \qquad (8)\\
&\text{Gross Production Average (GPA)} = \\
&\quad (1.8 \times \text{OBP}) + \text{SLG})/4,\\
&\text{Runs produced (RP)} = \text{R} + \text{RBI} - \text{HR},\\
&\text{Isolated Power (ISO)} = \text{SLG} - \text{AVG},\\
&\text{Secondary Average (SecA)} = \\
&\quad \frac{\text{BB}}{\text{AB}} + \frac{\text{TB}}{\text{AB}} - \frac{\text{H}}{\text{AB}} + \frac{\text{SB}}{\text{AB}} - \frac{\text{CS}}{\text{AB}}.
\end{aligned}
$$

Another more complicated statistic, which is similar in form to SecA, is Weighted On-Base Average (wOBA) that is scaled by PA rather than AB. In particular, OPS is touted by Albert (2010) as a modern sabermetric. Yet, OPS is merely a specific weighted aggregation of OBS and SLG. However, this may not the best combination of OBP and SLG to represent hitting performance for a particular group of hitters. For example, Grabiner (2014) mentions the linear combination should be $1.2 \times \text{OBP} + \text{SLG}$ where the value 1.2 is usually ignored. The second advantage of the multi-criteria approach mentioned in section 1 is that such combinations and weights do not have to be specified as the performance in terms of OBP and SLG can be simultaneously considered and evaluated.

Some managers have become quite accustomed to particular sabermetrics for measuring player performance. As mentioned previously, the multi-criteria approach can be applied to any collection of criteria. As a demonstration, consider the collection wOBA, wRC+, and WAR defined by Fangraphs (2018b). Weighted On-Base Average (wOBA) combines different aspects of hitting and weights them according to the actual run value. Weighted Runs Created Plus (wRC+) attempts to credit a batter for the value of a hitting outcome while controlling for park, league, and year effects. Wins Above Replacement (WAR) is designed to measure overall player contribution, beyond just hitting, by comparing team wins compared to a replacement player. The data for these three sabermetrics are taken from Fangraphs (2018a) for the 2016 season and contains 146 hitters who are qualifying hitters with more than 502 plate appearances (PA).

Table 8
Primary and Secondary Pareto optimal hitters based upon the sabermetrics wOBA, wRC+, WAR listed in order of the scored ranking (s.rank)

name	Team	G	PA	wOBA	wRC+	WAR	s.rank	r.rank	s.Batman	POS
Trout	Angels	159	681	0.418	170	9.6	1.36	1	0.9991	primary
Donaldson	Blue Jays	155	700	0.403	155	7.4	4.72	2	0.8903	secondary
Murphy	Nationals	142	582	0.408	155	5.7	6.60	3	0.8450	secondary
Bryant	Cubs	155	699	0.396	148	7.8	6.96	4	0.8812	secondary
Votto	Reds	158	677	0.413	159	5.3	7.20	6	0.8461	secondary
Ortiz	Red Sox	151	626	0.419	164	4.5	11.16	8	0.8385	primary
Betts	Red Sox	158	730	0.379	137	8.3	12.08	9	0.8578	secondary

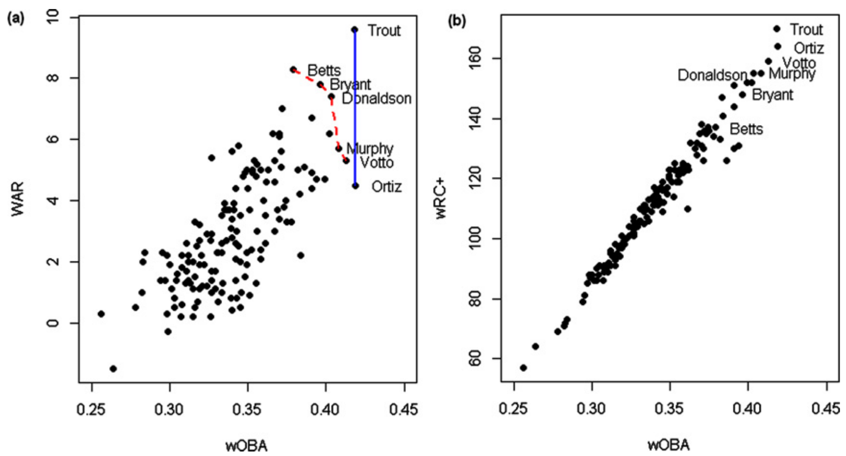


Fig. 3. Plots of the players in terms of (a) wOBA and WAR, (b) wOBA and wRC+with labels for those players on the Primary and Secondary front. In (a), players on the Primary Pareto front are connected with a solid line while players on the Secondary Pareto front are connected with a dotted line.

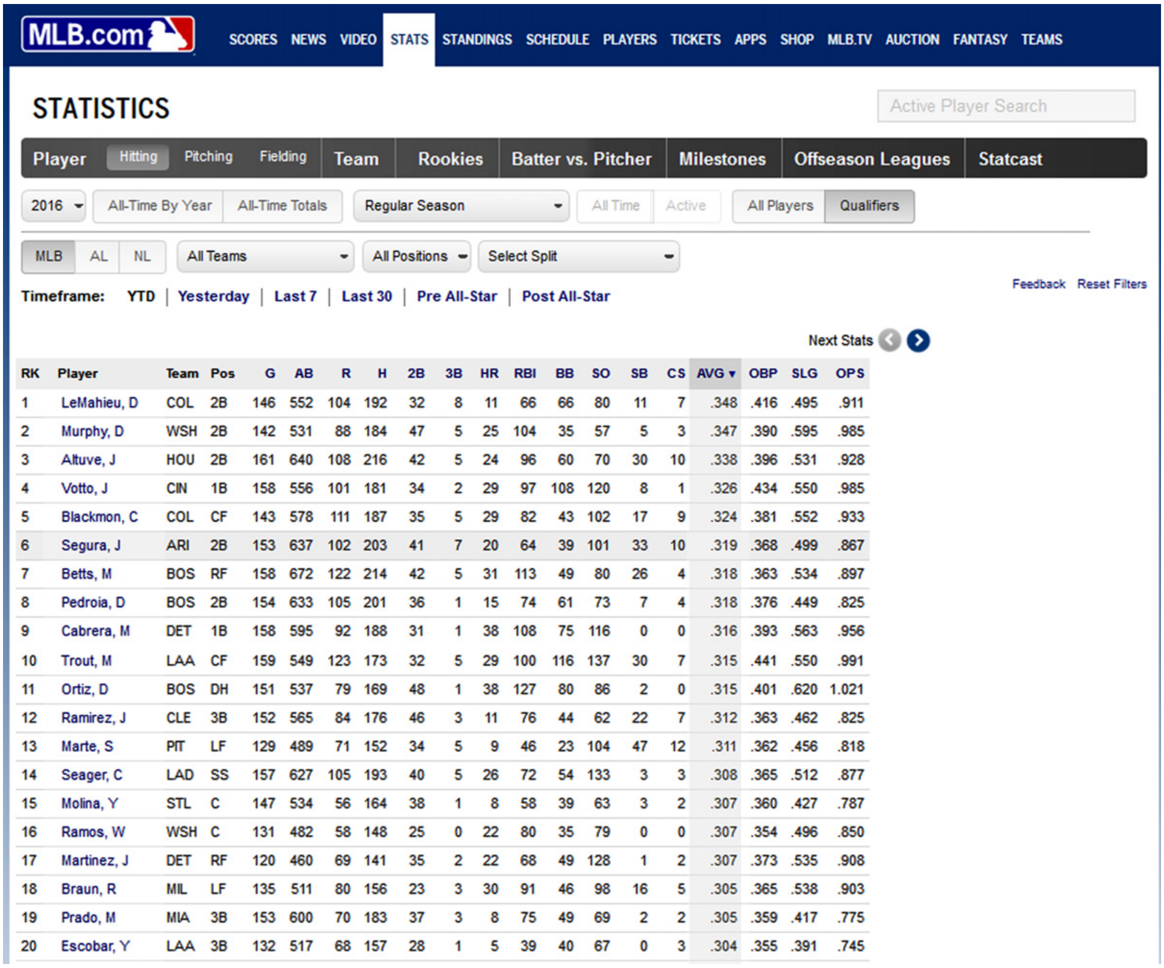
For these three sabermetrics, Table 8 gives the Primary Pareto optimal set (POSS1) and the Secondary Pareto optimal set (POSS2). The weights for these three criteria using EFA for the 2016 data were determined to be 0.36 for wOBA, 0.36 for wRC+, and 0.28 for WAR. The players are ordered in Table 8 according to the scored rankings. Ortiz is included on POSS1 since he had the highest observed wOBA which is slightly higher than that for Trout. If it were not for this value, then Trout would be the Batman with respect to these three criteria for which he still is very close (0.9991). POSS2 consists of Donaldson, Bryant, Murphy, Votto, and Betts. All these players are dominated by Trout and Ortiz who both have higher wOBA and wRC+ than any player on the secondary front. Figure 3 shows the players in POSS1 and POSS2 in terms of just two criteria. The pairwise correlation between wOBA and wRC+ is evident as expected since wRC+ is similar to wOBA, but controls for park, league, and year effects. Thus, player rankings for 2016 using wOBA are quite similar to those using wRC+. There is some conflict between wOBA and WAR since WAR measures additional player contributions beyond just hitting. The plot of wRC+ and WAR looks quite similar. The Pareto optimal sets POSS1 and POSS2 are much smaller than those presented in section 4 (POS1) and section 5 (POS2). This is due to fact that only three sabermetrics are used as the criteria and that two of these are highly correlated (0.9837). Nevertheless, all the players identified in POSS1 and POSS2 are included in POS1, except for Betts who is included in POS2. Betts is in POSS2 in large part due to his high value for WAR, which is the second highest.

It should be noted that there are also concerns about the use of these sabermetrics since wOBA does not adjust for hitting friendly parks, wRC+ does not differentiate positions, and WAR may not be developed enough to conduct player rankings (Fangraphs, 2018b). On the other hand, players have been ranked based upon WAR by the Baseball-Reference (2018). Due to these types of concerns, Fangraphs (2018) recommend in their discussion of WAR that one “should always use more than one metric at a time when evaluating players”. The proposed multi-criteria approach conducts this very task conveniently and efficiently.

7. Summary

A multi-objective approach is proposed in this paper that allows for informative comparisons of players in terms of multiple performance criteria, avoids complex combinations of the criteria into single metrics, and allows trade-offs among the criteria to be evaluated. The approach is demonstrated for evaluating baseball player batting performance through simultaneous consideration of multiple performance metrics. Traditional metrics, such as R, H, HR, RBI, AVG, OBP, SLG, are initially used for these evaluations. The primary Pareto optimal set (POS1) identifies those batters who are non-dominated or who cannot be beat with respect to these criteria. The secondary Pareto optimal set (POS2) identifies a second group of batters who are non-dominated apart from those batters in POS1.The Multiple Analysis of Variance (MANOVA) is used to generate predictions while also addressing the uncertainty in the predictions and the uncertainty associated with the criteria. The uncertainty can then be propagated to the Pareto optimal sets and to the scored rankings for predicting performance results in an upcoming season. Weighted rankings or the relative distance to the utopia point (Batman) are also shown to be helpful when it comes to ordering players with respect to the multiple criteria.

As an implementation example, the website *mlb.com/stats* contains statistics from which selected players can be ranked. Figure 4 shows a default view from this website for the 2016 MLB season. A few differences in the rankings can be observed from Fig. 4 and the previous rankings presented here due to their restriction to players who are qualifying hitters. However, simple adjustments to Fig. 4 can be made



RK	Player	Team	Pos	G	AB	R	H	2B	3B	HR	RBI	BB	SO	SB	CS	AVG ▾	OBP	SLG	OPS
1	LeMahieu, D	COL	2B	146	552	104	192	32	8	11	66	66	80	11	7	.348	.416	.495	.911
2	Murphy, D	WSH	2B	142	531	88	184	47	5	25	104	35	57	5	3	.347	.390	.595	.985
3	Altuve, J	HOU	2B	161	640	108	216	42	5	24	96	60	70	30	10	.338	.396	.531	.928
4	Votto, J	CIN	1B	158	556	101	181	34	2	29	97	108	120	8	1	.326	.434	.550	.985
5	Blackmon, C	COL	CF	143	578	111	187	35	5	29	82	43	102	17	9	.324	.381	.552	.933
6	Segura, J	ARI	2B	153	637	102	203	41	7	20	64	39	101	33	10	.319	.368	.499	.867
7	Betts, M	BOS	RF	158	672	122	214	42	5	31	113	49	80	26	4	.318	.363	.534	.897
8	Pedroia, D	BOS	2B	154	633	105	201	36	1	15	74	61	73	7	4	.318	.376	.449	.825
9	Cabrera, M	DET	1B	158	595	92	188	31	1	38	108	75	116	0	0	.316	.393	.563	.956
10	Trout, M	LAA	CF	159	549	123	173	32	5	29	100	116	137	30	7	.315	.441	.550	.991
11	Ortiz, D	BOS	DH	151	537	79	169	48	1	38	127	80	86	2	0	.315	.401	.620	1.021
12	Ramirez, J	CLE	3B	152	565	84	176	46	3	11	76	44	62	22	7	.312	.363	.462	.825
13	Marte, S	PIT	LF	129	489	71	152	34	5	9	46	23	104	47	12	.311	.362	.456	.818
14	Seager, C	LAD	SS	157	627	105	193	40	5	26	72	54	133	3	3	.308	.365	.512	.877
15	Molina, Y	STL	C	147	534	56	164	38	1	8	58	39	63	3	2	.307	.360	.427	.787
16	Ramos, W	WSH	C	131	482	58	148	25	0	22	80	35	79	0	0	.307	.354	.496	.850
17	Martinez, J	DET	RF	120	460	69	141	35	2	22	68	49	128	1	2	.307	.373	.535	.908
18	Braun, R	MIL	LF	135	511	80	156	23	3	30	91	46	98	16	5	.305	.365	.538	.903
19	Prado, M	MIA	3B	153	600	70	183	37	3	8	75	49	69	2	2	.305	.359	.417	.775
20	Escobar, Y	LAA	3B	132	517	68	157	28	1	5	39	40	67	0	3	.304	.355	.391	.745

Fig. 4. Screen shot of www.mlb.com/stats for 2016.

so that it can accommodate multiple criteria. That is, the user could be allowed to select multiple criteria items from among the hitting criteria such as R, HR, RBI, AVG, OBP, and SLG. Then the proposed calculations can be quickly applied so that players who are in POS1 are highlighted and players are then ordered according to their scored ranking across the criteria or according to their relative distance from Batman. As a result, Fig. 4 would more closely match that in Table 3.

The proposed approach is useful to casual fans, fantasy baseball players, and general managers for identifying the top players according to multiple criteria and to evaluate trade-offs among the criteria. The approach can be implemented rather easily with any collection of criteria that is perceived to best represent the type of player performance that is of interest. The criteria can be traditional, modern, or some combination. In addition, it is often necessary to fill out baseball rosters by position, in which case it makes more sense to implement the multi-criteria selection procedures separately for each position and likely with different criteria according to the expectations pertaining to that position. This approach could even involve a combination of hitting and fielding criteria. The proposed approach is also not limited to baseball. Baseball is often regarded as the sabermetric sport due to the wide availability of data, but other sports are now collecting various types of performance-based data. These multi-objective techniques would be useful tools to enhance sabermetrics.

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