

Tomography and Applications

Preface

1. Introduction

The 9th edition of the annual Meeting on Tomography and Applications took place in the period April 20-April 22, 2015. As always, it was organized by the Mathematical Department of Politecnico di Milano. It provided a broad overview of recent research on theory and applications of tomography. The event provided a forum for the participants, consisting of mathematicians and experimentalists, to discuss the state of the art of tomographic research from various points of view. In the internet era, when most recent results are easily accessible from the web, keeping the tradition of meetings such as this one is very important, as they provide forums for direct, personal contacts which are beneficial for solving challenging problems, defining new research directions, and for initiating new as well as strengthening established cooperations.

This special section of Fundamenta Informaticae consists of four selected papers from the meeting, which went through a thorough refereeing process. They provide a good representation of the topics discussed at the meeting.

The program of the meeting contained the following lectures.

MAIN LECTURES

- Ronny RAMLAU, Johannes Kepler University, Linz, Austria
Tomography for Image Improvement of Large Astronomical Telescopes.
- Stéphane ROUX, L.M.T. Cachan, CACHAN Cedex
Tomographic reconstruction: the challenge of “dark” information.
- Jeanpierre GUÉDON, Ecole Polytechnique de l’Université, Nantes
The Mojette transform: 20 years of (very) Discrete Tomography.

OTHER TALKS

- Wim van Aarle, University of Antwerp
Discrete Tomography in the real world.

- Andreas Alpers, TU München
Generalized power diagrams for tomography on polycrystalline microstructures.
- Francesco Brun, Elettra Trieste
From photons to scientific publications: a few examples of tomographic based scientific workflows performed at the SYRMEP beamline of Elettra.
- Rodolfo Fiorini, Politecnico di Milano
Exploiting Numerical Features in Computer Tomography Data Processing and Management by CICT.
- Richard Gardner, Western Washington University
Characterizing Steiner symmetrization.
- Viviana Ghiglione, TU München
The Prouhet-Tarry-Escott problem in Discrete Tomography.
- Peter Gritzmann, TU München
Dynamic Discrete Tomography for Particle tracking velocimetry.
- Esther Klann, Johannes Kepler University -TU Berlin
A weighted wavelet scheme for region of interest tomography.
- Tristan van Leeuwen, Mathematical Institute, Utrecht University
Automatic alignment for Tomographic reconstruction.
- Daniel Pelt, CWI-Amsterdam
Recent advances in filter-based tomographic reconstruction methods.
- Stefania Petra, Universität Heidelberg
Co-Sparse Tomographic Image Recovery: Performance Estimates and Applications.
- Robert Tijdemann, Leiden University
Dependencies in discrete tomography.
- Nicola Viganó, ESRF, Grenoble
Three-dimensional full-field X-ray orientation microscopy.
- Csaba Vincze, Debrecen University
Generalized conics' theory and its applications.

2. Overview of the papers presented in this special section

- S. Brunetti, and C. Peri, *On J -additivity and Bounded additivity*.

The problem of the faithful reconstruction of an unknown homogeneous object, regarded as a discrete set of points, from a series of projections along a fixed set S of discrete directions, is one of the main problems in Discrete Tomography. It is often pointed out that the problem may be ill-posed, meaning that, in general, there may exist a huge class of discrete sets sharing the same given projections. So, research on defining subclasses of binary matrices that are uniquely determined by projections becomes very important. P.C. Fishburn and L.A. Shepp defined the notion of *additivity* for a discrete set, and proved that *additive sets*, are unique with respect to a given set S of discrete directions. This class provides a deeper insight into the structural properties of unique sets, coinciding with them if $|S| \leq 2$, and being more demanding when $|S| > 2$. More precisely, in the former case additivity forces the absence of bad configurations in a discrete sets, providing its uniqueness, while in the latter case, it requires the absence of a wider set of configurations, say the weakly bad configurations. Here, the authors shown that the weakly bad configurations can be generalized to higher dimensions, linking it to the recently introduced notion of *J -additivity*. Then they define and study *bounded additivity*, i.e., additivity related to discrete sets that are bounded inside a fixed dimension grid: they demonstrate a strong relationship between the two notions, and, relying on a previous result by the same authors, they provide a bounded additive sets that is not additive.

- I. Gambini, and L. Vuillon, *Tiling the space by polycube analogues of Fedorovs polyhedra*.

A well known Theorem by D. Beauquier and M. Nivat provides the characterization of the two dimensional connected shapes that tile the plane by translation, and they split them in two non disjoint sets, say the pseudo squares and the pseudo hexagons, depending on the maximum number of neighborhood tiles, either four or six, respectively, needed to surround each in a tiling of the plane. It has been observed that each tiling of the plane by translation has one or two directions of periodicity, related to the characteristics of the involved tile. This paper is concerned with the generalization of these results to higher dimensions. In particular they show that there is no analogue to the Beauquier-Nivat characterization, by constructing, for each $n \in \mathcal{N}$, a $3D$ polycube surrounded by $2n + 6$ copies of itself in a lattice periodic tiling of the plane by translation; in this case the tiling is unique.

Then, the authors study the generalization to $3D$ of the pseudo square and pseudo hexagon $2D$ shapes: relying on an old theorem by Fedorov where he identified five different polytopes that tile the space by translation, they discretize the result and define a set of polycubes that act like the Fedorov's polyhedra. Finally they prove that these polycubes are the minimal volume's ones that tile the space by translation in a unique way. Some of these results are obtained by a brute force approach to generate all possible lattice periodic tilings.

- T. A. Smaglichenko, I. Th. Bjarnson, A. V. Smaglichenko, and W. R. Jacoby, *Method to find Minimum 1D Linear Gradient Model for Seismic Tomography*.

Computerized Tomography is usually concerned with the partial or total reconstruction of objects from a series of projections along a huge set of directions, and involve efficient analytical methods to compute the inversion of the Radon Transform. Theoretically, the considered objects do not play a prominent role, and may vary from internal parts of the human body (medical

imaging) to hardware's microchips or experiments' samples. As one can imagine, efforts have been done to adapt a so general paradigm to different frameworks, where an unknown and hardly reachable area of interest is present.

In this paper, the authors consider Cross-Borehole Tomography whose aim is to investigate the properties of the soil between two boreholes in order to predict earthquakes. This new model consists of a series of seismic sources and detectors located along the two boreholes that collect data about the heterogeneous structures between them. The proposed study is based on a gradient approach: for each region of interest, the velocity of the seismic waves internal to the soil is first detected by analysing their behaviour in case of guided explosions, and then a model of their propagation in the the area is defined. Changes in the model allow people to predict imminent strong earthquakes. The model consists of a $3D$ function over time that can be simplified in a $1D$ discretized gradient function due to the dependance of the waves velocity mainly on the soil depth. The paper proposes to consider a $1D$ gradient function that linearly depends on the soil depth z , i.e., of the form $f(z) = a + bz$. The parameters a and b are estimated by the Taylor expansion of a formula already present in literature. A test of the model is also proposed.

- N. Viganó, K. J. Batenburg, and W. Ludwig, *An orientation-space super sampling technique for six-dimensional diffraction contrast tomography*.

Again a different paradigm of the Computerized Tomography, say the Diffraction Contrast Tomography (DCT), is used here to study poly-crystalline materials, and to determine the physical and morphological properties of their parts, called grains, and their evolution over time. In particular, the DCT is a $3D$ technique that consists of placing an unknown granular object inside an incoming high energy beam and rotating it in order to produce diffraction spots in a detector behind the object due to the different orientations of the grains' surfaces. In this paper, the authors consider a non standard formulation of the DCT reconstruction problem that increases its dimensions to six by including both the real-space, and the orientation space occupied by the grains. Obviously, it is still possible to describe the reconstruction process in terms of an optimization over a six-dimensional space and use recent minimization techniques: such a formulation has been proven to be highly efficient in retrieving the local orientation of large textured regions.

The study carried on in this paper aims at verifying this model on a three dimensional non-real dataset of 60 grains each, with two different orientation space sampling resolutions (poor and average), in order to test the reconstruction performances and the accuracy of the obtained orientation space. It is explicitly pointed out that the reconstruction processes in the two spaces can be performed in parallel.

The obtained results confirm that, in both cases, the proposed method provides good results both in terms of grain shape reconstruction quality and in terms of local orientation determination accuracy. However, it is in the case of poor resolution of the orientation space, that the method performs at its best, allowing to retrieve useful information from memory limited cases.

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